Compiler Design

An Introduction to Equality Saturation and Its Applications

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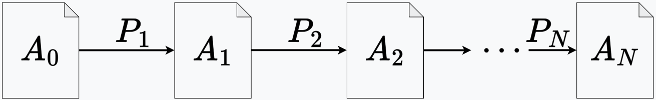
Table Of Contents

Pass Sequences

An optimizer performs a sequence of passes (a.k.a. transformations, rewrites) on the program to find the optimal version.



Optimization Passes

- Constant folding
- Dead code elimination
- Common subexpression elimination
- Loop unrolling
- Loop fusion
- Loop tilling
- Loop vectorization

Phase Ordering Problem

The order in which passes are applied affects the final program and its performance.

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- P_2. $x \cdot 2 \to x \ll 1$
- P_3. $xy \to yx$
- P_5. $x/x \to 1$
- P_6. $x \cdot 1 \to x$

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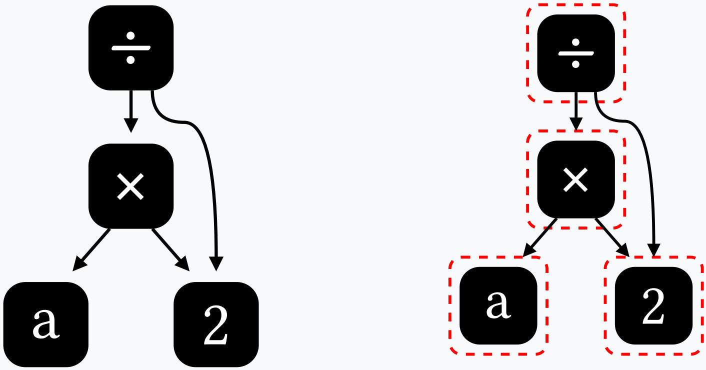
Problems:

1. The compiler gets stuck in a local optimum if it applies P_2 before P_1.
2. The passes are destructive: we lose the original and intermediate versions of the program.

Equivalence Graphs

We build an **e**-graph to store all defined equivalent versions of the program.

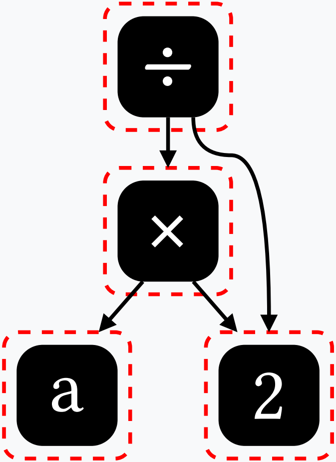


$$(a \cdot 2)/2$$

Equivalence Graphs

Store many equivalent versions of a program.

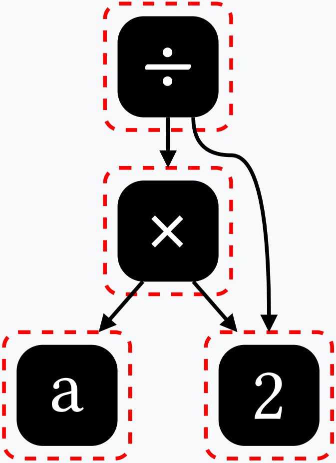
1. **e-node**: Operation node.
2. **e-class**: Set of equivalent e-nodes.
3. **e-graph**: Set of e-classes.



Equality Saturation

Process:

1. Build an **e**-graph of $(a \cdot 2)/2$.
2. Apply the rewrite rules to the e-graph until a fixed-point.
3. Extract the optimized expression via a cost model.



Equality Saturation

- P_1. $xy/z \to x\,(y/z)$
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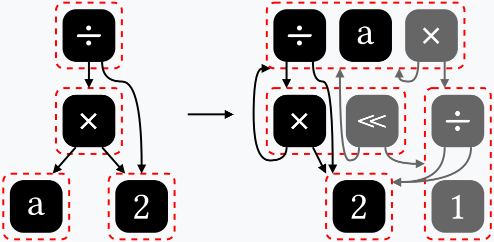
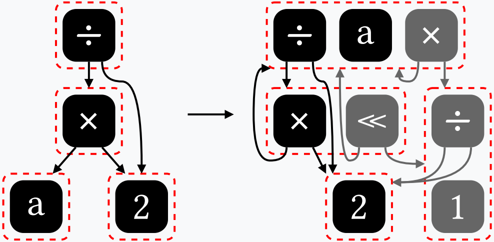


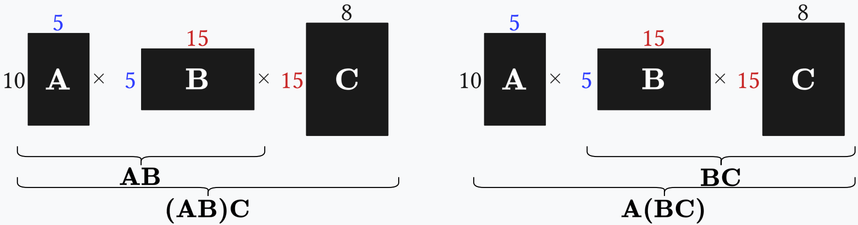
Equality Saturation



Problems are solved:

1. The global optimum is found by extracting the optimal expression via a cost model.
2. The original and intermediate versions of the program are preserved.

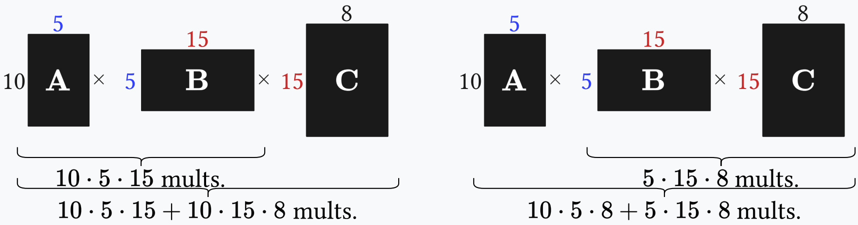
MatMul Associativity



$$\mathbf{(AB)\,C} = \mathbf{A\,(BC)}$$

How do we find out which one is more efficient?

The Order of Operations Matters

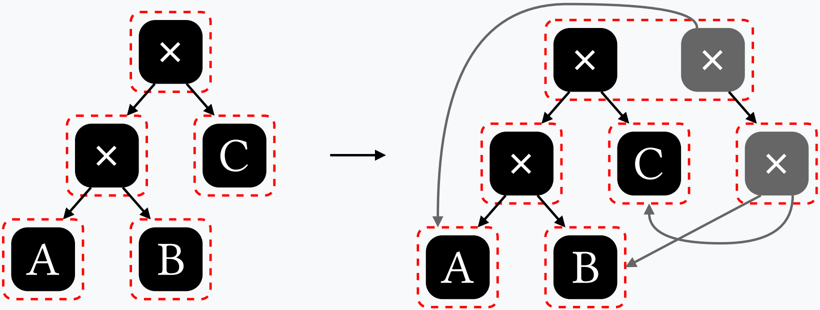


`cost` **(AB) C** = 1950 multiplications.

`cost` **A (BC)** = 1000 multiplications.

MatMul Associativity

$$\mathbf{(AB)\,C = A\,(BC)}$$



Polynomial Evaluation: Horner's Method

Reduce from $O(n^2)$ to $O(n)$ multiplications.

$$
\begin{aligned}
P(x) &= a_0 + a_1 x + a_2 x^2 + \cdots + a_{n-1} x^{n-1} + a_n x^n \\
&= a_0 + x(a_1 + x(a_2 + \cdots + x(a_{n-1} + x a_n)))
\end{aligned}
$$

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- Exponentiation: $\underline{x^0 \Rightarrow 1}$ and $\underline{x^n \Rightarrow x \cdot x^{n-1}}$

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- Exponentiation: $\underline{x^0 \Rightarrow 1}$ and $\underline{x^n \Rightarrow x \cdot x^{n-1}}$
- Commutativity: $\underline{x + y \Leftrightarrow y + x}$ and $\underline{x \cdot y \Leftrightarrow y \cdot x}$
- Associativity: $\underline{(x + y) + z \Leftrightarrow x + (y + z)}$ and $\underline{(x \cdot y) \cdot z \Leftrightarrow x \cdot (y \cdot z)}$
- Distributivity: $\underline{x \cdot (y + z) \Leftrightarrow x \cdot y + x \cdot z}$
- Identity: $\underline{x \cdot 1 \Rightarrow x}$

References

- Equality Saturation: a New Approach to Optimization. [Tate et al., 2009]
- DialEgg: Dialect-Agnostic MLIR Optimizer using Equality Saturation with Egglog. [Zayed and Dubach, 2025]
- Latent Idiom Recognition for a Minimalist Functional Array Language using Equality Saturation. [der Cruysse and Dubach, 2023]

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