80-110 Nature of Mathematical Reasoning

Spring 2002

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Homework 10 (B)

Wednesday, March 27
Due Monday, April 8

1. Read FOL, pages

- 91–103 (Sections 4.1–4.4),
- 112 (Section 4.7),
- 115–141 (Sections 5.1–5.10),
- and 152 (Section 5.13).

2. (4 points)

Let Γ be a set of sentences, and φ a sentence (in first order logic).

$\Gamma \vdash \varphi$ means that "Γ proves φ," i.e. that there is a proof of φ from premisses in Γ.

$\Gamma \models \varphi$ means that "φ is a semantic consequence of Γ," i.e. that any model for Γ (that is, for all sentences in Γ at the same time), is a model for φ.

Assume Gödel's Completeness Theorem: $\Gamma \vdash \varphi$ iff $\Gamma \models \varphi$.

- Show that for any given set of sentences Γ, Γ has no model iff Γ is inconsistent (i.e. $\Gamma \vdash \bot$)

(Hint: If Γ has no model, then $\Gamma \models \varphi$ for any φ whatsoever (why?). Also, there is no model for $\bot$.)

3. (3 points)

Write out in words what the following sentences mean classically and intuitionistically. Here's an example for the sentence $A \to B$:

1. Classical meaning: If A is true, then B is true.

2. Intuitionistic meaning: There is a construction that will turn any proof of A into a proof of B.

a) $A \vee B$

b) $\neg A$

c) $\forall x \ P(x)$ (Assume the domain we are quantifying over is the natural numbers.)

4. (3 points)

Discuss the following questions using the informal explanations of the intuitionistic connectives and quantifiers presented in class. You may assume that the domain is the natural numbers. Keep your answers short.

a) Would the proof presented in class that the number of primes is not finite be valid for an intuitionist?

b) Would a classically minded mathematician accept that

$$\forall x \exists y \ (P(x) \to P(y)),$$

where P is an arbitrary predicate? Would an intuitionist?

c) How about

$$\exists x \forall y \ (P(x) \to P(y))?$$

Would an intuitionist accept this sentence?