STUDENT NAME: ____________________

STUDENT ID: ____________________

FIRST MIDTERM

COMP-250: Introduction to Computer Science - Winter 2008

March 10, 2008

You are allowed one double sided cheat sheet.

There are 4 questions, for a total of 100 points. Please read all the questions first. Please make sure to **write your name** and ID number on the exam booklet!

Answer all questions on the exam booklet

Good luck!

1. [20 points] **Big-oh**

   For each of the questions below, provide a true or false answer and explain your reason.

   (a) $1000000n + 10^{10} \in O(n)$

   (b) $2^{n+1} \in O(2^n)$?

   (c) $2^{2n} \in O(2^n)$?

   (d) $2^{\log_{10}(n)} \in O(n)$?

2. [30 points] **More Big-Oh**

   For each of the pieces of pseudocode below, state what $O(n)$ is.

   (a) **Algorithm** f1(n)
   $i \leftarrow 1$
   while $i < n$
       print(i)
       $i \leftarrow i + 10$

   (b) **Algorithm** f2(n)
   $i \leftarrow 1$
   while $i < n$
       print(i)
       $i \leftarrow i * 10$

   (c) **Algorithm** f3(n)
   $i \leftarrow 1$
   while $i < n$
       print(i)
       $i \leftarrow i * 10 + 37$

(d) **Algorithm** f4(n)
$i \leftarrow n$
while $i! = 0$
  print(i)
  $i \leftarrow i$ **mod** 10

(e) **Algorithm** f5(n)
if $n = 0$ **return**
print(n)
f5($n - 1$)

(f) **Algorithm** f6(n)
if $n = 0$ **return**
print(n)
f5($n/10$)

(g) **Bonus 5 points**
Algorithm f7(n)
if $n = 0$ **or** $n = 1$ **return**
print(n)
f7($n - 1$)
f7($n - 2$)

3. [30 points] **Pseudocode**

   Write the pseudocode for an algorithm which receives as input an array of positive integers a and a positive integer x. If there are two integers p and q in the array such that $2p+q=x$, the algorithm should return p and q. Otherwise it should return $(-1,-1)$. *Your algorithm should work in* $O(n\log n)$. Hint: you can call as subroutines any of the algorithms we discussed in class.

   Example: CrazyFind($\{2,5,1,7\}$, 9) should return (1, 7)
   Example: CrazyFind($\{2,5,1,7\}$, 100) should return (-1, -1)
   Algorithm CrazyFind (a,n, x)
   Input: a is an array of positive integers of size n and x is a positive integer
   Output: A pair (p,q) of numbers from array a such that $2p+q=x$, if such a pair exists; $(-1,-1)$ otherwise

4. [20 points] **Crazy sort**

   Consider the sorting algorithm described by the following pseudocode:

   Algorithm CrazySort (a,i,j)
   Input: An array of integers a and indices i and j in the array
   Output: The array a will be sorted
   if $i+1 > j$ **then return**
   if $a[i] > a[j]$ **then** swap($a[i]$,$a[j]$)
   $k \leftarrow \lfloor \frac{j-i+1)}{3} \rfloor$
   CrazySort(a,i,$j-k$) //recursive call on the first two-thirds of the array
   CrazySort(a,$i+k$,j) //recursive call on the last two-thirds of the array
   CrazySort(a,i,$j-k$) // recursive call again on the first two-thirds of the array

   The algorithm is called with: CrazySort(a,1,n)
   where n is the length of the array.

   (a) [10 points] Prove by induction that the algorithm is correct.
   (b) [5 points] Write down a recurrence for the running time of the algorithm, $T(n)$. You may use a constant, C, to cover for all the $O(1)$ operations.
   (c) [5 points] What is $O()$ for the algorithm? Hint: to justify this, you may need to use the fact that, for a constant k, $1+k+k^2+\cdots+k^m = \frac{k^{m+1}-1}{k-1}$.