Simpson's paradox (2)

The paradox lies in ignoring the context in which the results are given.

If we derive correct conditional probabilities based on this data (assuming 50% males in the population) we get:

$$P(\text{recovery} \mid \text{drug}) = \frac{1}{2}\frac{15}{15+40} + \frac{1}{2}\frac{90}{90+50} \approx 0.46$$

$$P(\text{recovery} \mid \text{no drug}) = \frac{1}{2}\frac{20}{20+40} + \frac{1}{2}\frac{20}{20+10} = 0.5$$

Three prisoners dilemma (2)

Let I_B be the proposition “Prisoner B will be declared innocent”.
Let G_A be the proposition “Prisoner A will be declared guilty”.
If we compute $P(G_A|I_B)$, we obtain:

$$P(G_A|I_B) = \frac{P(I_B|G_A)P(G_A)}{P(I_B)} = \frac{1 \cdot 1/3}{2/3} = \frac{1}{2}$$

But there is a fallacy in what we are computing: we are not considering all the possible range of answers to the question.
Let I'_B be the sentence “The guard says that B will be declared innocent”. What we really need to compute is:

$$P(G_A|I'_B) = \frac{P(I'_B|G_A)P(G_A)}{P(I'_B)} = \frac{1/2 \cdot 1/3}{1/2} = \frac{1}{3}$$

Note that I'_B implies I_B, but the reverse is not true!