Probabilistic Reasoning in AI - Assignment 1

Due Wednesday, January 16, 2008

1. [10 points] **Simpson's paradox**

   The following table describes the effectiveness of a certain drug on a population:

	Male		Female		Overall	
	Recovered	Died	Recovered	Died	Recovered	Died
Drug used	15	40	90	50	105	90
No drug	20	40	20	10	40	50

   If you look at the data, the good news is that the ratio of recovery for the whole population increases from 40/50 to 105/90. Unfortunately, if you look at the male group and the female group separately, in each group, the ratio decreases. Explain what would be a good way to reason about this data, and show that it avoids this apparent problem.

2. [10 points] **Reasoning by cases** A rule that is often useful in dealing with probabilities is known as reasoning by cases. Let X, Y, Z be discrete random variables. Then:

$$p(X|Y) = \sum_{z \in \Omega_Z} p(X, z|Y)$$

   Prove this property using the chain rule and basic properties of conditional distributions.

3. [40 points] **Properties of conditional independence**

   Recall that the conditional independence property that $A \perp\!\!\!\perp B|C$ is a statement that the conditional probability distribution satisfies

$$p(A|B,C) = p(A|C)$$

   (a) [5 points] Show that, if $A \perp\!\!\!\perp B|C$, then $p(A,B|C) = p(A|C)p(B|C)$

   (b) [5 points] Show that conditional independence is symmetric:

$$A \perp\!\!\!\perp B|C \Leftrightarrow B \perp\!\!\!\perp A|C$$

   (c) [5 points] Show the decomposition property:

$$A \perp\!\!\!\perp (B \cup D)|C \Rightarrow A \perp\!\!\!\perp B|C \text{ and } A \perp\!\!\!\perp D|C$$

   (d) [5 points] Show the weak union property:

$$A \perp\!\!\!\perp (B \cup D)|C \Rightarrow A \perp\!\!\!\perp B|(C \cup D) \text{ and } A \perp\!\!\!\perp D|(C \cup B)$$

(e) [10 points] Show the contraction property:

$$A \perp\!\!\!\perp B | (C \cup D) \text{ and } A \perp\!\!\!\perp D | C \Rightarrow A \perp\!\!\!\perp (B \cup D) | C$$

(f) [10 points] Show that, for strictly positive distributions (i.e., distributions in which there are no zero-probability events), we have the following intersection property:

$$A \perp\!\!\!\perp B | (C \cup D) \text{ and } A \perp\!\!\!\perp C | (B \cup D) \Rightarrow A \perp\!\!\!\perp (B \cup C) | D$$

4. [10 points] (from Russell & Norvig) This exercise investigates the way in which conditional independence relationships affect the amount of information needed for probabilistic calculations.

   (a) Suppose we wish to compute $P(H|E_1,E_2)$ and we have no conditional independence information. Which of the following sets of numbers are sufficient for the calculation?

      i. $P(E_1,E_2), P(H), P(E_1|H), P(E_2|H)$
      ii. $P(E_1,E_2), P(H), P(E_1,E_2|H)$
      iii. $P(H), P(E_1|H), P(E_2|H)$

   (b) Suppose that we know that $P(E_1|H,E_2) = P(E_1|H)$ for all values of H, E_1, E_2. Now which of the above sets is sufficient?

   (c) Assuming H, E_1, E_2 are all Boolean, how many numbers are sufficient to represent the joint distribution in the two cases?

5. [10 points] **Bayes Ball**

   Using the Bayes Ball algorithm, determine for the following graph which variables can be reached starting at node A. The evidence nodes are shaded.

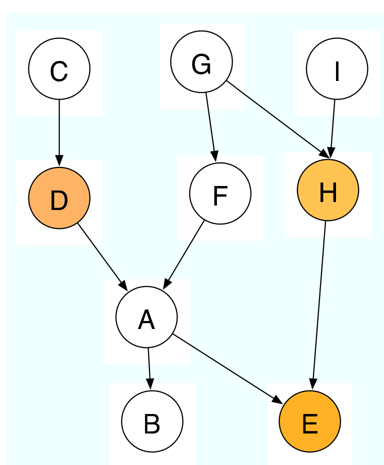

Figure 1: Bayes Ball Example

6. [20 points] **Bayes nets**

   The starship Enterprise is under fire from the Borg and Data the android is trying to save it. He knows that if the ship is close to the Borg vessel and the power is high, he will hit the Borg with probability 90%. If the Enterprise is close but the power is low, the probability drops to 70%. If the Enterprise is far and the battery is high, the chances of a successful hit are 50%. If the Enterprise is far and the battery is low, the chance of success is only 10%. Data has a sensor which tells him whether the Enterprise is far or close, but this sensor sometimes malfunctions: it indicates the correct distance only with probability 80%.

   (a) [10 points] Draw the Bayesian network corresponding to this problem (including the probability tables).

   (b) [10 points] Data observes that the power is low and that the sensor tells him the Enterprise is close. Compute the probability that he will be able to hit the Borg.