80-110 Nature of Mathematical Reasoning

Spring 2002

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Homework 9 (B)

Wednesday, March 20
Due Monday, March 25

1. Read FOL, pages

- 44–46 (Section 3.5),
- 51–64 (Sections 3.7–3.8),
- and 88–90 (3.12).

2. (2 points) Give a semantic proof, using truth tables, for the validity of the inference of *dilemma*:

- Premises: $P \vee Q$, $P \to R$, $Q \to S$
- Conclusion: $R \vee S$

3. (1 point) Prove that if Γ is consistent, then every finite $\psi \subseteq \Gamma$ is consistent.

4. (7 points) A partially ordered set A is *well-ordered* if every nonempty subset B has a least element:

$$\exists y \in B\ \forall x \in B\ (y \leq x).$$

Suppose a set Γ of sentences axiomatizes well-orderings, that is to say, the models of Γ are just the well-orderings. Now add sentences $\{c_{i+1} < c_i | i \in \mathbb{N}\}$.

1. Show that $\Gamma \cup \{c_{i+1} < c_i | i \in \mathbb{N}\}$ has a model.
2. Conclude that that model is not well-ordered.