

More on list intersection & search

a 7 1 4 10 } common elements
b 3 4 9 1 } (e.g. 2)

For each elem in $a \rightarrow$ look for it in b
list intersection $\rightarrow$ search abstraction

Suppose a, b sorted (increasing)

$$a \quad 1 \quad 4 \quad 7 \quad 10$$

6 1 3 4 9

If arrays sorted - should be able to get a faster algorithm!

Binary search

Example: 1 3 7 8 9 12 13
 ↑ ↑ ≤ ≥ ↑
 l r = found r

 either left
 or right

val = 14

Look at some elements but still
get right result

Algorithm

binary Search (a, n, val)

(L3-2)

Input: sorted array a of n elements; val (value we're looking for)

Output: true if val found, false otherwise

int $l \leftarrow 0$, $r \leftarrow n-1$

while $l \leq r$ // array not empty

int $mid \leftarrow \frac{l+r}{2}$

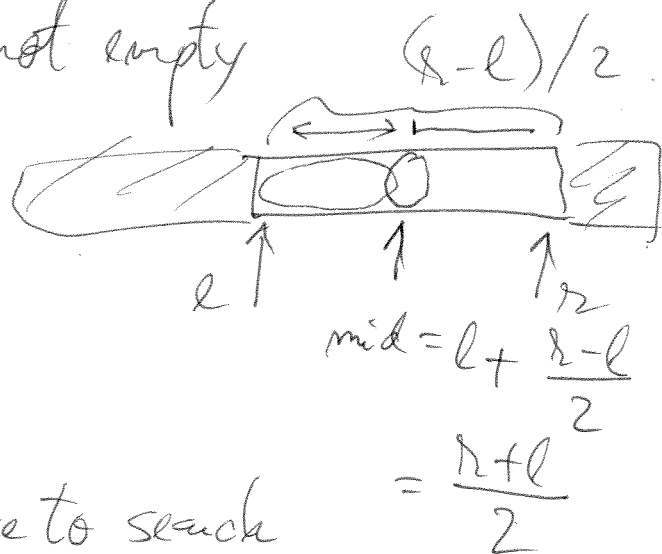
if $a[mid] = val$ then
return true

if $a[mid] > val$ // have to search
left of the middle

$r \leftarrow mid - 1$

else $l \leftarrow mid + 1$

return false // I was left with an "empty"
array to search



Size n ... 1 } (2 ~~see~~ comparisons
Size $\frac{n}{2}$... 1 } really)

Size $\frac{n}{2^2}$... 1 } stop! $\Rightarrow O(\log_2 n)$

Size $\frac{n}{\log_2 n}$

Size of sorted array

Algorithm listIntersection (a, n, b, m)

Input: Array a of size n , b of size m

Output: return no. of elements in common

$\Rightarrow$ sort (b, m) Abstraction!!

int intersect $\leftarrow 0$

for $i \leftarrow 0$ to $n-1$

if binarySearch (~~a~~ , $b, m, a[i]$) then
 $\nearrow$ intersect $\leftarrow$ intersect + 1

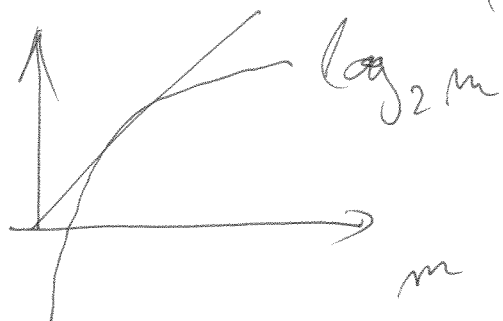
return intersect

n • running time of binary Search + Sorting
 (size of a) On array b time

$$\underline{m \log_2 m}$$

Previous search: $O(m)$

$$\begin{aligned} n \cdot O(\log_2 m) + O(m \log_2 m) & \neq (n+m) \log_2 m \\ & = O((m+n) \log_2 m) \end{aligned}$$



$m, m \simeq 1 \text{ mil}$
 binary Search version
 $\simeq 25000$ times
 faster!

"Look in the middle, then
look either left or right

L3-4

Algorithm binarySearch (a, val, l, r)

Input: Array a, value val, left & right indices between
which we search.

// Call binarySearch (a, val, 0, n-1)

Output: same

// DO I have an array still? // Recursion

if $l > r$ return false "base case"

int mid $\leftarrow \frac{l+r}{2}$ // as before

if $a[mid] = val$ then return true

// Search again in one side

if $a[mid] > val$ then // search left

return binarySearch (a, val, l, mid-1)

return binarySearch (a, val, mid+1, r)

There is always an iterative algorithm that does
same thing but recursion often

- more natural
- easier to analyze