CP: Conditional Probabilities and Cherry Pie

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I'm sorry for the confusion in class in the last lecture. But there's always something to learn from such situations: Do not present examples that you thought about in the bus on the morning before the lecture.

Anyway, here's what I think is the right way to look at the problem.

Recall the situation: You are presented with a cherry pie, which is cut into eight pieces, and you also know that in two of these pieces there is a pit.

1. You are offered a piece of the pie. What is the probability that you find a pit in it?

$$P(\text{pit in first piece}) = \frac{\textit{\# of favorable outcomes}}{\textit{total \# of outcomes}} = \frac{2}{8} = \frac{1}{4}.$$

2. Now the situation is different. First, I take a piece of the pie and find a pit in it. Now you take a piece. What is the probability that you find a pit in it?

$$P(\text{pit in second piece} \mid \text{pit in first piece}) = \frac{\textit{\# of favorable outcomes}}{\textit{total \# of outcomes}} = \frac{1}{7}.$$

If you know that the first piece had a pit, $P(\text{pit in first piece})=1$, if you do not know, then it is $= 1/4$.

Using the definition of conditional probabilities,

$$P(A|B) = \frac{P(A \text{ \& } B)}{P(B)},$$

we can now also calculate the value of P(pit in first piece & pit in second piece).

Let $A =$"pit in first piece", $B =$"pit in second piece." Then we have

$$\begin{aligned} P(A \;\&\; B) &= \\ P(B \;\&\; A) &= P(B|A) \cdot P(A) \\ &= \frac{1}{7} \cdot \frac{1}{4} = \frac{1}{28}. \end{aligned}$$

3. Assume now, that you do not know whether there was a pit in the first piece. Then, to determine the probability that you find a pit in the second piece you have to use the Law of total probability:

$$P(A) = P(A|B)P(B) + P(A|\neg B)P(\neg B).$$

Let A and B as above, and consequently $\neg A =$"no pit in first piece." Then we have:

$$\begin{aligned} P(A) &= 1/4, \\ P(\neg A) &= 1 - P(A) = 1 - 1/4 = 3/4, \\ P(B|A) &= 1/7, \\ P(B|\neg A) &= 2/7. \\ P(B) &= P(B|A)P(A) + P(B|\neg A)P(\neg A), \\ &= \frac{1}{7} \cdot \frac{1}{4} + \frac{2}{7} \cdot \frac{3}{4} = \frac{1}{28} + \frac{6}{28} = \frac{7}{28} = \frac{1}{4}. \end{aligned}$$

Notice that the result is the same as in situation 1. This means that it does not matter how the piece you get is chosen. Even if you get the last piece of the pie (without knowing whether a pit was in the other pieces) the probability that you find a pit would still be 1/4.

That's it with regard to cherry pies and pits.