Probabilistic Reasoning in AI - Problem set 1

Due Wednesday, January 16, 2002, in class
This assignment is worth 5% of the final grade

1. [20 points]
   (a) Prove that the two different ways of stating the third axiom of probability are equivalent.
   (b) If $P(B) = 1$, then $P(A|B) = P(A)$.
   (c) If $A \to B$, then $P(B|A) = 1$ and $P(A|B) = \frac{P(A)}{P(B)}$
   (d) Given $P(A), P(B) > 0$, if A and B are disjoint (i.e. $P(A \vee B) = P(A) + P(B)$, then they cannot be independent, and if A and B are independent, then they cannot be disjoint.)

2. [10 points] Express the statement that X and Y are conditionally independent given Z as a constraint on the joint distribution entries for $P(X, Y, Z)$.

3. [10 points] Pearl, pg. 73, problem 2.1

4. [20 points] Pearl, pg. 73, problem 2.3

5. [10 points] Pearl, pg. 75, problem 2.6

6. (a) [5 points] Prove the conditionalized version of the product rule:
$$P(A, B|E) = P(A|B, E)P(B|E)$$
   (b) [5 points] Prove the conditionalized version of Bayes rule:
$$P(A|B, C) = \frac{P(B|A, C)P(A|C)}{P(B|C)}$$

7. [10 points] This exercise investigates the way in which conditional indep3endence relationships affect the amount of information needed for probabilistic calculations.
   (a) Suppose we wish to compute $P(H|E_1, E_2)$ and we have no conditional independence information. Which of the following sets of numbers are sufficient for the calculation?
      i. $P(E_1, E_2), P(H), P(E_1|H), P(E_2|H)$
      ii. $P(E_1, E_2), P(H), P(E_1, E_2|H)$
      iii. $P(H), P(E_1|H), P(E_2|H)$
   (b) Suppose that we know that $P(E_1|H, E_2) = P(E_1|H)$ for all values of H, E_1, E_2. Now which of the above sets is sufficient?
   (c) Assuming H, E_1, E_2 are all Boolean, how many numbers are sufficient to represent the joint distribution in the two cases?

8. [10 points] Which of the following properties are satisfied by the probabilistic independence relation? Prove formally or disprove with a counterexample.
   (a) If $I(X, Y \wedge Z)$ then $I(X, Y)$ and $I(X, Z)$.
   (b) If $I(X, Y)$ and $I(X, Z)$ then $I(X, Y \wedge Z)$.