80-110 The Nature of Mathematical Reasoning
Summer 2000
Dirk Schlimm

Homework No. 8

Wednesday, July 19 2000
Due Friday, July 21 2000

• You may do either problem 3 or 4 – you don't have to do both of them! (Problem 4 is harder).

1. J. B. Shaw (Handout #13) distinguishes four views with regard to the nature of geometry. Summarize each of these views in a few sentences.
State in a paragraph which one of these you find most convincing and explain why.

2. Russell (Handout #15) accepts Dedekind and Peano's analysis of the natural numbers, but wants to go further than them. At the end of the Chapter he raises two objections against considering Peano's axioms "as an adequate basis for arithmetic". State Russell's objections in your own words.
(If you're curious about Russell's own definition of number, here it is: "A number is anything which is the number of some class", where "the number of a class is the class of all those classes that are similar to it".)

3. FOL, Problem 14 on page 22.

4. Consider the following axiom system:
Undefined term: *points*.
Definition: Let X be a set of points, a certain subset of X is called *line*.
Definition: Two lines that have no points in common are said to be *parallel*.
Axioms:

1. There exists at least one point and at least one line.
2. If X and Y are two distinct points, there is a unique line containing them both.

3. Through a given point not on a given line, there is a unique line parallel to the given line.

4. Not all points are on the same line.

Questions:

a) Say which lines are parallel in each of the figures and give a reason for your answer.

b) Which of the figures below is a model for this axiom system? Justify your answer.

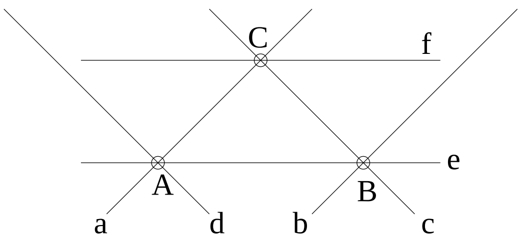

Figure 1: Points are represented as little circles in this figure. If something is not a circle, it's not a point! Points are labeled with capital letters, lines with lowercase letters. For example, line c consists exactly of points B and C.

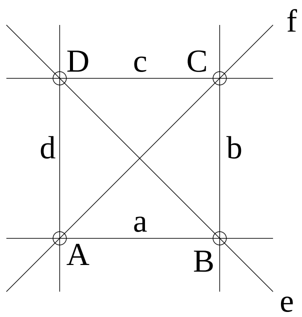

Figure 2: Points are represented as little circles in this figure. If something is not a circle, it's not a point! Points are labeled with capital letters, lines with lowercase letters.