COMP-424 - Assignment 2

Posted Monday, February 4, 2013
Due Wednesday, February 13, 2013

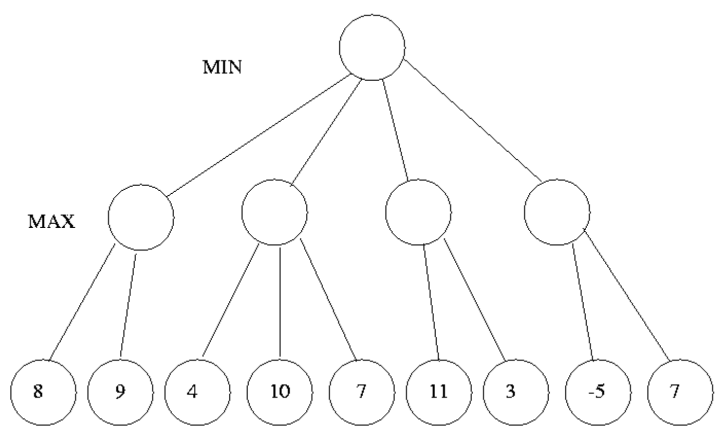
Please submit your assignment by e-mail to cs424@cs.mcgill.ca, by 11:59pm

1. [5 points] **Reading**

   We will continue our reading of important AI papers by looking at the paper by Arthur Samuel, "Some studies in machine learning, using the game of checkers II", published in 1967. The text of the paper (in pdf) is linked from the class web page. Summarize in one paragraph the main ideas in the paper (note that we will dwell more on some of these ideas in the second part of the course).

2. [15 points] **Game tree search**

   Consider the following game tree (note that min is the root node, not max)



   (a) [5 points] Show the minimax values at each node, and the optimal move at the root.

   (b) [5 points] Show the result of alpha-beta pruning on this tree

   (c) [5 points] Show what would be the optimal ordering of all nodes in the tree, so alpha-beta pruning eliminates the maximum number of nodes possible.

3. [25 points] **Optimization**

   Suppose you are given an undirected graph $G = (V, E)$ and you want to partition its vertices, into two disjoint sets, V_1 and V_2, such that:

   - The number of vertices in V_1 and V_2 is as close as possible
   - The number of edges where one end is a node in V_1 and the other end is a node in V_2 is as small as possible.

   (a) [5 points] Give a function which expresses this optimization problem.

(b) [5 points] Explain how you would perform gradient ascent on this function: what are the states, and what is the set of neighbours of a given state?

(c) [10 points] Explain how you would encode this problem for a genetic algorithm: how are the individuals encoded, what is the fitness function, what are the mutation and crossover operations, and any other details if needed.

(d) [5 points] Do you expect gradient ascent, simulated annealing or genetic algorithms to work better on this problem? Justify your answer

4. [20 points] **Constraint satisfaction** (adapted form Russell & Norvig)

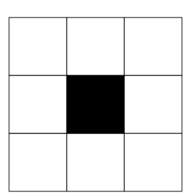
In CSPs, a non-binary constraint is a constraint that involves more than 2 variables. For example, consider the variables X, Y , Z where the domain of X and Y is $\{Red, Green\}$ and the domain of Z is $\{Red, Green, Blue\}$. An example of a non-binary constraint is that exactly two of these variables have the color $Green$.

Sometimes we want to convert a non-binary CSP to a binary CSP (because a lot of work has been put into producing good binary CSP solvers). To do that we add a new extra variable for every constraint. In detail, to replace a non-binary constraint C (over variables $X_1, \ldots X_k$) with only binary constraints, we create a new variable W . The variable W can take on as many values as there are legal assignments to $X_1, \ldots X_k$ that satisfy C . We then remove the original constraint, and replace it with a set of binary constraints between the new variable W and the original variables X_i participating in the original constraint. For simplicity, you may assume that the original CSP does not contain any unary constraints.

(a) [5 points] Complete the definition for a transformation of this type when it is executed for a constraint C over a set of variables $X_1, \ldots X_k$. You may assume that C is represented as a set of legal tuples of values for $X_1, \ldots X_k$. Specifically, how would you define W, and what are the binary constraints associated with W ?

(b) [5 points] Apply the algorithm to the example given above (where exactly two variables out of X, Y , Z have the color $Green$). Define exactly the domain of the new extra variable by writing down all the possible values in the domain. Show the new constraints created by writing down all the constraints that involve Z and the new extra variable.

(c) [5 points] Consider the problem of creating a crossword puzzle in which all the words must appear in some given dictionary D. We use the notation $D = \{w^1, w^2, \ldots w^n\}$ where w^i is the i-th word in the dictionary. Let w^i_j be the jth letter of word i and l_i be the length of node i. As an example, consider the following instance of the problem:



Assuming that CAP,CAT,PEN,TIN $\in D$ (which can, of course, also contain other words), a possible solution could be:

C	A	P
A	■	E
T	I	N

Formalize this instance of the problem as a CSP problem, such that each variable corresponds to one empty square in the crossword puzzle (you will need 8 variables). What are the constraints in this formalization?

(d) [5 points] Note that the constraints are non-binary. Explain what you obtain if you reformulate the problem as a binary CSP using the method you gave above.

5. [10 points] **Simple propositional logic**

(a) [6 points] For each of the formulas below, determine if it is valid, satisfiable or unsatisfiable. If the formula is valid or unsatisfiable, provide a proof. If it is satisfiable but not valid, show a model (assignment of values to the literals) which makes it true and one that makes it false.

- $P \wedge \neg(P \vee Q)$
- $[(P \wedge Q) \rightarrow Q] \rightarrow (P \wedge Q)$
- $(P \wedge Q) \Leftrightarrow \neg(\neg P \wedge \neg Q)$

(b) [4 points] How many models are there for the following sentences (i.e., in how many worlds are they true):

- $X_1 \wedge X_2 \wedge \ldots \wedge X_n$
- $(X_1 \wedge X_2) \vee (X_2 \wedge X_3) \vee \ldots \vee (X_{n_1} \wedge X_n)$

6. [10 points] **Propositional logic and text**

Consider the following propositional symbols:
$A =$ Mary is in class
$B =$ Mary has the flu
$C =$ Mary is bored
$D =$ Mary sees the doctor

(a) [6 points] Express each of the following English sentences in propositional logic:

- Mary is in class but she is not bored.
- Mary has to see a doctor if she has the flu.
- Mary does not go to class if she has the flu.

(b) [4 points] Suppose that the fact above constitute your knowledge base. Can you infer if Mary has to see the doctor? If so, show a formal proof. If not, explain what you would need to add to the knowledge base to get the answer.

7. [15 points] **First-order logic**

   (a) [6 points] Formalize the following text in first-order logic:

   All Star Fleet cadets obey their superiors all the time. All Star Fleet officers obey their superiors some of the time. A Start Fleet member is either a cadet or an officer, but not both. All cadets wear blue uniforms. All officers wear red uniforms. Data does not wear a blue uniform.

   (b) [6 points] Is the knowledge base above sufficient to prove that Data is an officer? If so, show a proof (using resolution and unification). If not, add a minimal number of statements which would allow you to prove that Data is an officer, and give the proof (using resolution and unification).

   (c) [3 points] Can you now prove formally if Data obeys orders all the time? If yes, show a proof. If no, explain why not.