Compiler Design

Lecture 19:
Instruction Selection via Tree-pattern matching

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(EaC-11.3)

The Concept

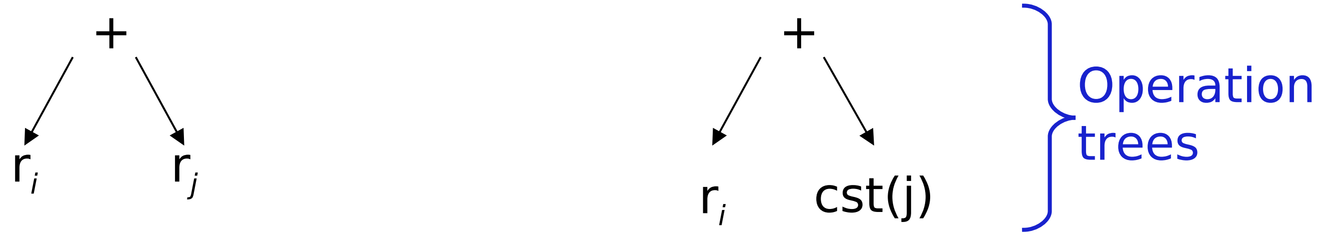
Many compilers use tree-structured IRs

- Abstract syntax trees generated in the parser
- Trees or DAGs for expressions

These systems might well use trees to represent target ISA

Consider the add operators

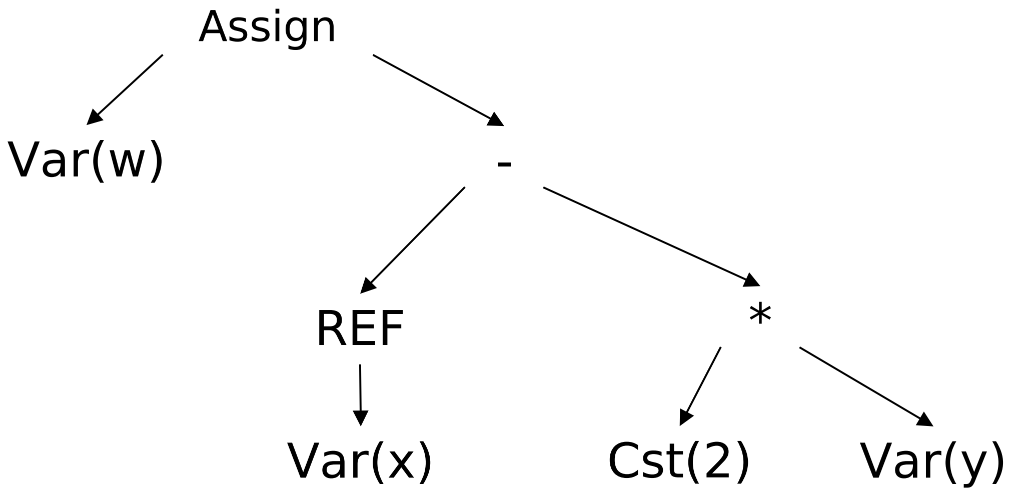
`add` $r_i, r_j \Rightarrow r_k$ `addI` $r_i, j \Rightarrow r_k$



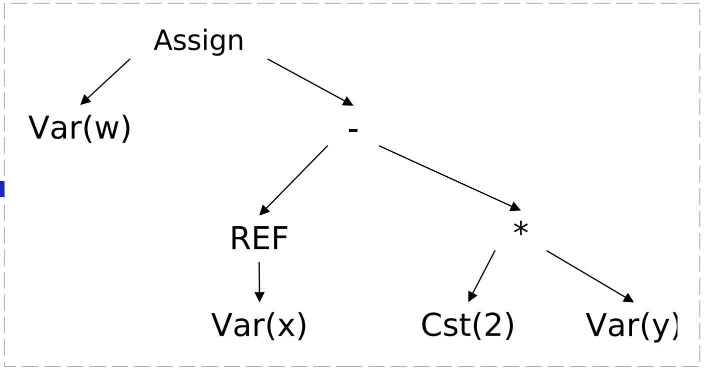
What if we could match these “operation trees” against IR tree?

The Concept

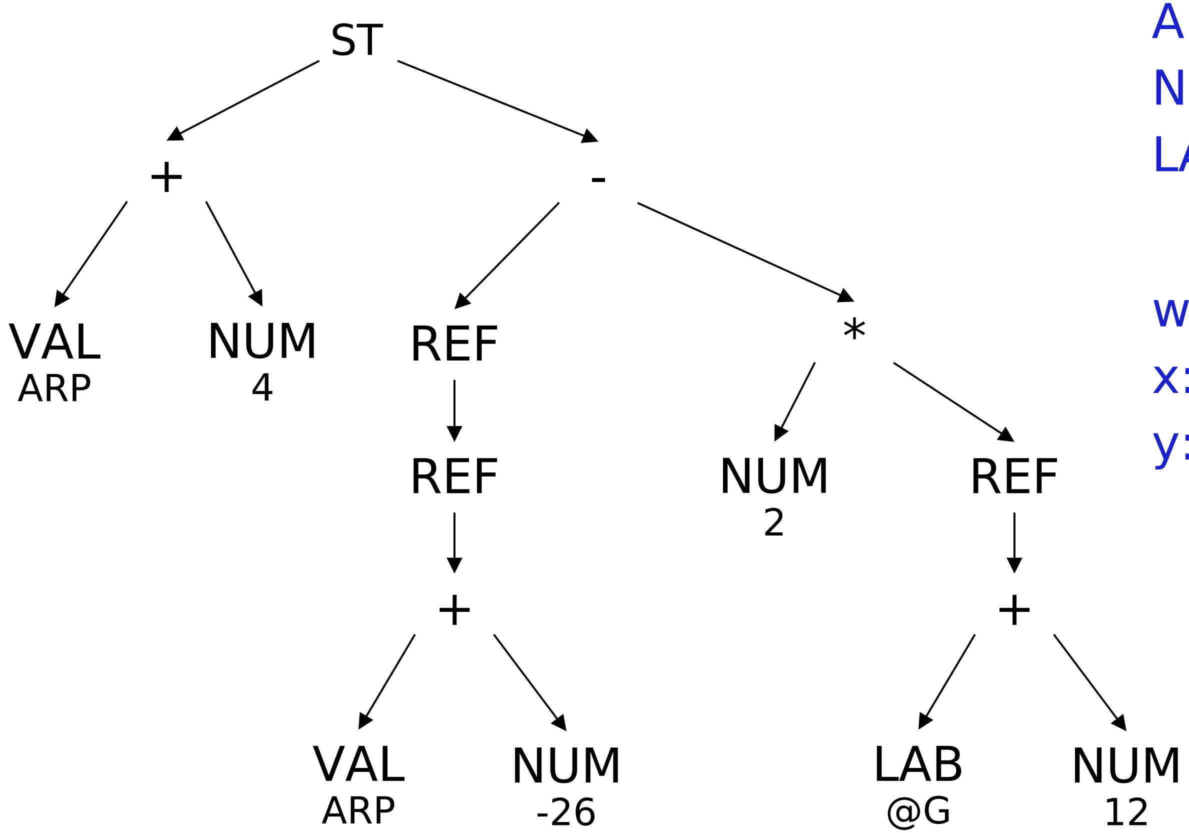
AST for w ← (*x) - 2 * y



The Concept



Low-level AST for w ← (*x) - 2 * y



ARP: $fp

NUM: constant

LAB: ASM label

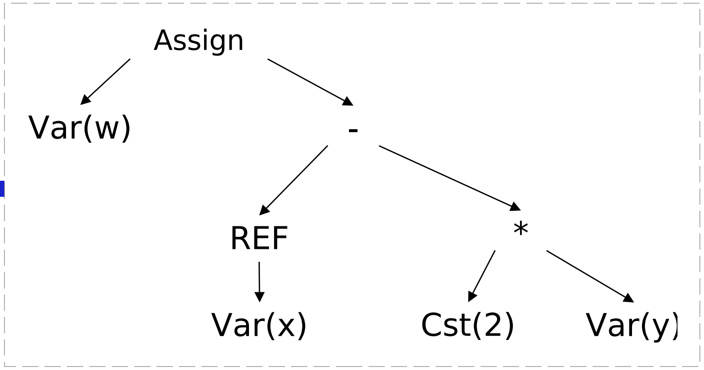
w: at ARP+4

x: at ARP-26

y: at @G+12

ARP = Activation Record Pointer = **frame pointer**

The Concept



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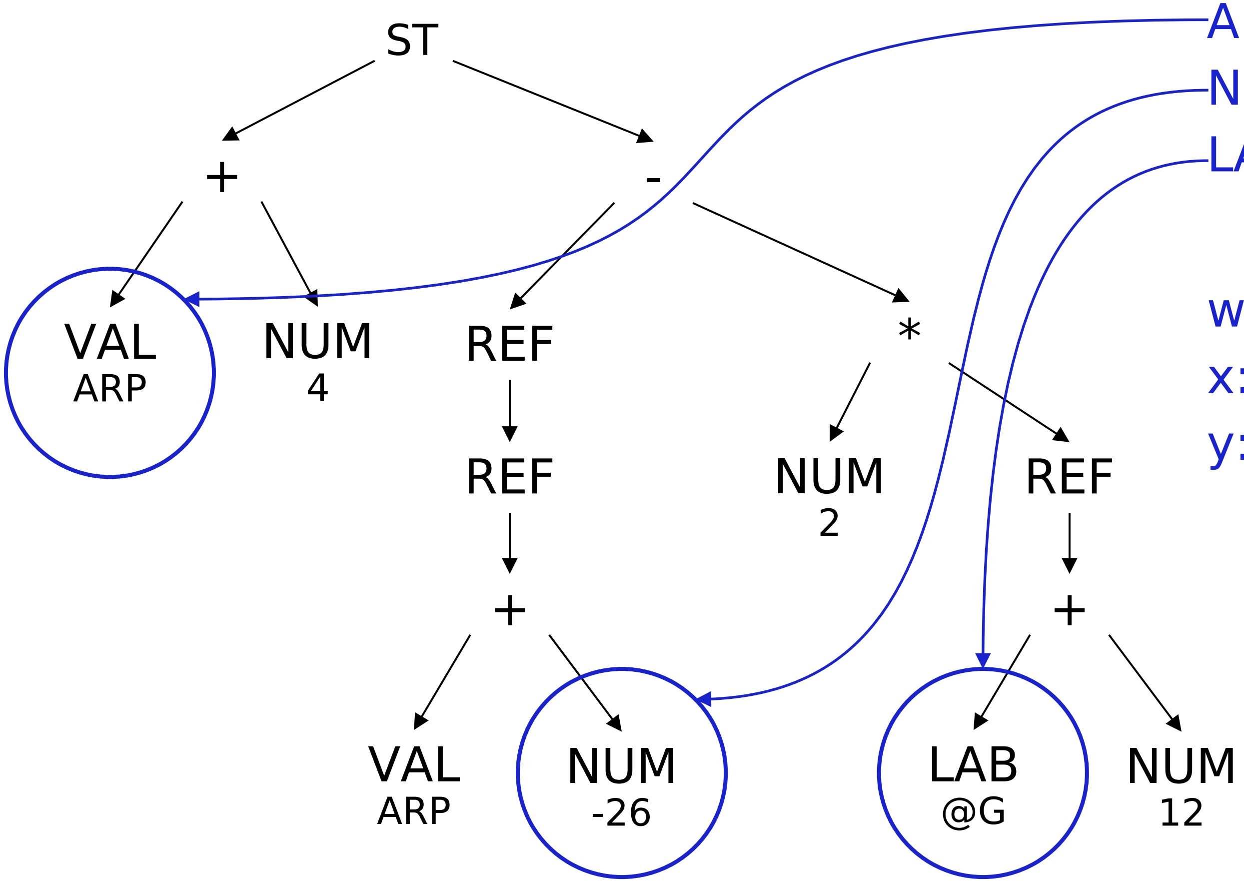

ARP: $fp
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LAB: ASM label

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y: at @G+12

ARP = Activation Record Pointer = **frame pointer**

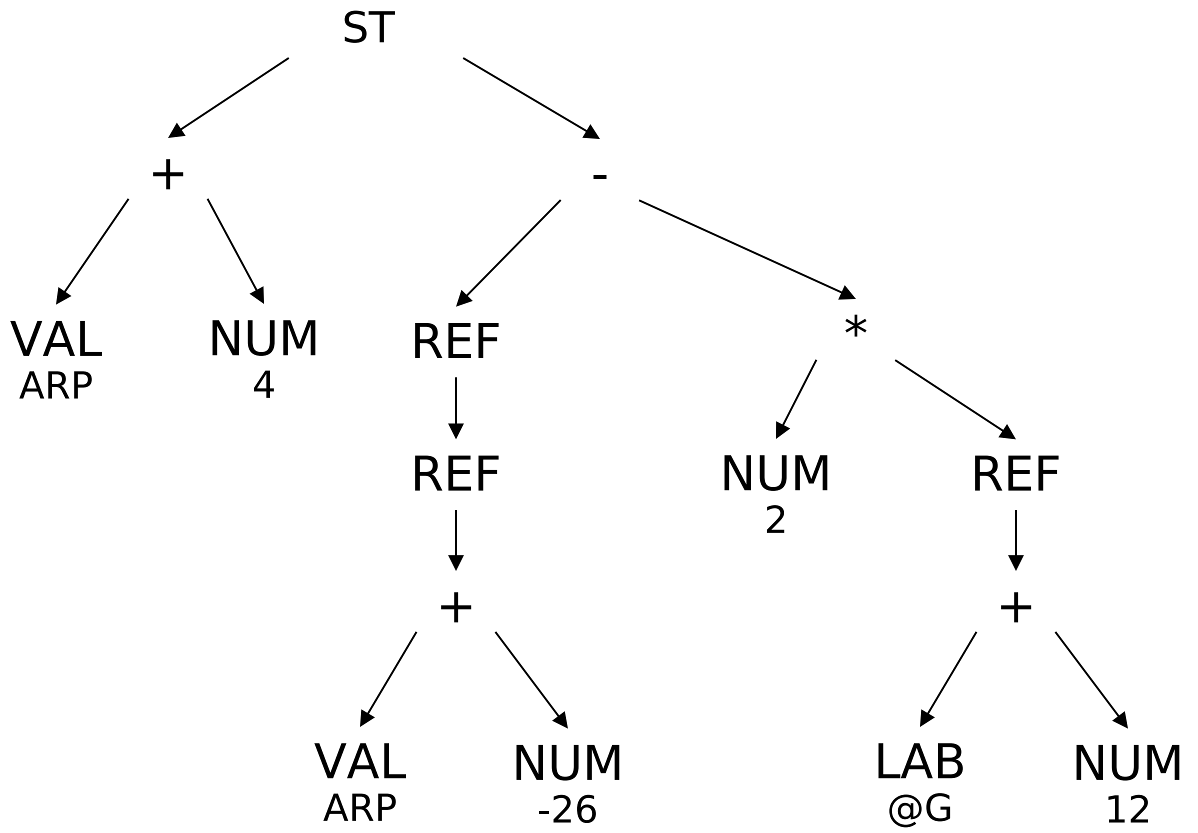
Tree-pattern matching

Goal is to “tile” AST with operation trees

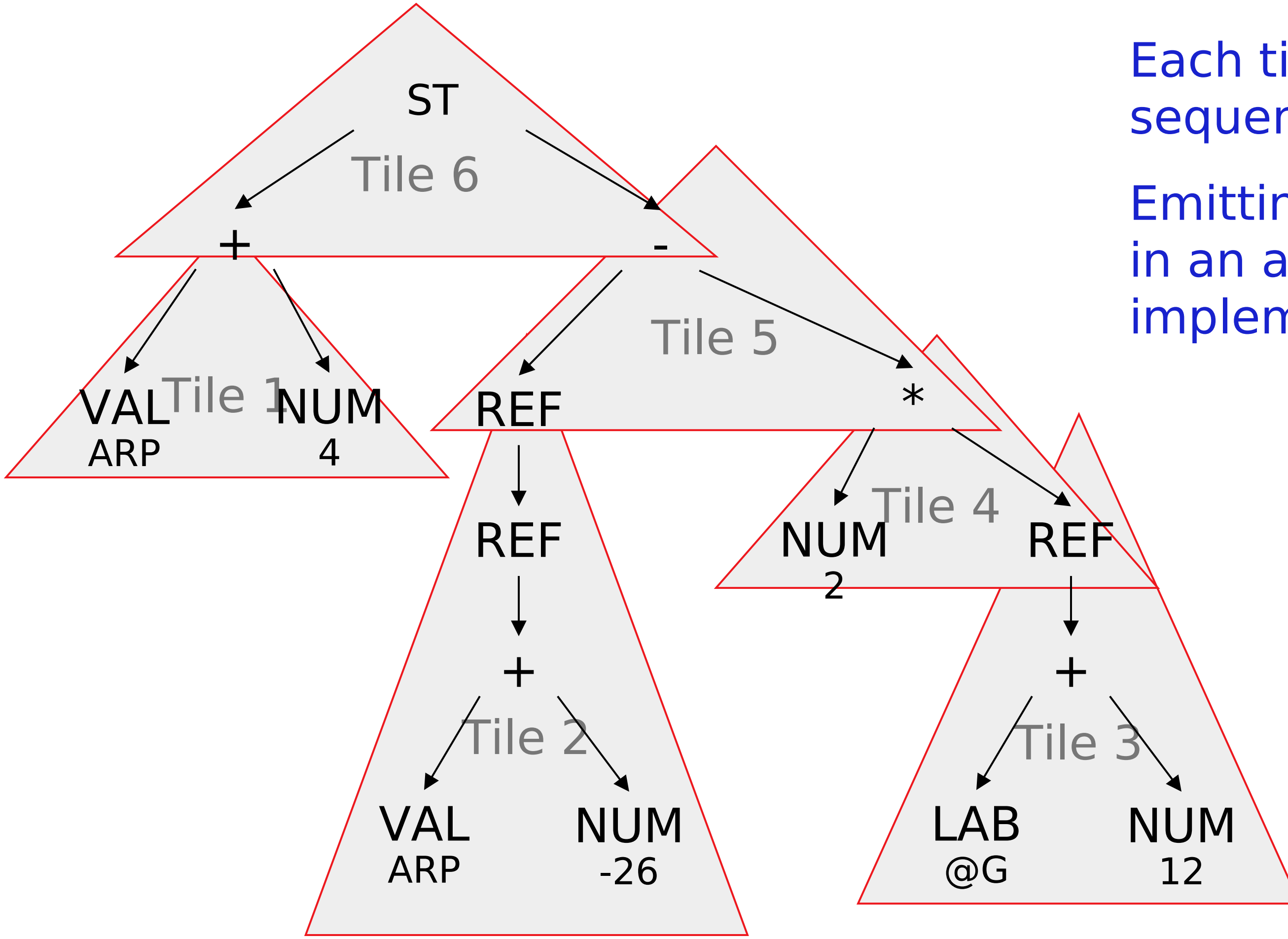
- A tiling is collection of <*ast*,*op* > pairs
  - → *ast* is a node in the low-level AST
  - → *op* is an operation tree
  - → <*ast*, *op* > means that *op* could implement the subtree at *ast*

- A tiling ‘implements” an AST if it covers every node in the AST and the overlap between any two trees is limited to a single node
  - → <*ast*, *op*> ∈ *tiling* means *ast* is also covered by a leaf in another operation tree in the tiling, unless it is the root
  - → Where two operation trees meet, they must be compatible (expect the value in the same location)

Tiling the Tree



Tiling the Tree



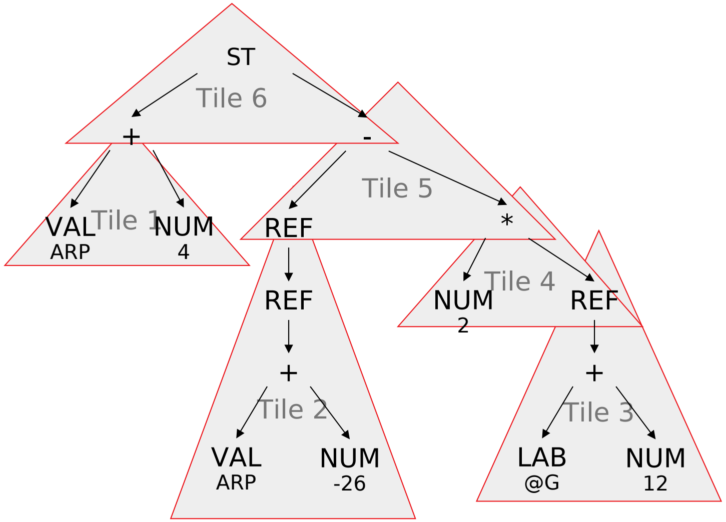
Each tile corresponds to a sequence of operations

Emitting those operations in an appropriate order implements the tree.

Generating Code

Given a tiled tree:

- Postorder treewalk, with node-dependent order for children
- Emit code sequence for tiles, in order
- Tie boundaries together with register names



→ Tile 6 uses registers produced by tiles 1 & 5

→ Tile 6 emits "`store` $r_{\texttt{tile 5}} \Rightarrow r_{\texttt{tile 1}}$"

→ Can incorporate a "real" register allocator or just use virtual registers

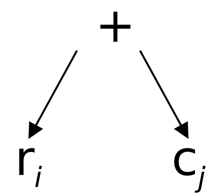
So, What’s Hard About This?

Finding the matches to tile the tree

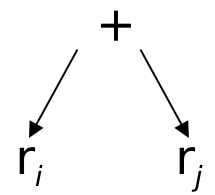
- Compiler writer connects operation trees to AST subtrees
  - → Encode tree syntax, in linear form
  - → Provides a set of rewrite rules
  - → Associated with each is a code template

Notation

To describe these trees, we need a concise notation



$+(r_i,c_j)$



$+(r_i,r_j)$

Linear prefix form

Notation

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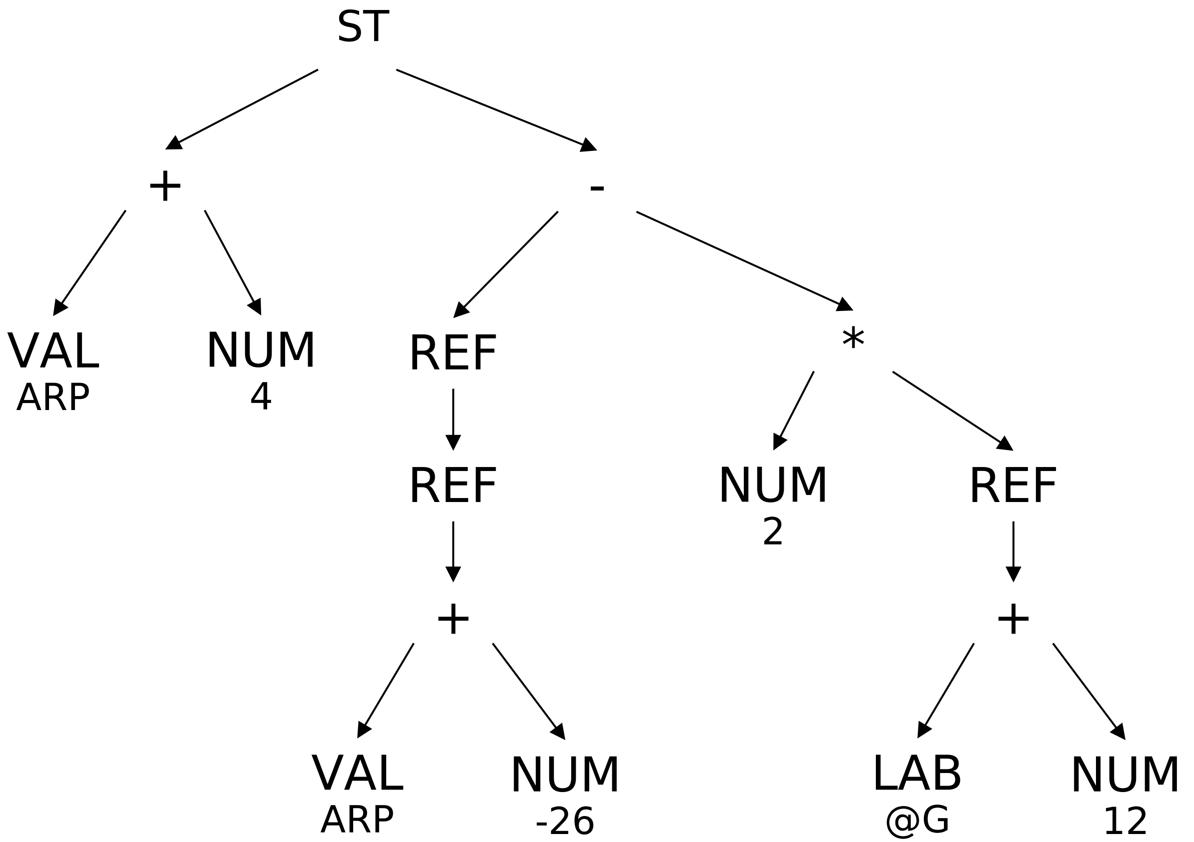

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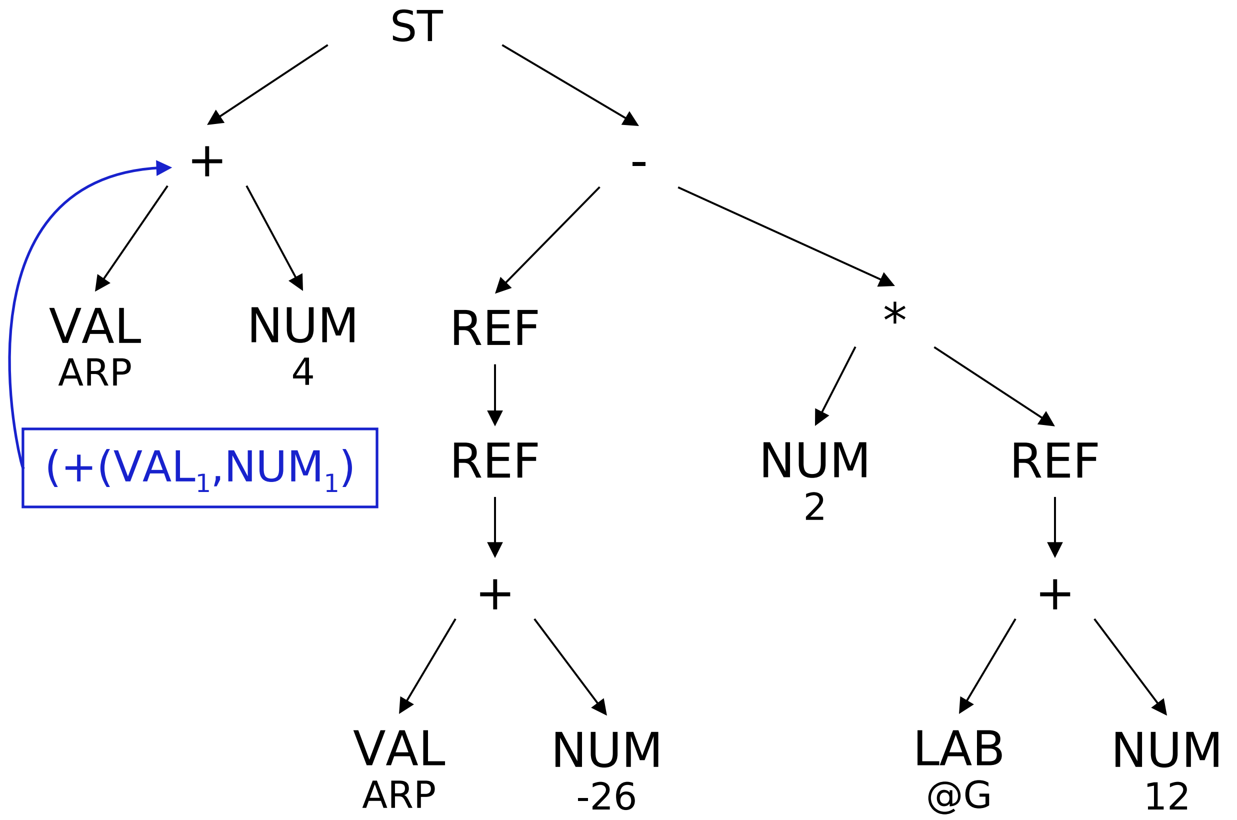

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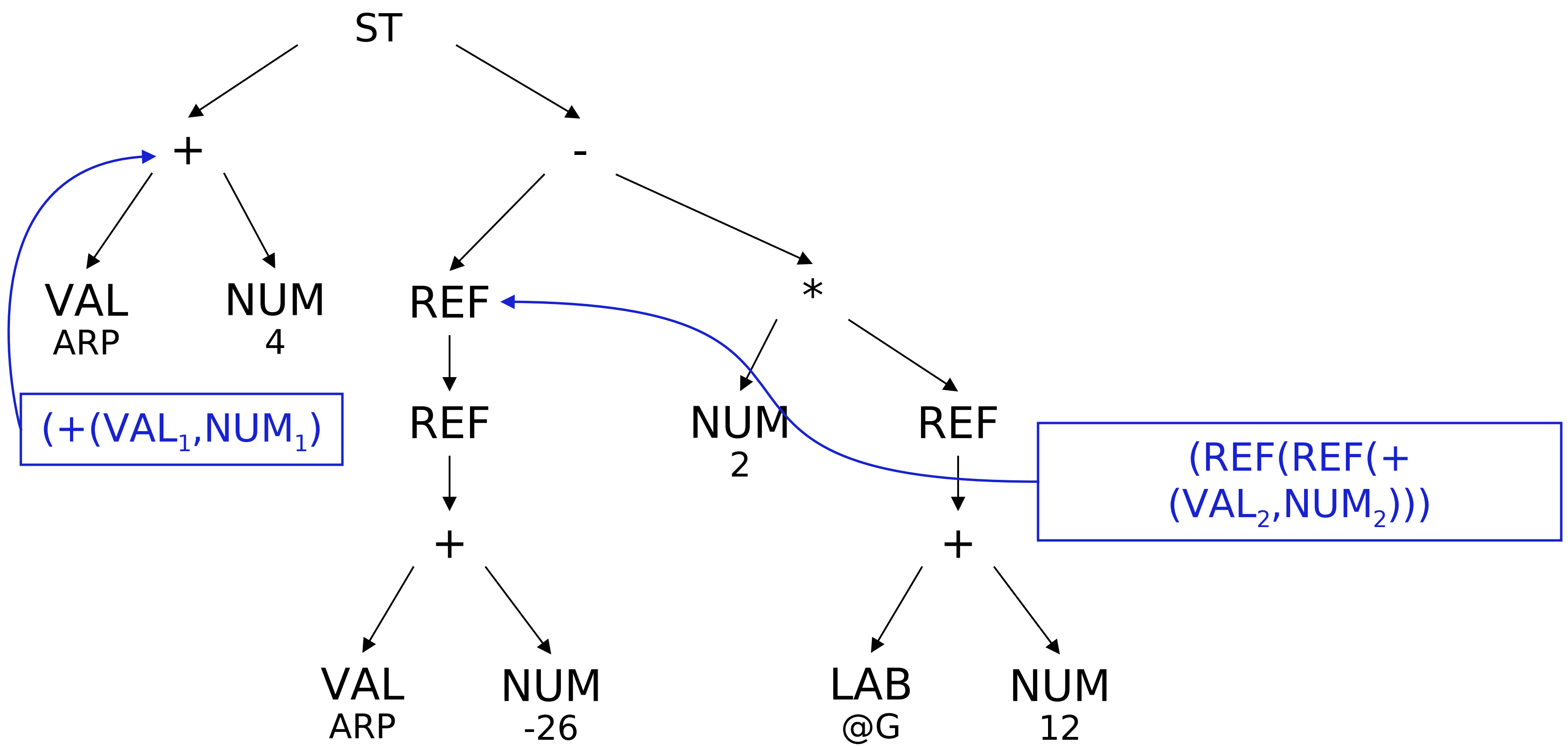

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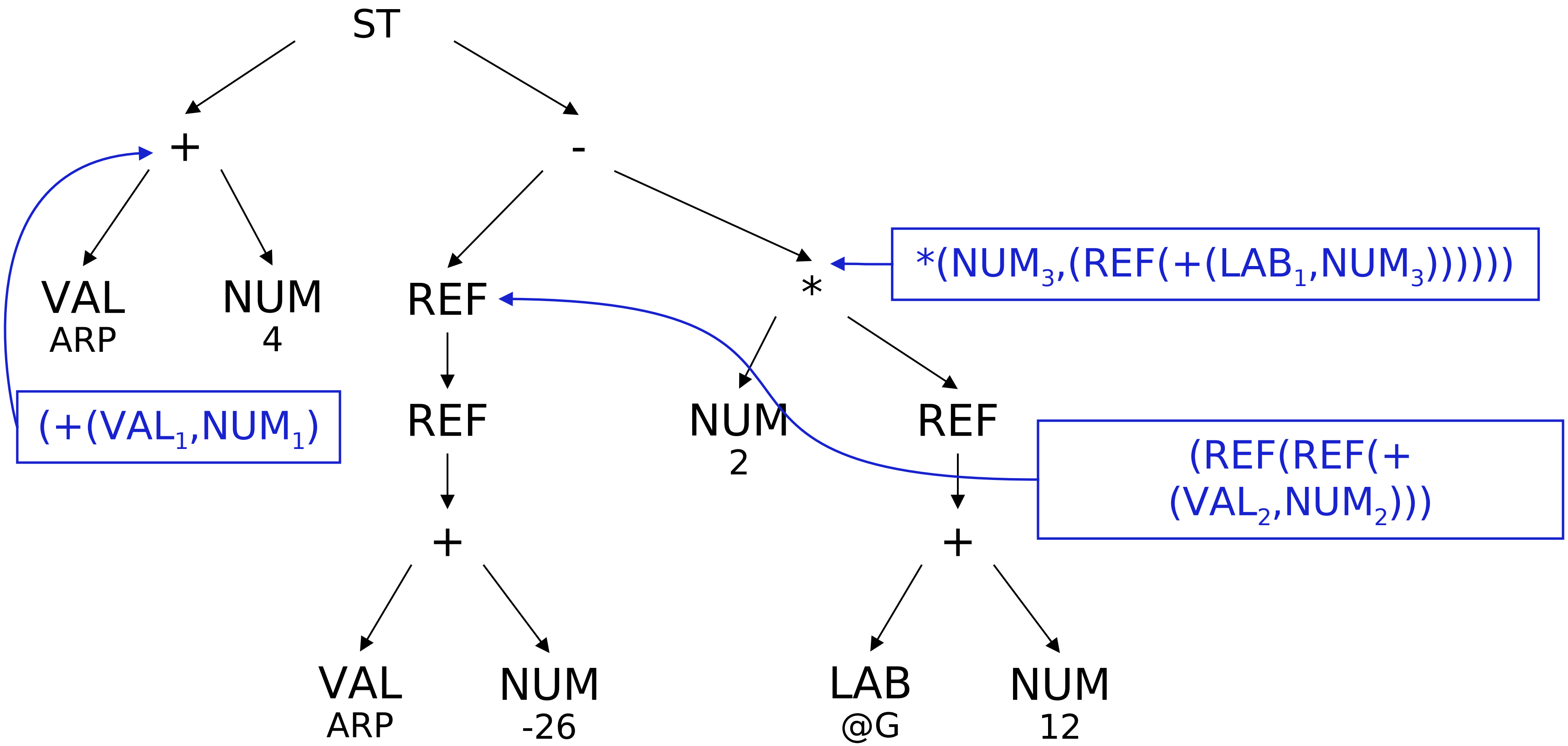

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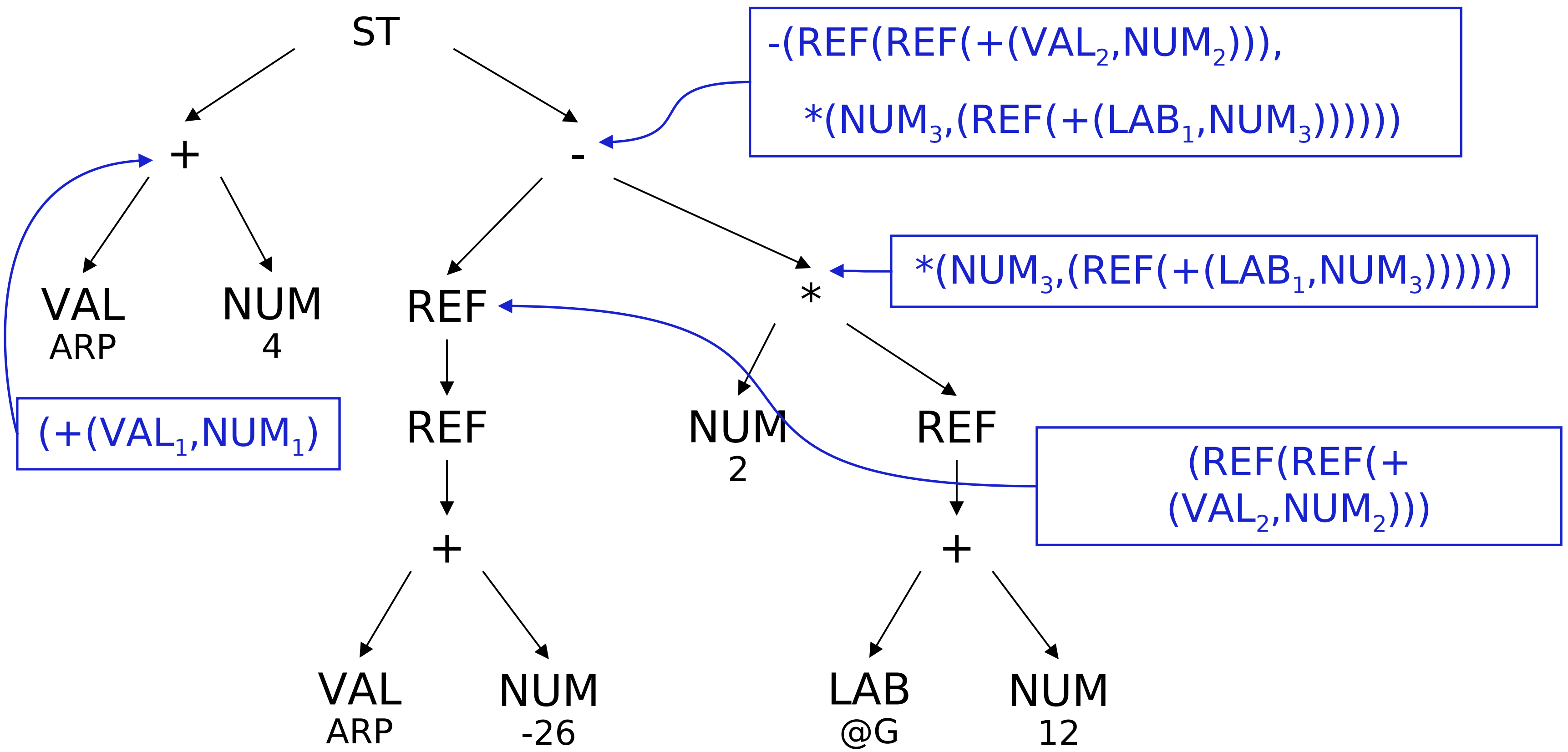

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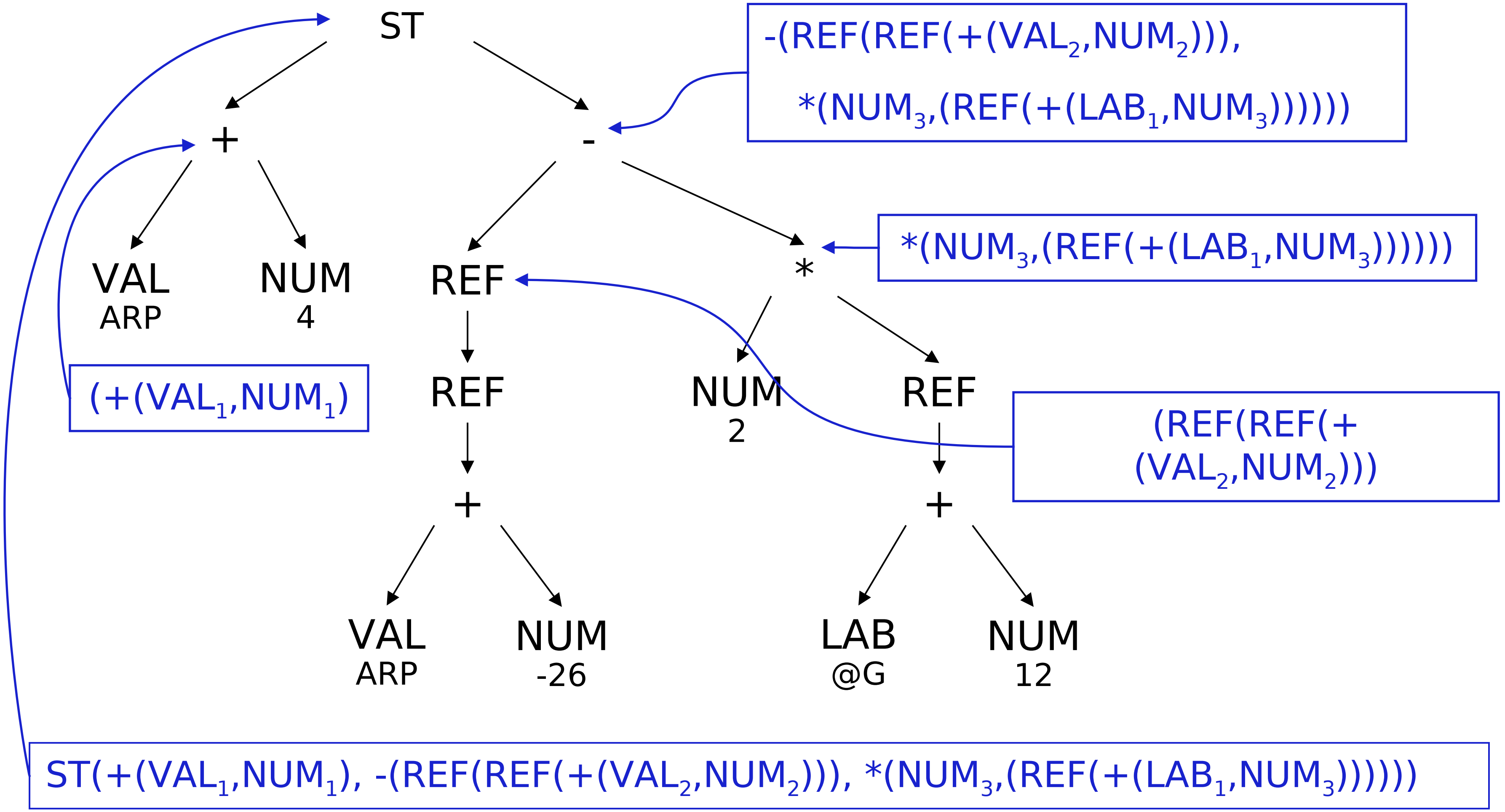


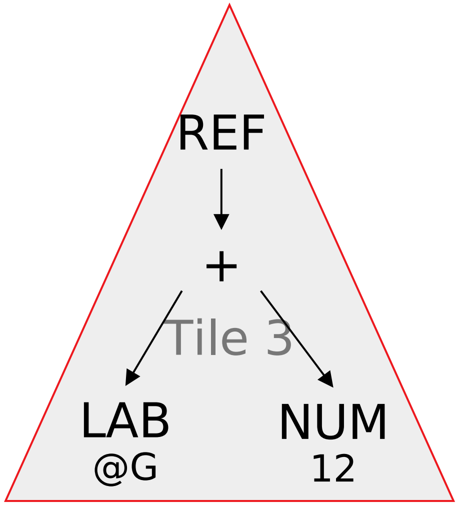
Rewrite rules: LL Integer AST into ILOC

	Rule	Cost	Template
1	Goal → Assign	0	
2	Assign → ST(Reg_1,Reg_2)	3	`store` $r_2 \Rightarrow r_1$
3	Assign → ST(+(Reg_1,Reg_2),Reg_3)	3	`storeAO` $r_3 \Rightarrow r_1, r_2$
4	Assign → ST(+(Reg_1,NUM_2),Reg_3)	3	`storeAI` $r_3 \Rightarrow r_1, n_2$
5	Assign → ST(+(NUM_1,Reg_2),Reg_3)	3	`storeAI` $r_3 \Rightarrow r_2, n_1$
6	Reg → LAB_1	1	`loadI` $l_1 \Rightarrow r_{new}$
7	Reg → VAL_1	0	
8	Reg → NUM_1	1	`loadI` $n_1 \Rightarrow r_{new}$
9	Reg → REF(Reg_1)	10	`load` $r_1 \Rightarrow r_{new}$
10	Reg → REF(+ (Reg_1,Reg_2))	10	`loadAO` $r_1, r_2 \Rightarrow r_{new}$
11	Reg → REF(+ (Reg_1,NUM_2))	10	`loadAI` $r_1, n_2 \Rightarrow r_{new}$
12	Reg → REF(+ (NUM_1,Reg_2))	10	`loadAI` $r_2, n_1 \Rightarrow r_{new}$
13	Reg → REF(+ (Reg_1,Lab_2))	10	`loadAI` $r_1, l_2 \Rightarrow r_{new}$
14	Reg → REF(+ (Lab_1,Reg_2))	10	`loadAI` $r_2, l_1 \Rightarrow r_{new}$
15	Reg → + (Reg_1,Reg_2)	1	`addI` $r_1, r_2 \Rightarrow r_{new}$
16	Reg → + (Reg_1,NUM_2)	1	`addI` $r_1, n_2 \Rightarrow r_{new}$
17	Reg → + (NUM_1,Reg_2)	1	`addI` $r_2, n_1 \Rightarrow r_{new}$
18	Reg → + (Reg_1,Lab_2)	1	`addI` $r_1, l_2 \Rightarrow r_{new}$
19	Reg → + (Lab_1,Reg_2)	1	`addI` $r_2, l_1 \Rightarrow r_{new}$
20	Reg → - (NUM_1,Reg_2)	1	`subI` $r_2, n_1 \Rightarrow r_{new}$
…	…	…	…

A real set of rules would cover more than signed integers …

So, What's Hard About This?

Consider tile 3 in our example



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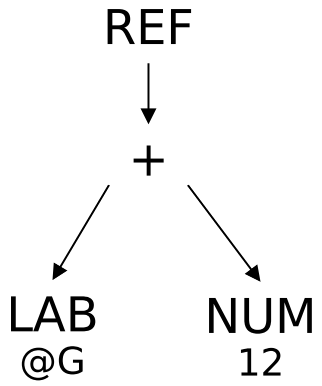
Consider tile 3 in our example

What rules match tile 3?



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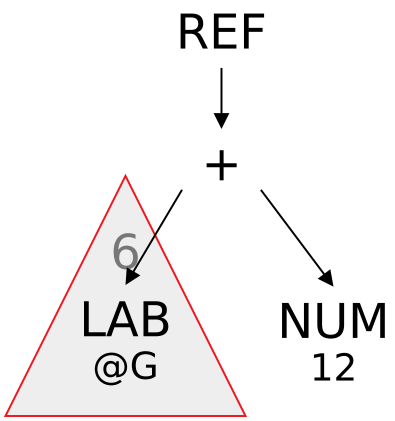

What rules match tile 3?

6: Reg $\rightarrow$ LAB_1 tiles the lower left node

So, What's Hard About This?

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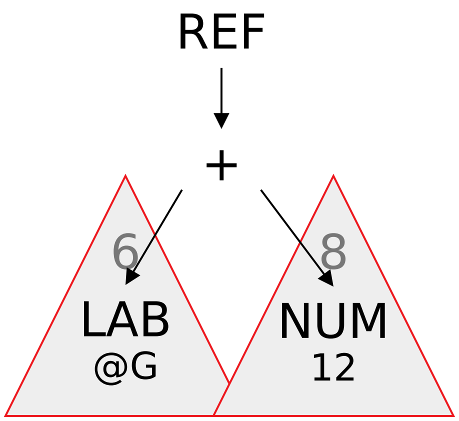


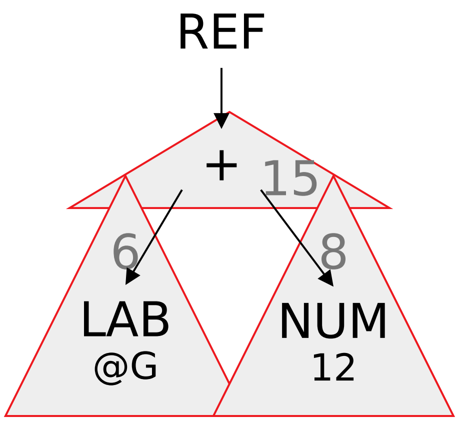
What rules match tile 3?

6: Reg $\rightarrow$ LAB_1 tiles the lower left node

8: Reg $\rightarrow$ NUM_1 tiles the bottom right node

So, What's Hard About This?

Consider tile 3 in our example



What rules match tile 3?

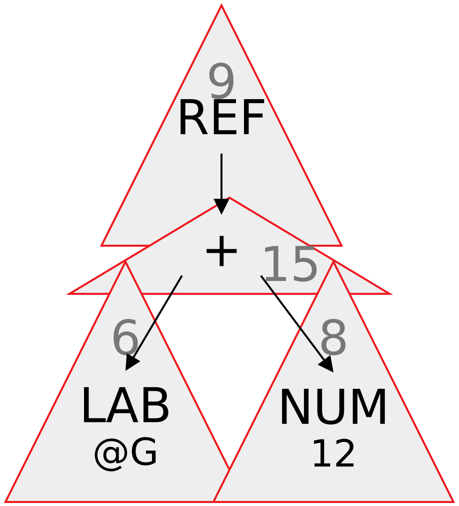
6: Reg $\rightarrow$ LAB_1 tiles the lower left node

8: Reg $\rightarrow$ NUM_1 tiles the bottom right node

15: Reg $\rightarrow$ + (Reg_1,Reg_2) tiles the + node

So, What's Hard About This?

Consider tile 3 in our example



What rules match tile 3?

6: Reg $\rightarrow$ LAB_1 tiles the lower left node

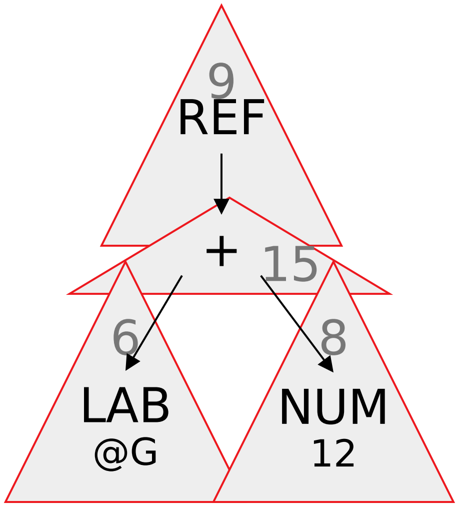
8: Reg $\rightarrow$ NUM_1 tiles the bottom right node

15: Reg $\rightarrow$ + (Reg_1,Reg_2) tiles the + node

9: Reg $\rightarrow$ REF(Reg_1) tiles the REF

So, What's Hard About This?

Consider tile 3 in our example



What rules match tile 3?

- 6: Reg $\rightarrow$ LAB_1 tiles the lower left node
- 8: Reg $\rightarrow$ NUM_1 tiles the bottom right node
- 15: Reg $\rightarrow$ + (Reg_1,Reg_2) tiles the + node
- 9: Reg $\rightarrow$ REF(Reg_1) tiles the REF

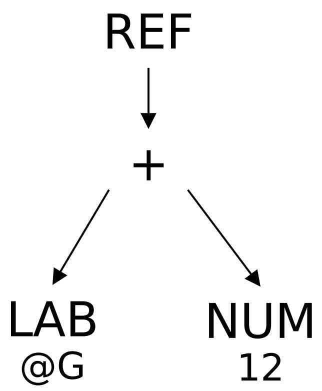
We denote this match as <6,8,15,9>

Of course, it implies <8,6,15,9>

Both have a cost of 13

Finding matches

Many Sequences Match Our Subtree



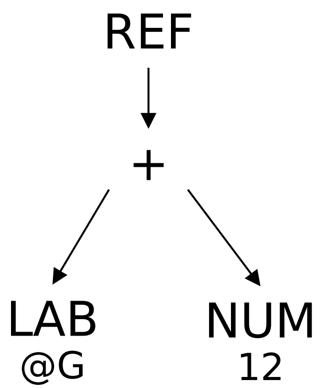
Cost	Sequences			
11	6,11	8,14		
12	6,8,10	8,6,10	6,16,9	8,19,9
13	6,8,15,9	8,6,15,9		

In general, we want the low cost sequence

- Each unit of cost is an operation (1 cycle)
- We should favour short sequences

Finding matches

Low Cost Matches



Sequences with Cost of 11	
6: Reg $\rightarrow$ LAB_1 11: Reg $\rightarrow$ REF(+(Reg_1,NUM_2))	`loadI  @G   ⇒ r`$_i$ `loadAI r`$_i$`,12 ⇒ r`$_j$
8: Reg $\rightarrow$ NUM_1 14: Reg $\rightarrow$ REF(+(LAB_1,Reg_2))	`loadI 12    ⇒ r`$_i$ `loadAI r`$_i$`,@G ⇒ r`$_j$

These two are equivalent in cost

6,11 might be better, because @G may be longer than the immediate field

Tiling the Tree

- Assume each rule implements one operator
- Assume operator takes 0, 1, or 2 operands

- Need to “massage” the rules, e.g.:

	Rule	$	Template
…	…	…	…
10	Reg → REF(+ (Reg_1,Reg_2))	10	`loadAO` $r_1, r_2 \Rightarrow r_{new}$
…	…	…	…

becomes:

	Rule	$	Template
…	…	…	…
10	Reg → REF(R10a)	10	`loadAO` $r_1, r_2 \Rightarrow r_{new}$
10a	R10a → + (Reg_1,Reg_2)	0	
…	…	…	…

Algorithm to tile the tree

```
Tile(n)
 Label(n) ← Ø

 if n is a leaf
   Label(n) = {n → n} // dummy rule
   Label(n) ++= all rules where rhs == n

 else if n is a unary node
   Tile(child(n))
   foreach rule r that matches n’s op
     if (child(r) in Label(child(n))
       Label(n) ++= r

 else // n is binary node
  Tile(left(n))
  Tile(right(n))
  foreach rule r that matches n’s op
    if (left (rhs(r)) in lhs(Label(left (n)) &
        right(rhs(r)) in lhs(Label(right(n))))
      Label(n) ++= r
```

Notes:

- child, left and right refer to the children of an AST node or of the rhs of a rule

Tiling the Tree

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```

	Rule	$	Template
…	…	…	…
6	Reg → LAB_1	1	loadI l_1 ⇒ r_{new}
7	Reg → VAL_1	0	
8	Reg → NUM_1	1	loadI n_1 ⇒ r_{new}
9	Reg → REF(Reg_1)	10	load r_1 ⇒ r_{new}
10	Reg → REF(R10a)	10	loadAO r_1, r_2 ⇒ r_{new}
10a	R10a → + (Reg_1,Reg_2)	0	
11	Reg → REF(R11a)	10	loadAI r_1, n_2 ⇒ r_{new}
11a	R11a → + (Reg_1,NUM_2)	0	
12	Reg → REF(R12a)	10	loadAI r_2,n_1 ῶ r_{new}
12a	R12a → + (NUM_1,Reg_2)	0	
13	Reg → REF(R13a)	10	loadAI r_1,l_2 ῶ r_{new}
13a	R13a → + (Reg_1,Lab_2)	0	
14	Reg → REF(R14a)	10	loadAI r_2,l_1 ῶ r_{new}
14a	R14a → + (Lab_1,Reg_2)	0	
15	Reg → + (Reg_1,Reg_2)	1	addI r_1, r_2 ⇒ r_{new}
16	Reg → + (Reg_1,NUM_2)	1	addI r_1, n_2 ⇒ r_{new}
17	Reg → + (NUM_1,Reg_2)	1	addI r_2, n_1 ⇒ r_{new}
18	Reg → + (Reg_1,Lab_2)	1	addI r_1, l_2 ⇒ r_{new}
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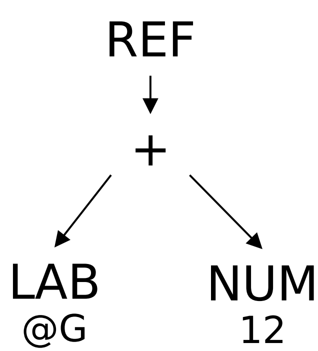
…	…	…	…



Label(Ref) =

Label(+) =

Label(Lab) =

Label(Num) =

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```

This algorithm

- Finds all matches in rule set
- Labels node n with that set
- Can keep lowest cost match at each point for each type of nodes → Dynamic programming
- Spends its time in the two matching loops

The Big Picture

- Tree patterns represent AST and ASM
- Can use matching algorithms to find low-cost tiling of AST
- Can turn a tiling into code using templates for matched rules
- Techniques (& tools) exist to do this efficiently

Hand-coded matcher	Lots of work
Encode matching as an automaton	O(1) cost per node Tools like BURS (bottom-up rewriting system), BURG
Use parsing techniques	Uses known technology Very ambiguous grammars
Linearize tree into string and use string searching algorithm (Aho-Corasick)	Finds all matches

Next Lecture

- Instruction scheduling