Compiler Design

Lecture 19:
Instruction Selection via Tree-pattern matching

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(EaC-11.3)

The Concept

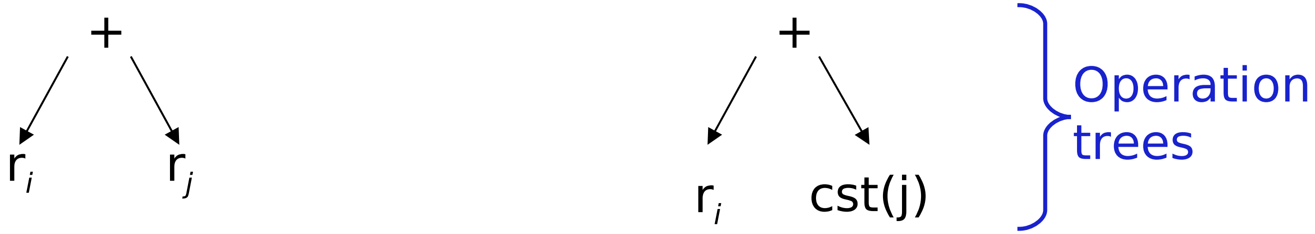
Many compilers use tree-structured IRs

- Abstract syntax trees generated in the parser
- Trees or DAGs for expressions

These systems might well use trees to represent target ISA

Consider the add operators

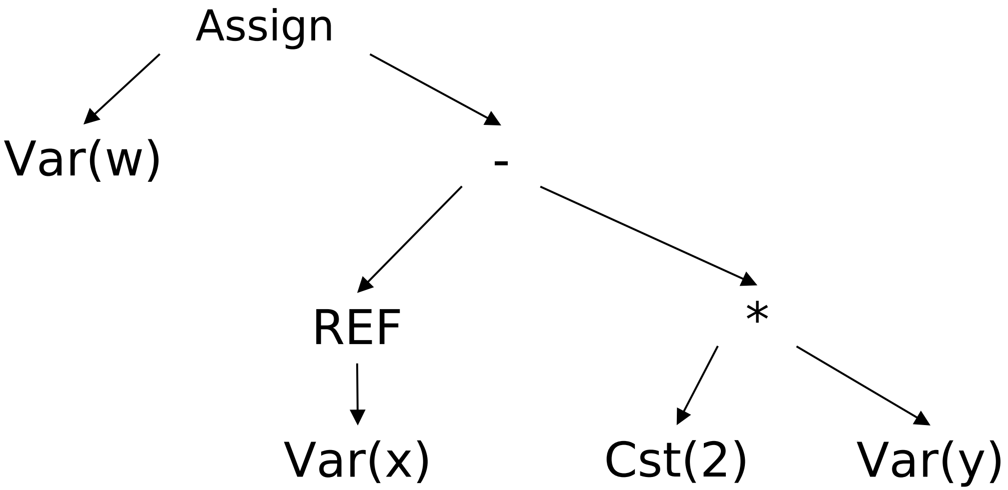
add $r_i, r_j \Rightarrow r_k$ addI $r_i, j \Rightarrow r_k$



What if we could match these “operation trees” against IR tree?

The Concept

AST for w ← (*x) - 2 * y



Assign

Var(w)

-

REF

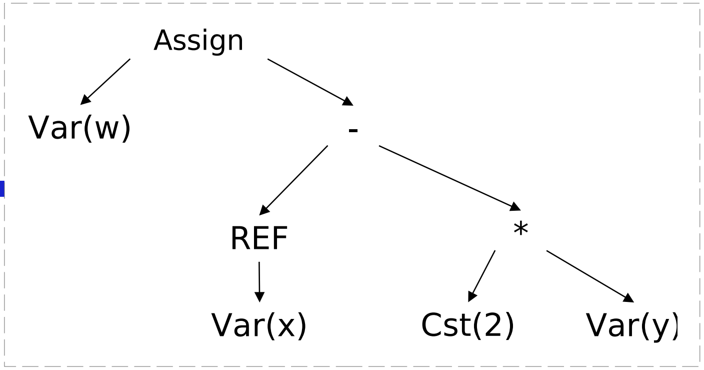
*

Var(x)

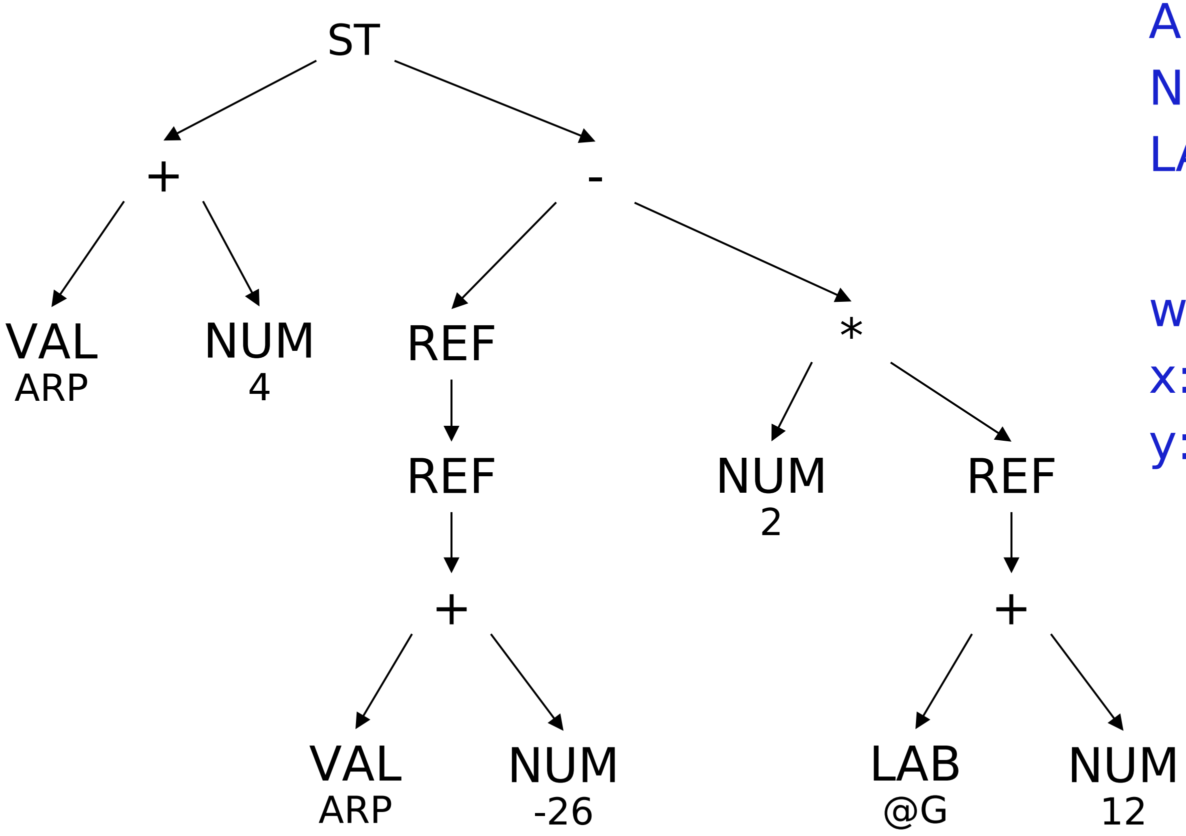
Cst(2)

Var(y)

The Concept



Low-level AST for w ← (*x) - 2 * y

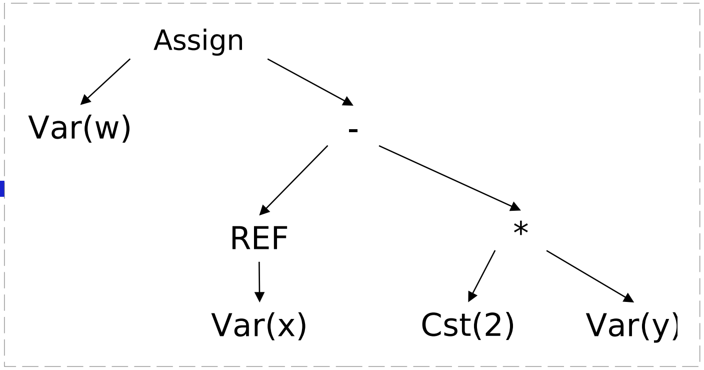


ARP: $fp
NUM: constant
LAB: ASM label

w: at ARP+4
x: at ARP-26
y: at @G+12

ARP = Activation Record Pointer = **frame pointer**

The Concept



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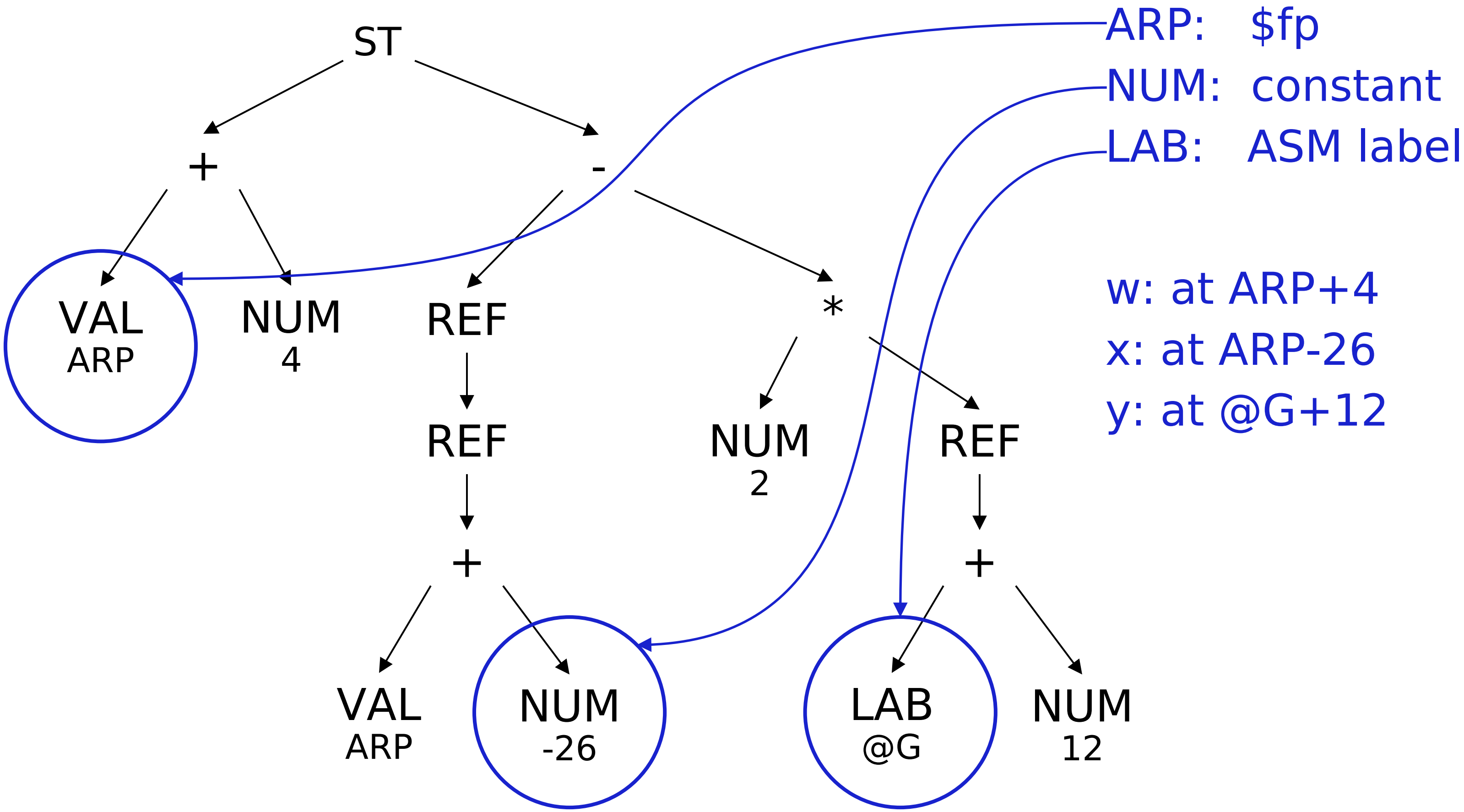

ARP: $fp
NUM: constant
LAB: ASM label

w: at ARP+4
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y: at @G+12

ARP = Activation Record Pointer = **frame pointer**

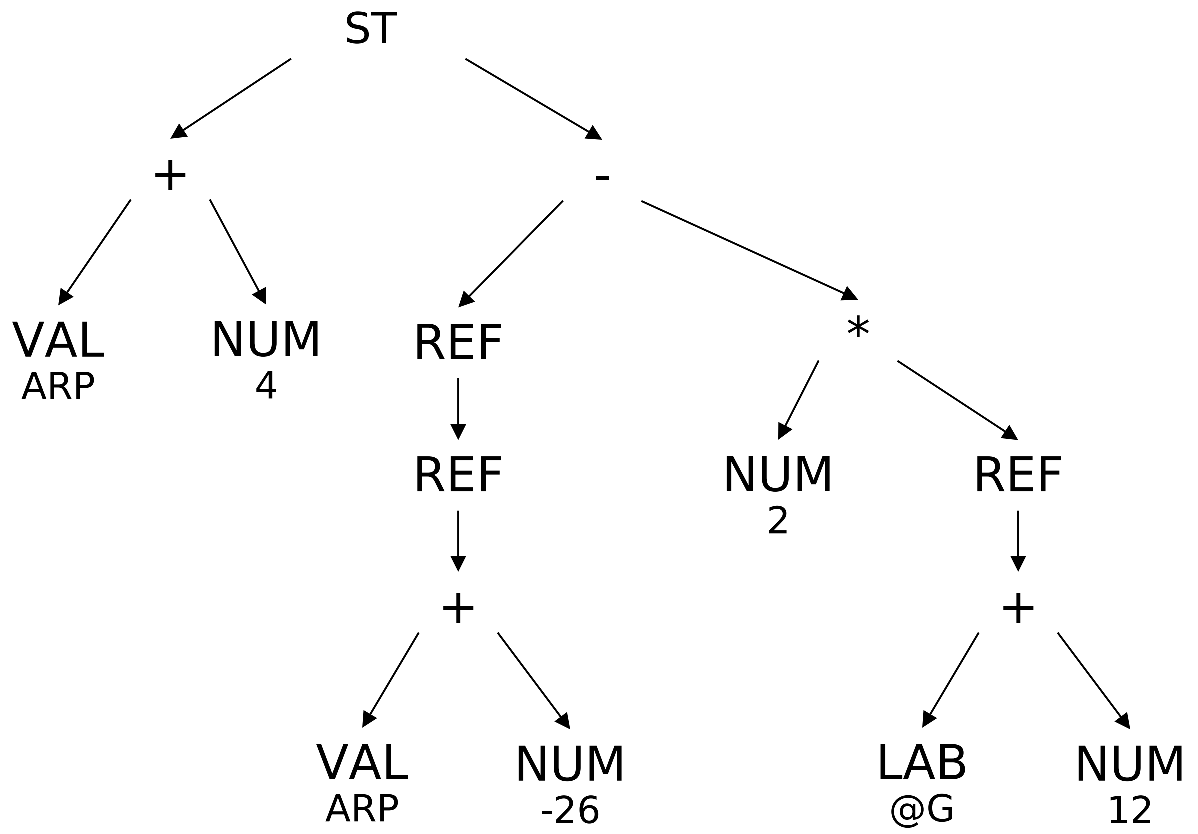
Tree-pattern matching

Goal is to “tile” AST with operation trees

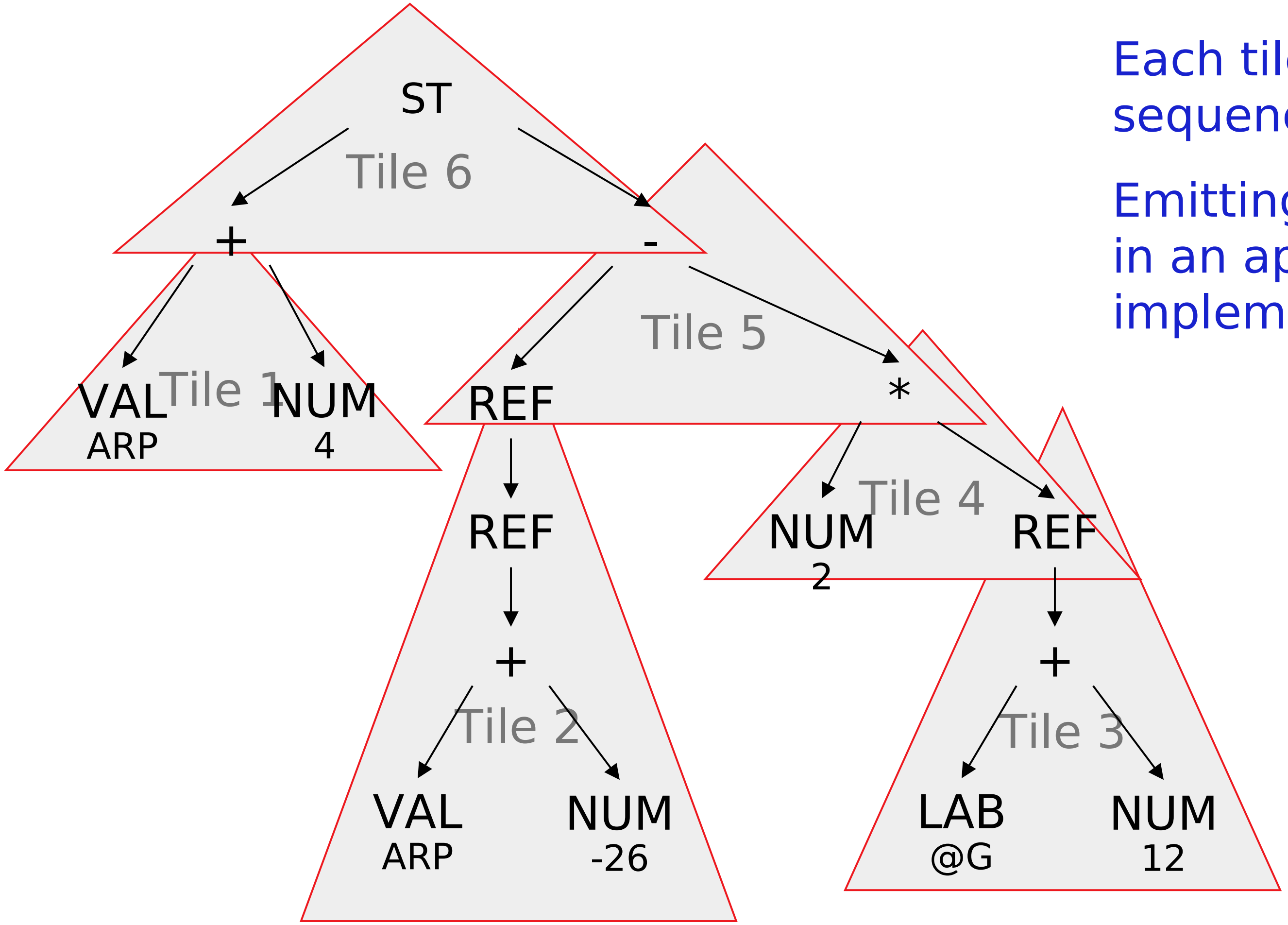
- A tiling is collection of <*ast*,*op* > pairs
  - → *ast* is a node in the low-level AST
  - → *op* is an operation tree
  - → <*ast*, *op* > means that *op* could implement the subtree at *ast*

- A tiling ‘implements” an AST if it covers every node in the AST and the overlap between any two trees is limited to a single node
  - → <*ast*, *op*> ∈ *tiling* means *ast* is also covered by a leaf in another operation tree in the tiling, unless it is the root
  - → Where two operation trees meet, they must be compatible (expect the value in the same location)

Tiling the Tree



Tiling the Tree



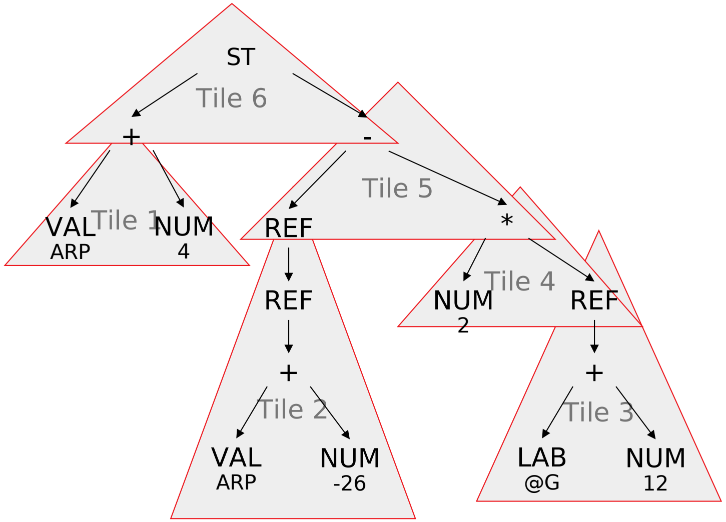
Each tile corresponds to a sequence of operations

Emitting those operations in an appropriate order implements the tree.

Generating Code

Given a tiled tree:

- Postorder treewalk, with node-dependent order for children
- Emit code sequence for tiles, in order
- Tie boundaries together with register names



→ Tile 6 uses registers produced by tiles 1 & 5

→ Tile 6 emits “`store` $r_{\texttt{tile 5}} \Rightarrow r_{\texttt{tile 1}}$”

→ Can incorporate a “real” register allocator or just use virtual registers

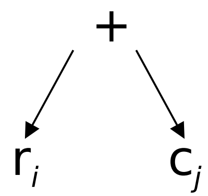
So, What’s Hard About This?

Finding the matches to tile the tree

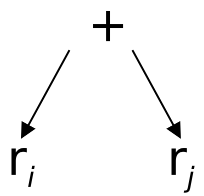
- Compiler writer connects operation trees to AST subtrees
    - → Encode tree syntax, in linear form
    - → Provides a set of rewrite rules
    - → Associated with each is a code template

Notation

To describe these trees, we need a concise notation



$+(r_i,c_j)$

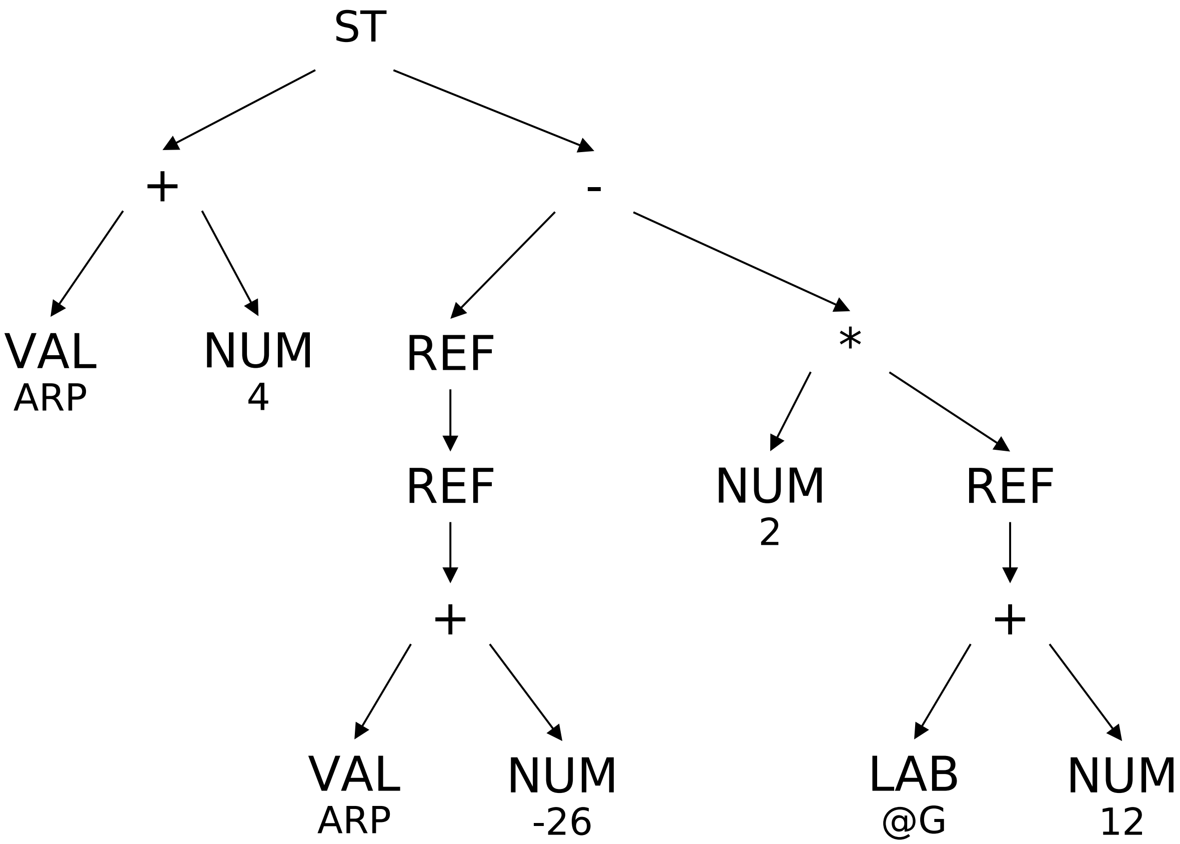


$+(r_i,r_j)$

Linear prefix form

Notation

To describe these trees, we need a concise notation



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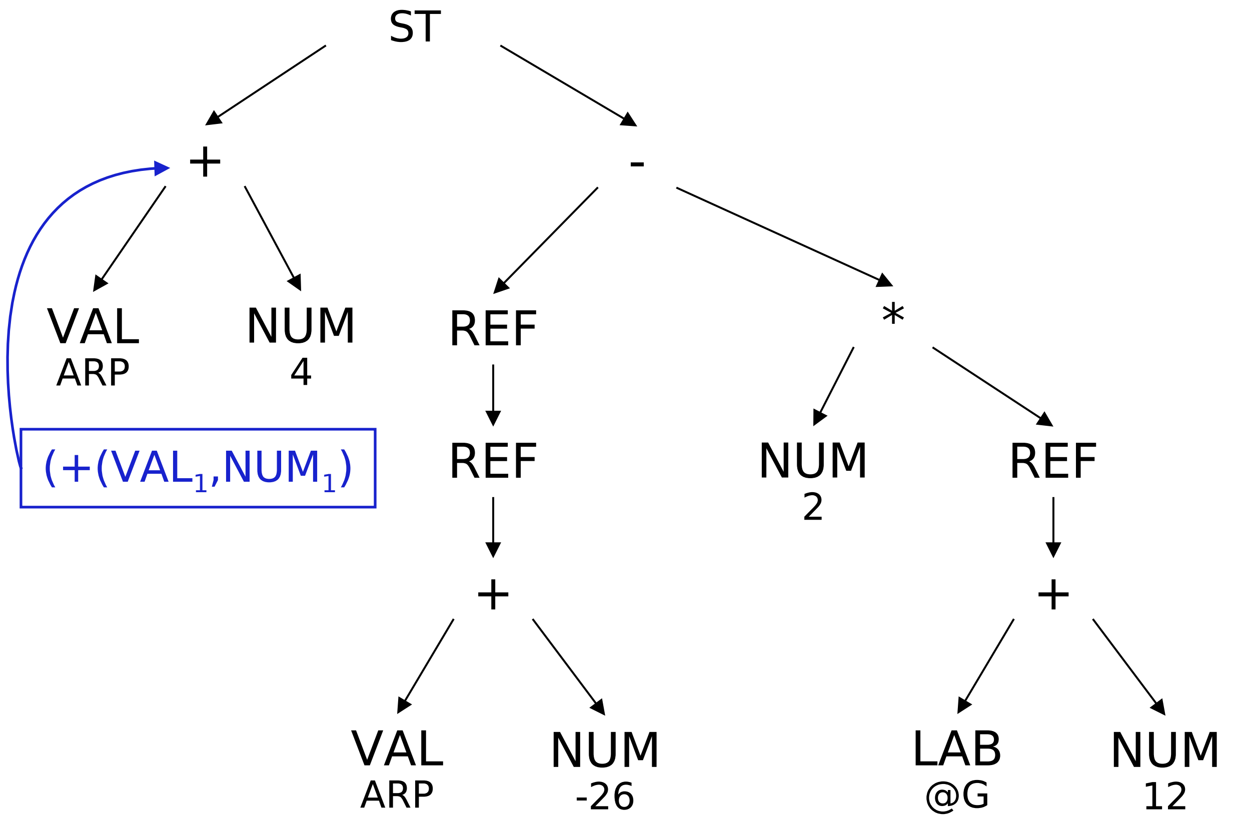

ST

\+ -

VAL ARP

NUM 4

REF

*

REF

NUM 2

REF

\+ +

VAL ARP

NUM -26

LAB @G

NUM 12

$(+(VAL_1,NUM_1)$

Notation

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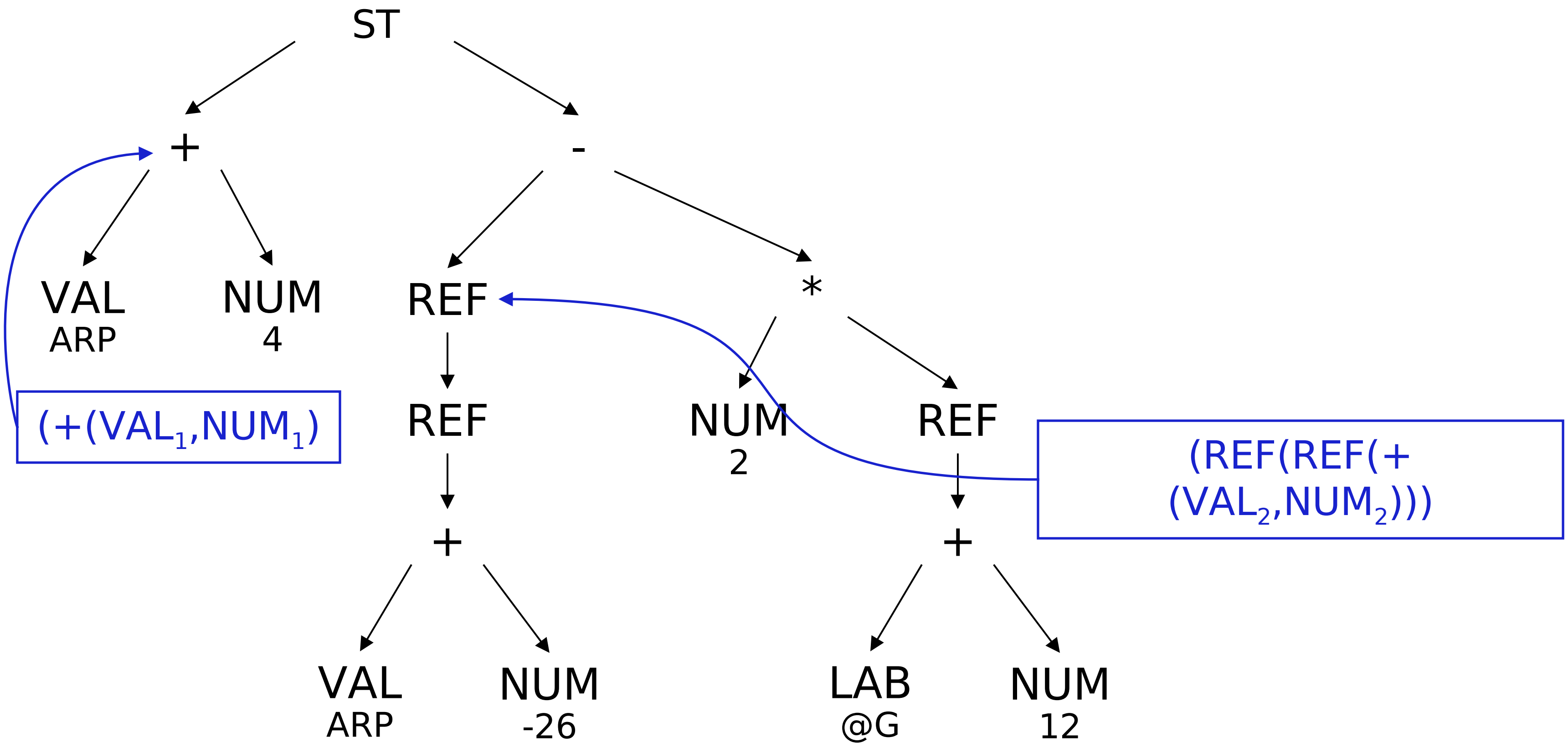
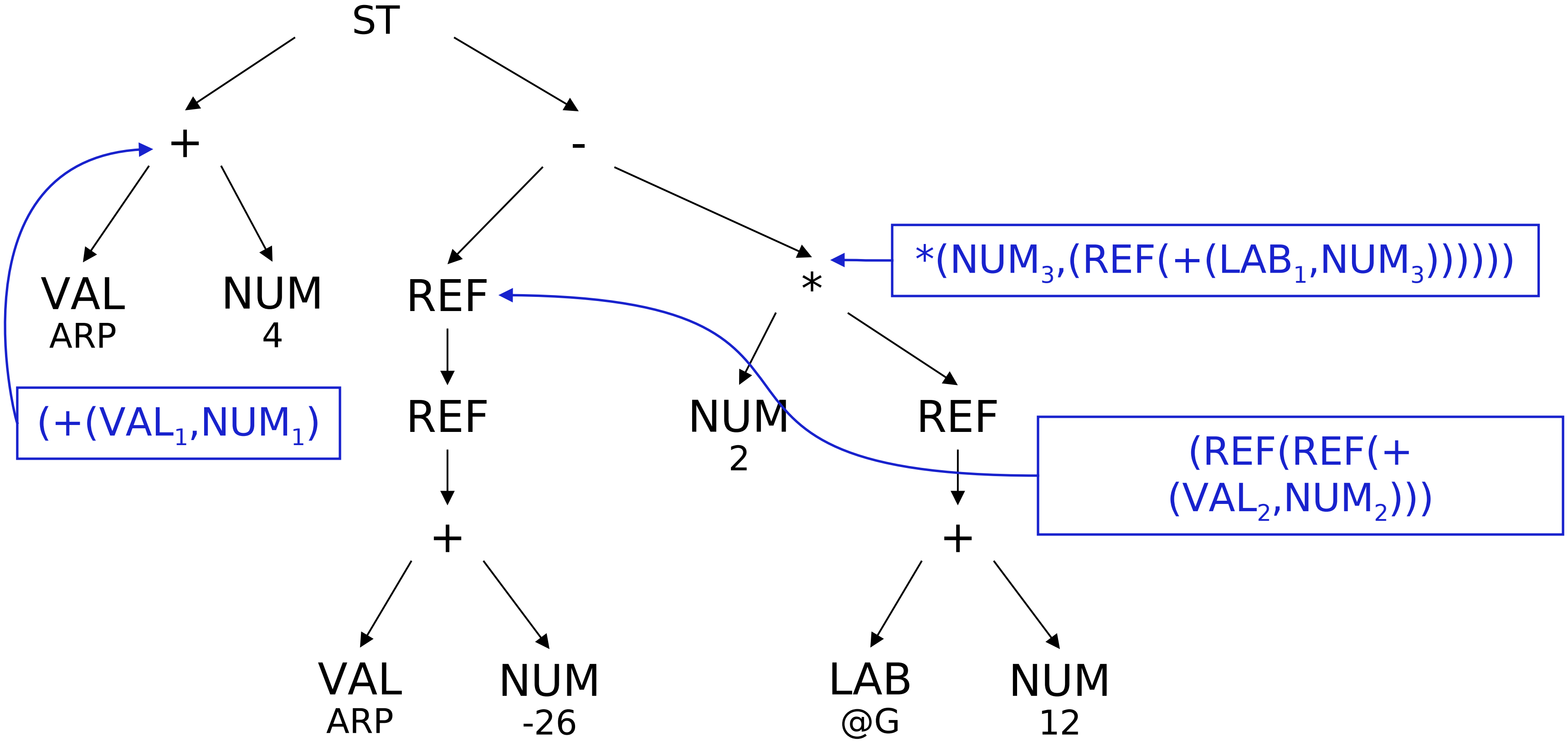


(+(VAL$_1$,NUM$_1$)

(REF(REF(+
(VAL$_2$,NUM$_2$)))

Notation

To describe these trees, we need a concise notation



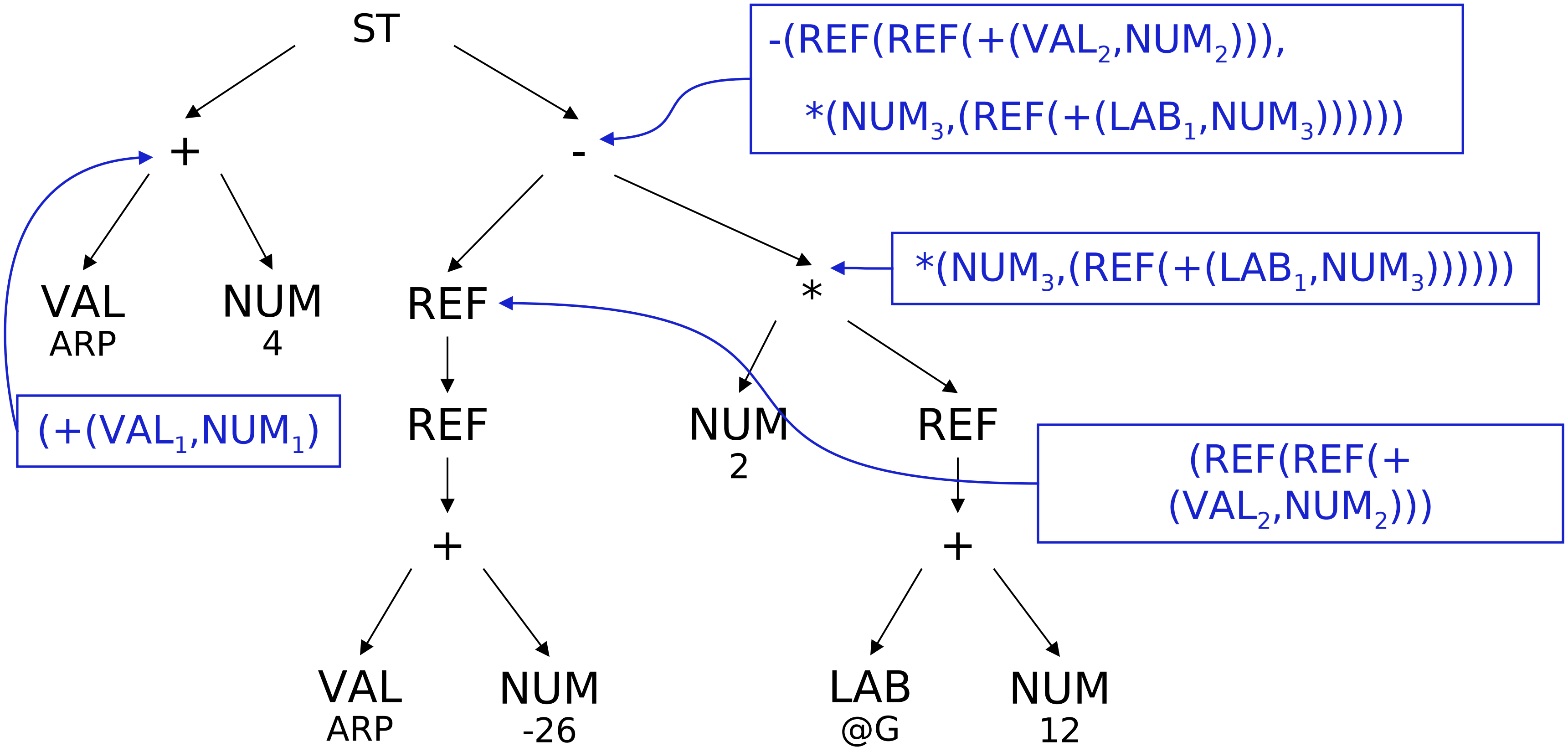
$*(NUM_3,(REF(+(LAB_1,NUM_3))))))$

$(+(VAL_1,NUM_1)$

$(REF(REF(+(VAL_2,NUM_2)))$

Notation

To describe these trees, we need a concise notation



$-(REF(REF(+(VAL_2,NUM_2))),$

$*(NUM_3,(REF(+(LAB_1,NUM_3))))))$

$*(NUM_3,(REF(+(LAB_1,NUM_3))))))$

$(+(VAL_1,NUM_1)$

$(REF(REF(+$
$(VAL_2,NUM_2)))$

Notation

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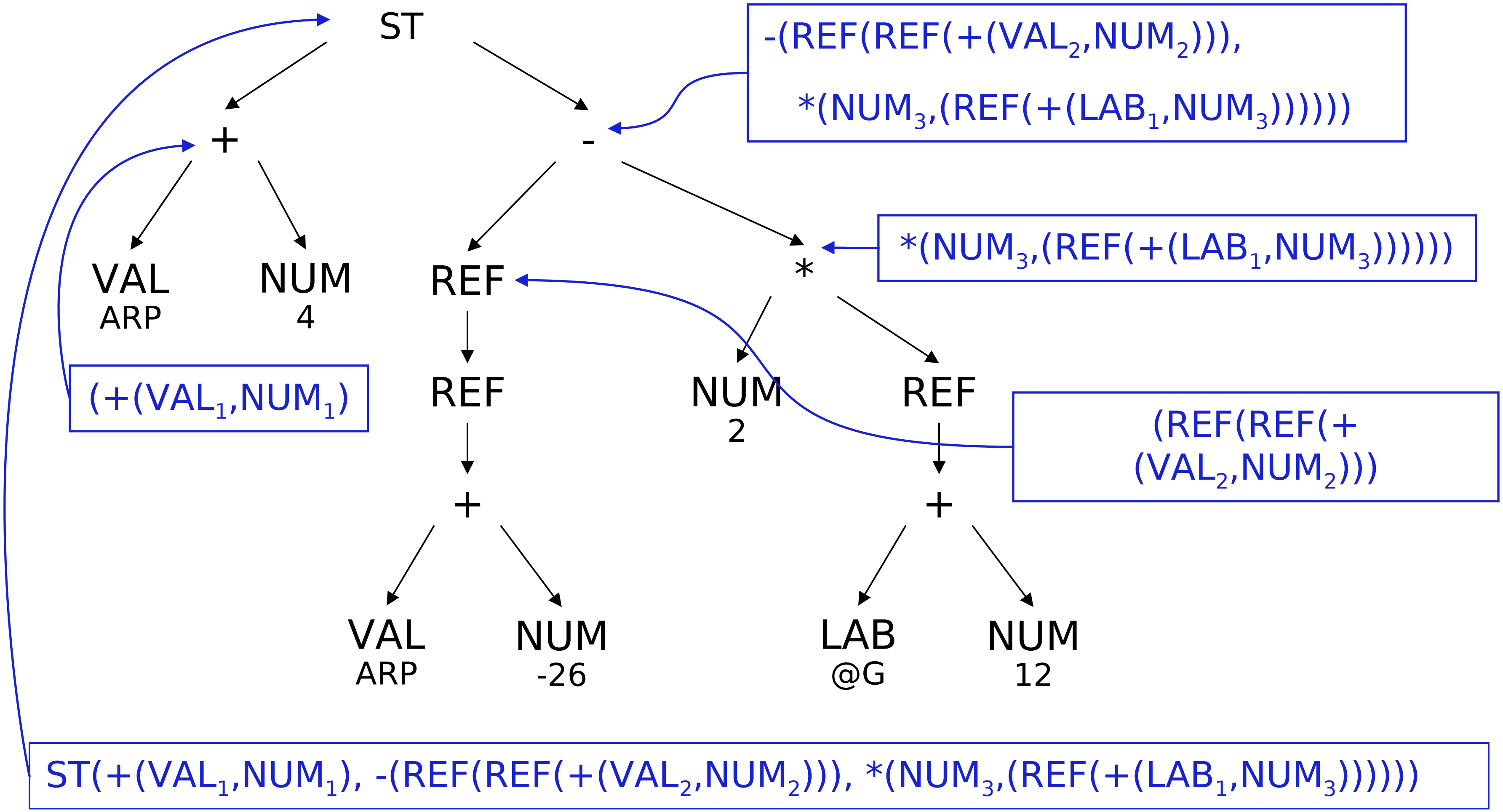


-(REF(REF(+(VAL$_2$,NUM$_2$))),
*(NUM$_3$,(REF(+(LAB$_1$,NUM$_3$))))))

*(NUM$_3$,(REF(+(LAB$_1$,NUM$_3$))))))

(+(VAL$_1$,NUM$_1$)

(REF(REF(+
(VAL$_2$,NUM$_2$)))

ST(+(VAL$_1$,NUM$_1$), -(REF(REF(+(VAL$_2$,NUM$_2$))), *(NUM$_3$,(REF(+(LAB$_1$,NUM$_3$))))))

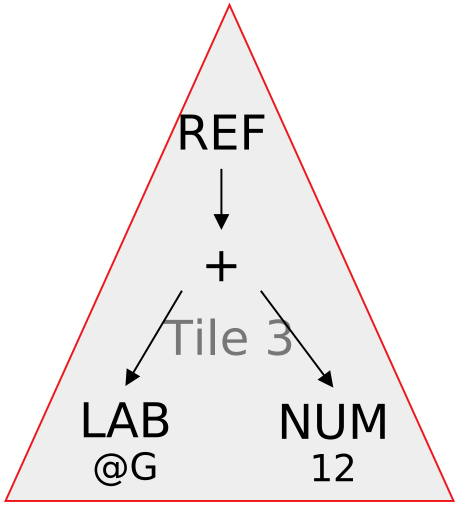
Rewrite rules: LL Integer AST into ILOC

	Rule	Cost	Template
1	Goal → Assign	0	
2	Assign → ST(Reg$_1$,Reg$_2$)	1	store r_2 ⇒ r_1
3	Assign → ST(+(Reg$_1$,Reg$_2$),Reg$_3$)	1	storeAO r_3 ⇒ r_1,r_2
4	Assign → ST(+(Reg$_1$,NUM$_2$),Reg$_3$)	1	storeAI r_3 ⇒ r_1,n_2
5	Assign → ST(+(NUM$_1$,Reg$_2$),Reg$_3$)	1	storeAI r_3 ⇒ r_2,n_1
6	Reg → LAB$_1$	1	loadI l_1 ⇒ r_{new}
7	Reg → VAL$_1$	0	
8	Reg → NUM$_1$	1	loadI n_1 ⇒ r_{new}
9	Reg → REF(Reg$_1$)	1	load r_1 ⇒ r_{new}
10	Reg → REF(+ (Reg$_1$,Reg$_2$))	1	loadAO r_1,r_2 ⇒ r_{new}
11	Reg → REF(+ (Reg$_1$,NUM$_2$))	1	loadAI r_1,n_2 ⇒ r_{new}
12	Reg → REF(+ (NUM$_1$,Reg$_2$))	1	loadAI r_2,n_1 ⇒ r_{new}
13	Reg → REF(+ (Reg$_1$,Lab$_2$))	1	loadAI r_1,l_2 ⇒ r_{new}
14	Reg → REF(+ (Lab$_1$,Reg$_2$))	1	loadAI r_2,l_1 ⇒ r_{new}
15	Reg → + (Reg$_1$,Reg$_2$)	1	addI r_1,r_2 ⇒ r_{new}
16	Reg → + (Reg$_1$,NUM$_2$)	1	addI r_1,n_2 ⇒ r_{new}
17	Reg → + (NUM$_1$,Reg$_2$)	1	addI r_2,n_1 ⇒ r_{new}
18	Reg → + (Reg$_1$,Lab$_2$)	1	addI r_1,l_2 ⇒ r_{new}
19	Reg → + (Lab$_1$,Reg$_2$)	1	addI r_2,l_1 ⇒ r_{new}
20	Reg → - (NUM$_1$,Reg$_2$)	1	rsubI r_2,n_1 ⇒ r_{new}
…	…	…	…

A real set of rules would cover more than signed integers …

So, What’s Hard About This?

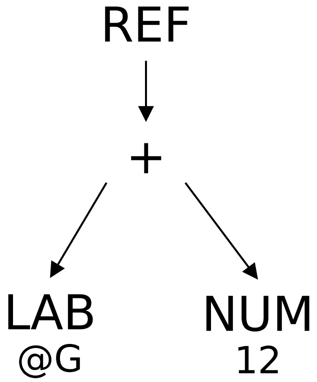
Consider tile 3 in our example



So, What’s Hard About This?

Consider tile 3 in our example

What rules match tile 3?



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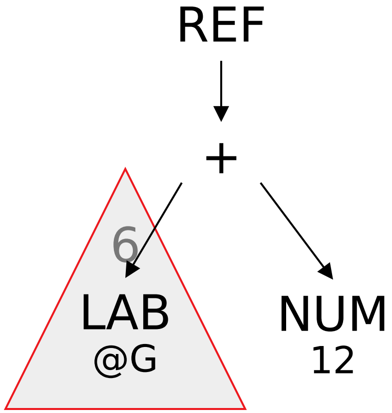

What rules match tile 3?

6: Reg → LAB_1 tiles the lower left node

So, What's Hard About This?

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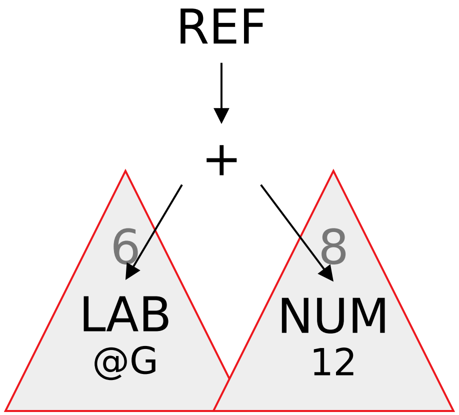


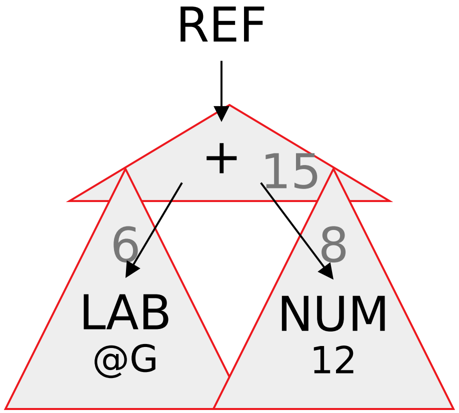
What rules match tile 3?

6: Reg → LAB$_1$ tiles the lower left node

8: Reg → NUM$_1$ tiles the bottom right node

So, What’s Hard About This?

Consider tile 3 in our example



What rules match tile 3?

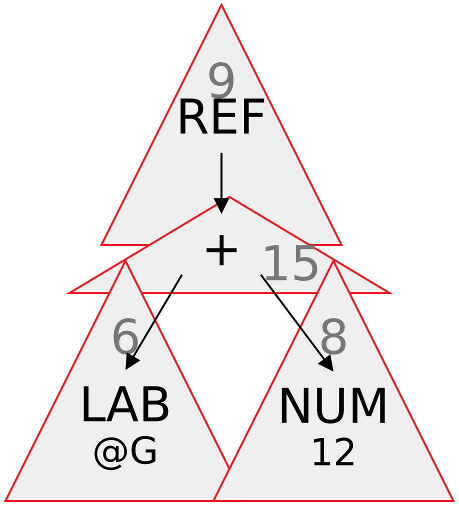
6: Reg → LAB_1 tiles the lower left node

8: Reg → NUM_1 tiles the bottom right node

15: Reg → + (Reg_1,Reg_2) tiles the + node

So, What’s Hard About This?

Consider tile 3 in our example



What rules match tile 3?

6: Reg → LAB_1 tiles the lower left node

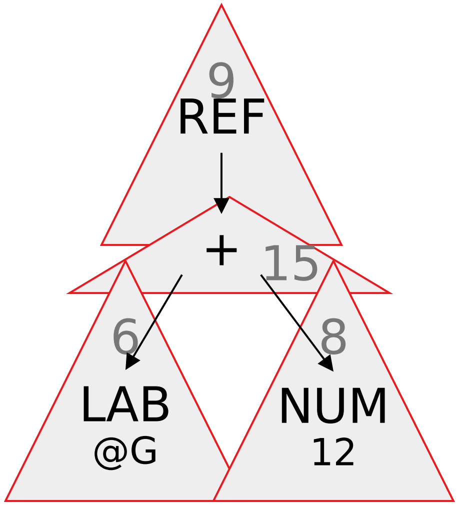
8: Reg → NUM_1 tiles the bottom right node

15: Reg → + (Reg_1,Reg_2) tiles the + node

9: Reg → REF(Reg_1) tiles the REF

So, What’s Hard About This?

Consider tile 3 in our example



What rules match tile 3?

6: Reg → LAB_1 tiles the lower left node

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9: Reg → REF(Reg_1) tiles the REF

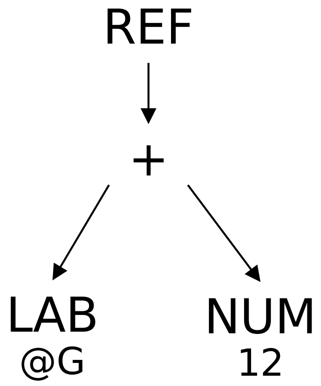
We denote this match as <6,8,15,9>

Of course, it implies <8,6,15,9>

Both have a cost of 4

Finding matches

Many Sequences Match Our Subtree



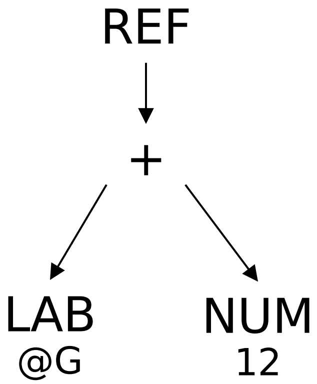
Cost	**Sequences**			
2	6,11	8,14		
3	6,8,10	8,6,10	6,16,9	8,19,9
4	6,8,15,9	8,6,15,9		

In general, we want the low cost sequence

- Each unit of cost is an operation (1 cycle)
- We should favour short sequences

Finding matches

Low Cost Matches



Sequences with Cost of 2	
6: Reg → LAB_1 11: Reg → $REF(+(Reg_1,NUM_2))$	loadI @G ⇒ r_i loadAI r_i,12 ⇒ r_j
8: Reg → NUM_1 14: Reg → $REF(+(LAB_1,Reg_2))$	loadI 12 ⇒ r_i loadAI r_i,@G ⇒ r_j

These two are equivalent in cost

6,11 might be better, because @G may be longer than the immediate field

Tiling the Tree

- Assume each rule implements one operator
- Assume operator takes 0, 1, or 2 operands

Now, …

Algorithm to tile the tree

```
Tile(n)
   Label(n) ← Ø
   if n has two children then
       Tile (left child of n)
       Tile (right child of n)
       for each rule r that implements n
           if (left(r) ∈ Label(left(n)) and
              (right(r) ∈ Label(right(n))
             then Label(n) ← Label(n) ∪ { r }
  else if n has one child
       Tile(child of n)
       for each rule r that implements n
           if (left(r) ∈ Label(child(n))
              then Label(n) ← Label(n) ∪ { r }
  else  /* n is a leaf */
       Label(n) ← {all  rules that implement n }
```

Match binary nodes against binary rules

Match unary nodes against unary rules

Handle leaves with lookup in rule table

Notes:
- left and right refer to the children of the AST node or left/right-hand sides of a rule
- implements: e.g. rule 9 implements REF

Tiling the Tree

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	Rule	**$**	**Template**
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3	Assign → ST(+(Reg$_1$,Reg$_2$),Reg$_3$)	1	storeAO $r_3 \Rightarrow r_1, r_2$
4	Assign → ST(+ (Reg$_1$,NUM$_2$),Reg$_3$)	1	storeAI $r_3 \Rightarrow r_1, n_2$
5	Assign → ST(+ (NUM$_1$,Reg$_2$),Reg$_3$)	1	storeAI $r_3 \Rightarrow r_2, n_1$
6	Reg → LAB$_1$	1	loadI $l_1 \Rightarrow r_{new}$
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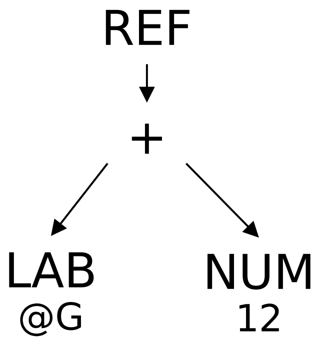

Label(Ref) =

Label(+) =

Label(Lab) =

Label(Num) =

Tiling the Tree

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```

This algorithm

- Finds all matches in rule set
- Labels node n with that set
- Can keep lowest cost match at each point for each type of nodes → Dynamic programming
- Spends its time in the two matching loops

The Big Picture

- Tree patterns represent AST and ASM
- Can use matching algorithms to find low-cost tiling of AST
- Can turn a tiling into code using templates for matched rules
- Techniques (& tools) exist to do this efficiently

Hand-coded matcher	Lots of work
Encode matching as an automaton	O(1) cost per node Tools like BURS (bottom-up rewriting system), BURG
Use parsing techniques	Uses known technology Very ambiguous grammars
Linearize tree into string and use string searching algorithm (Aho-Corasick)	Finds all matches

Next Lecture

- Instruction scheduling