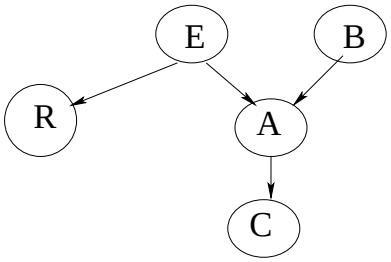
Probabilistic Reasoning in AI - Assignment 2

Due Friday, January 25, 2008

1. [30 points] **I-maps**

   Usually we think of Bayes nets as representing causal influences, but of course this need not be the case. This question involves transforming a Bayes net G into another Bayes net G' with a different structure, but representing the same distribution Therefore, G' will need to ba an I-map of the original distribution

   (a) [10 points] Consider the alarm network:

   Suppose that we want to remove node A (by marginalizing over it). What arcs do you need to add in order to obtain the correct joint distribution over the remaining variables?

   (b) [10 points]

   Suppose we want to reverse the arc $B \rightarrow A$. What additional minimal modifications to the structure of the network are necessary to ensure that the new network is an I-map of the original one? You do not need to prove that the modifications are minimal.

   (c) [10 points] Now consider a general Bayes net G. Assume for simplicity that the arc $X \rightarrow Y$ is the only directed path form X to Y. If you reverse the arc, what additional minimal modifications to G are needed in order to ensure that the new network G' is an I-map of the original distribution? Hint: Consider the algorithm for constructing a minimal I-map that we discussed in class.

2. [20 points] **Undirected graphical models**

   Consider the 2D spin glass model we discussed in lecture 5 (slide 19).

   (a) [10 points] Suppose that instead of connecting pixels in a 4-neighborhood, we want to connect them in an 8-neighborhood. Describe what the parameters of the undirected graphical model will be.

   (b) [10 points] Suppose that we want to use such a model to capture natural scenes in images. Describe the advantages and disadvantages of this model compared to connecting a pixel only to 4 neighbors.

3. [20 points] **Directed and undirected graphical models**

(a) [5 points] Draw a directed graphical model on 4 variables which can capture any probability distribution.

(b) [5 points] Can any edges be removed from the graph while still preserving this property? If so, show which one. If not explain why.

(c) [5 points] Draw an undirected graphical model over 4 variables which can capture any probability distribution

(d) [5 points] Can any edges be removed from the graph while still preserving this property? If so, show which one. If not explain why.

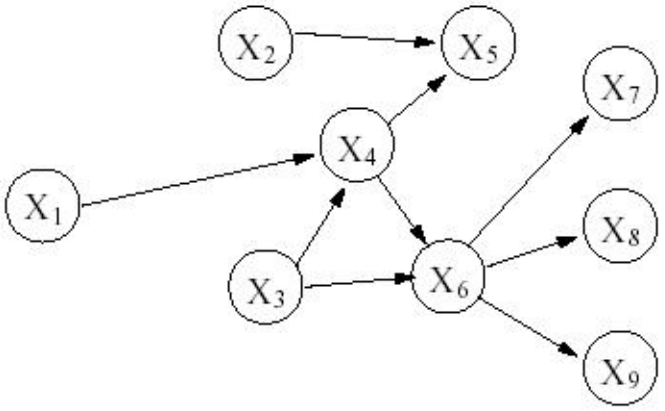
4. [20 points] **Moralization and Variable Elimination**

(a) [5 points] Moralize the graph below and show the resulting undirected graph.

(b) [5 points] Show the cliques that are formed using the elimination ordering (7,8,9,6,3,5,4,2,1).

(c) [5 points] Show the cliques that are formed using the elimination ordering (7,6,8,9,4,3,2,5,1).

(d) [5 points] Which of the two elimination orderings result in a more time-efficient calculation, if we were to compute $p(x_1|x_7)$ using variable elimination? Which would result in a more space-efficient calculation?. Briefly explain why.



5. [10 points] **Clique trees**

Assume that we have constructed a clique tree for a Bayesian network, and each clique has at most k nodes. Suppose we add an arc between two nodes in the original network. Give an upper bound on the maximum clique size in the clique tree for the new network. Justify your answer.