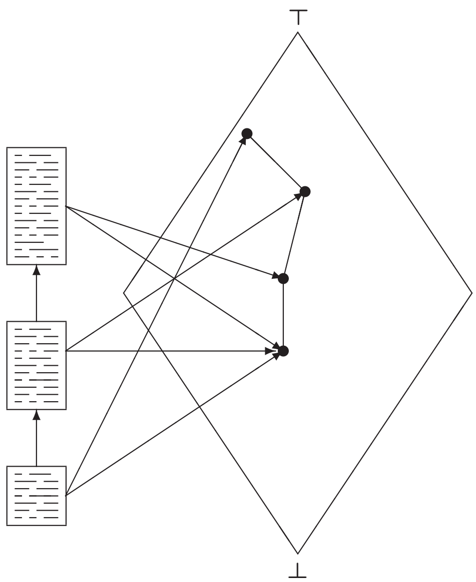
Static analysis



Static analysis determines interesting properties of programs to enable some optimizations.

All interesting properties are actually undecidable, so the analysis computes a conservative approximation:

- if we say *yes*, then the property definitely holds;
- if we say *no*, then the property may or may not hold;
- only the *yes* answer will help us to perform the optimization;
- a trivial analysis will say *no* always; so
- the art is to say *yes* as often as possible.

Properties need not be simply *yes* or *no*, in which case the notion of *approximation* is more subtle.

Static analysis may take place:

- at the source code level;
- at some intermediate level; or
- at the machine code level.

Static analysis may look at:

- basic blocks only;
- an entire function (intraprocedural); or
- the whole program (interprocedural).

In each case, we are maximally pessimistic at the boundaries.

The precision and cost of an analysis rises as we include more information.

Simple static analysis:

- is merely advanced weeding;
- uses symbol and type information; and
- is recursive in the program syntax.

An example is the *definite assignment* requirement in Java and JOOS:

- local variables must be assigned before they are read;
- this is undecidable; but
- the language specification dictates a specific conservative approximation.

For each program point, compute a set of local variables that:

- contains only variables that have definitely been assigned;
- may be too small, since the analysis is conservative; and
- depends on the set computed for the previous program point.

It accepts:

```
{ int k;
  if (flag) k = 3; else k = 4;
  System.out.println(k);
}
```

but rejects:

```
{ int k;
  if (flag) k = 3;
  if (!flag) k = 4;
  System.out.println(k);
}
```


JOOS code for statements:

```
ASNSET *defasnSTATEMENT(STATEMENT *s, ASNSET *before)
{ if (s!=NULL) {
     switch (s->kind) {
       case skipK:
            return before;
       case expK:
            return defasnEXP(s->val.expS,before);
       case returnK:
            if (s->val.returnS!=NULL)
               (void)defasnEXP(s->val.returnS,before);
            return setUniversal();
       case sequenceK:
            return
              defasnSTATEMENT(s->val.sequenceS.second,
                 defasnSTATEMENT(s->val.sequenceS.first,
                                 before)
              );
       case ifelseK:
            return
              setIntersect(
                defasnSTATEMENT(s->val.ifelseS.thenpart,
                  defasnEXPassume(s->val.ifelseS.condition,
                                  before,1)
                ),
                defasnSTATEMENT(s->val.ifelseS.elsepart,
                  defasnEXPassume(s->val.ifelseS.condition,
                                  before,0)
                )
              );
       case ...
  } }
```


To make the analysis more precise, it considers boolean expressions in more detail.

The procedure `defasnEXPassume(...,b)` assumes the expression evalutes to `b`.

This refinement handles a case like:

```
{ int k;
  if (a>0 && (k=b)>0) System.out.println(k);
}
```

which would otherwise be rejected.

In general, a static analysis becomes more precise when it may make further assumptions about the context.

The definite assignment analysis is particularly simple:

there are no recursive dependencies between the computed sets.

This allows a simple implementation:
a top-down traversal of the parse tree.

For more sophisticated analyses, we generate equations and compute the solution as a fixed point.

For the JIT compiler, we want to optimize the use of registers:

```
mov 1,R3      ==>   mov 1,R1
mov R3,R1
```

This requires knowledge about the future uses of registers:

The optimization is only sound if the value of `R3` is not used later on.

For basic block register allocation, which variables need to be written back to memory?

The naïve scheme:

- must write all those variables that are *only* in registers.

A better scheme:

- write all those variables that are only in registers *and* whose values might be used later on.

This could avoid many useless spills.

In both examples, we need to know if some $\mathtt{R}_i$ might be used later on. If so, it is called *live*; otherwise, it is called *dead*.

A static analysis can conservatively approximate liveness at each program point.

Exact liveness is of course undecidable.

A trivial analysis will call everything live, which precludes all optimizations.

A superior analysis will identify more dead variables.

Liveness analysis for VirtualRISC:

- build a control flow graph (goto graph);
- define dataflow equations for each node;
- compute the least solution of these equations.

For basic blocks the computation is trivial.

For intraprocedural analysis we must compute a minimal fixed point in a lattice.

Consider a simple basic block:

```
mov 3,R1
mov 4,R2
add R1,R2,R3
mov R3,R0
return
```

The <u>underlined</u> registers are written (defined), the others are merely read (used).

The control flow graph is:

```
S1:  mov 3,R1
       |
       v
S2:  mov 4,R2
       |
       v
S3:  add R1,R2,R3
       |
       v
S4:  mov R3,R0
       |
       v
S5:  return
```


Each instruction uses some registers and defines some registers:

	uses(S_i)	defines(S_i)
S1: mov 3,R1	{}	{R1}
S2: mov 4,R2	{}	{R2}
S3: add R1,R2,R3	{R1,R2}	{R3}
S4: mov R3,R0	{R3}	{R0}
S5: return	{R0}	{}

The register R0 is implicitly used for the return value.

Let out(S_i) be the variables that are live just *after* S_i and in(S_i) those that are live just *before* S_i:

```
          |  in(Si)
          v
Si:  op X,Y,Z
          |  out(Si)
          v
```

Then we have the dataflow equation:

$$\text{in}(S_i) = \text{uses}(S_i) \cup (\text{out}(S_i) - \text{defines}(S_i))$$

We add those registers that are used in the current instruction and delete those that are defined here.

Since out(S5) = {}, it follows that:

$$\text{in}(S5) = \text{uses}(S5) = \{R0\}$$

We can continue to unravel the equations:

$$\begin{aligned}
\text{out}(S4) &= \text{in}(S5) = \{R0\}\\
\text{in}(S4) &= \text{uses}(S4) \cup (\text{out}(S4) - \text{defines}(S4))\\
&= \{R3\} \cup (\{R0\} - \{R0\})\\
&= \{R3\}\\
\text{out}(S3) &= \text{in}(S4) = \{R3\}\\
\text{in}(S3) &= \text{uses}(S3) \cup (\text{out}(S3) - \text{defines}(S3))\\
&= \{R1,R2\} \cup (\{R3\} - \{R3\})\\
&= \{R1,R2\}
\end{aligned}$$

and so on:

	uses(S_i)	defines(S_i)	in(S_i)
S1: mov 3,R1	{}	{R1}	{}
S2: mov 4,R2	{}	{R2}	{R1}
S3: add R1,R2,R3	{R1,R2}	{R3}	{R1,R2}
S4: mov R3,R0	{R3}	{R0}	{R3}
S5: return	{R0}	{}	{R0}

In basic blocks we use the equation:

$$\text{out}(\mathtt{S}_i) = \text{in}(\mathtt{S}_{i+1})$$

If we have branches, then a node in the control flow graph may have several successors.

In this case, we must use the equation:

$$\text{out}(\mathtt{S}_i) = \bigcup_{x \,\in\, \text{succ}(\mathtt{S}_i)} \text{in}(x)$$

But now the equations are cyclic and cannot simply be unraveled.

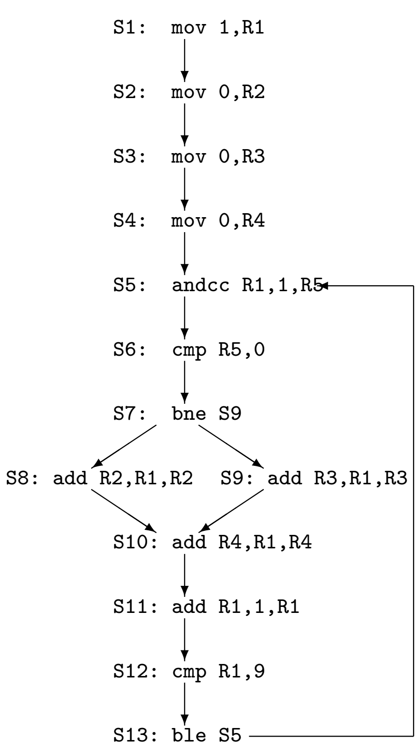
Consider the small piece of C code:

```
{ int i, sum_even, sum_odd, sum;
  i = 1;
  sum_even = 0;
  sum_odd = 0;
  sum = 0;
  while (i < 10)
   { if (i%2 == 0) sum_even = sum_even + i;
     else sum_odd = sum_odd + i;
     sum = sum + i;
     i++;
   }
}
```

It yields the following VirtualRISC code:

```
        mov 1,R1           // R1 is i
        mov 0,R2           // R2 is sum_even
        mov 0,R3           // R3 is sum_odd
        mov 0,R4           // R4 is sum
loop:
        andcc R1,1,R5     // R5 = R1 & 1
        cmp R5,0
        bne else          // if R5 != 0 goto else
        add R2,R1,R2      // R2 = R2 + R1; even case
        b endif
else:
        add R3,R1,R3      // R3 = R3 + R1; odd case
endif:
        add R4,R1,R4      // R4 = R4 + R1; update sum
        add R1,1,R1       // R1 = R1 + 1;  increment i
        cmp R1,9
        ble loop          // if i <= 9 goto loop
```


The control flow graph:



To unravel the liveness equations, we should start with:

$$\text{out}(\mathtt{S13}) = \text{in}(\mathtt{S5})$$

but we have not computed in(S5) yet, so this will not work!

If in(S1),…,in(S13) are known, then we can unravel the code as before and obtain the sets in(S1),…,in(S13) once again.

But this means that unraveling is a function:

$$f : \mathcal{P}(R)^{13} \rightarrow \mathcal{P}(R)^{13}$$

where $R = \{\mathtt{R1}, \mathtt{R2}, \ldots, \mathtt{R5}\}$. A solution is a fixed point, and we want the minimal one.

Two fundamental observations:

- the set $D = \mathcal{P}(R)^{13}$ is a finite *lattice*:

$$\forall x, y \in D : x \sqcap y \in D \ \wedge \ x \sqcup y \in D$$

  where $\sqsubseteq$ is point-wise set inclusion; and

- the unraveling function f is *monotonic*:

$$\forall x, y \in D : x \sqsubseteq y \Rightarrow f(x) \sqsubseteq f(y)$$

  since $g(x) = A \cup (x - B)$ is monotonic.

The fixed point theorem:

Any monotonic function f on a finite lattice D has the unique minimal fixed point:

$$\bigsqcup_i f^i(\bot)$$

which is always obtained after finitely many iterations.

For $D = \mathcal{P}(R)^{13}$ we have that:

$$\bot = (\emptyset, \emptyset, \ldots, \emptyset)$$

so we start with the sets in($\mathtt{S}_i$) = {} and keep unraveling until they no longer change.

Note that:

$$\top = (R, R, \ldots, R)$$

is always a safe answer, but clearly useless and pessimistic.

Observe that the maximal fixed-point:

$$\sqcap_i f^i(\top)$$

may in general be smaller than $\top$.

Computing the minimal fixed point:

	uses	defs	succ	$\bot$	$f(\bot)$	$f^2(\bot)$
S1		R1	S2	{}	{}	{}
S2		R2	S3	{}	{}	{}
S3		R3	S4	{}	{}	{}
S4		R4	S5	{}	{}	{R1}
S5	R1	R5	S6	{}	{R1}	{R1}
S6	R5		S7	{}	{R5}	{R5}
S7			S8,S9	{}	{}	{R1,R2,R3}
S8	R1,R2	R2	S10	{}	{R1,R2}	{R1,R2,R4}
S9	R1,R3	R3	S10	{}	{R1,R3}	{R1,R3,R4}
S10	R1,R4	R4	S11	{}	{R1,R4}	{R1,R4}
S11	R1	R1	S12	{}	{R1}	{R1}
S12	R1		S13	{}	{R1}	{R1}
S13			S5	{}	{}	{R1}

The function is:

$$f(X_1, X_2, \ldots, X_{13}) = (Y_1, Y_2, \ldots, Y_{13})$$

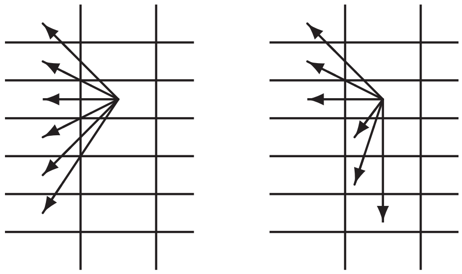
where:

$$Y_i = \text{uses}(\mathtt{S}_i) \cup \Big(\bigcup_{\mathtt{S}_j \in \text{succ}(\mathtt{S}_i)} X_j - \text{defs}(\mathtt{S}_i) \Big)$$

	$f^3(\bot)$	$f^4(\bot)$	$f^5(\bot)$
S1	{}	{}	{}
S2	{}	{R1}	{R1}
S3	{R1}	{R1}	{R1}
S4	{R1}	{R1}	{R1,R2,R3}
S5	{R1}	{R1,R2,R3}	{R1,R2,R3,R4}
S6	{R1,R2,R3,R5}	{R1,R2,R3,R4,R5}	{R1,R2,R3,R4,R5}
S7	{R1,R2,R3,R4}	{R1,R2,R3,R4}	{R1,R2,R3,R4}
S8	{R1,R2,R4}	{R1,R2,R4}	{R1,R2,R4}
S9	{R1,R3,R4}	{R1,R3,R4}	{R1,R3,R4}
S10	{R1,R4}	{R1,R4}	{R1,R4}
S11	{R1}	{R1}	{R1}
S12	{R1}	{R1}	{R1,R2,R3}
S13	{R1}	{R1,R2,R3}	{R1,R2,R3,R4}

	$f^6(\bot)$	$f^7(\bot)$	$f^8(\bot)$
S1	{}	{}	{}
S2	{R1}	{R1}	{R1}
S3	{R1,R2}	{R1,R2}	{R1,R2}
S4	{R1,R2,R3}	{R1,R2,R3}	{R1,R2,R3}
S5	{R1,R2,R3,R4}	{R1,R2,R3,R4}	{R1,R2,R3,R4}
S6	{R1,R2,R3,R4,R5}	{R1,R2,R3,R4,R5}	{R1,R2,R3,R4,R5}
S7	{R1,R2,R3,R4}	{R1,R2,R3,R4}	{R1,R2,R3,R4}
S8	{R1,R2,R4}	{R1,R2,R4}	{R1,R2,R3,R4}
S9	{R1,R3,R4}	{R1,R3,R4}	{R1,R2,R3,R4}
S10	{R1,R4}	{R1,R2,R3,R4}	{R1,R2,R3,R4}
S11	{R1,R2,R3}	{R1,R2,R3,R4}	{R1,R2,R3,R4}
S12	{R1,R2,R3,R4}	{R1,R2,R3,R4}	{R1,R2,R3,R4}
S13	{R1,R2,R3,R4}	{R1,R2,R3,R4}	{R1,R2,R3,R4}

A turbo fixed point technique:



The improved function is:

$$f_\Delta(X_1, X_2, \ldots, X_{13}) = (Y_1, Y_2, \ldots, Y_{13})$$

where:

$$Y_i = \text{uses}(\mathtt{S}_i) \cup \left(\bigcup_{\mathtt{S}_j \in \text{succ}(\mathtt{S}_i)} Z_j - \text{defs}(\mathtt{S}_i) \right)$$

$$Z_j = \begin{cases} Y_j & \text{if } j > i \\ X_j & \text{otherwise} \end{cases}$$

Improved fixed point computation:

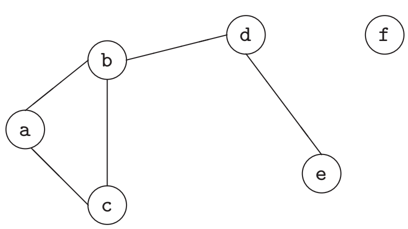
	$\perp$	$f_\Delta(\perp)$	$f_\Delta^2(\perp)$	$f_\Delta^3(\perp)$
S1	{}	{}	{}	{}
S2	{}	{}	{R1}	{R1}
S3	{}	{}	{R1,R2}	{R1,R2}
S4	{}	{}	{R1,R2,R3}	{R1,R2,R3}
S5	{}	{R1}	{R1,R2,R3,R4}	{R1,R2,R3,R4}
S6	{}	{R5}	{R1,R2,R3,R4,R5}	{R1,R2,R3,R4,R5}
S7	{}	{}	{R1,R2,R3,R4}	{R1,R2,R3,R4}
S8	{}	{R1,R2}	{R1,R2,R4}	{R1,R2,R3,R4}
S9	{}	{R1,R3}	{R1,R3,R4}	{R1,R2,R3,R4}
S10	{}	{R1,R4}	{R1,R4}	{R1,R2,R3,R4}
S11	{}	{R1}	{R1}	{R1,R2,R3,R4}
S12	{}	{R1}	{R1}	{R1,R2,R3,R4}
S13	{}	{}	{R1}	{R1,R2,R3,R4}

Number of iterations is down from 8 to 3.

Liveness analysis is used for register allocation in optimizing compilers.

In the basic block case, reduce spills to those variables that are only in registers *and* live.

In the intraprocedural case, construct a graph whose nodes are variables:



and where edges connect nodes that are live at the same time.

Register allocation is now reduced to finding a minimal graph coloring:

```
{ {a,d,f}, {b,e}, {c} }
```

and assigning a register to each color.

Liveness analysis is a *backwards* analysis, since we unravel from the future towards the past.

An example of a *forwards* analysis is constant propagation:

```
S1:  mov 3,R1         {(R0,?),(R1,?),(R2,?),(R3,?)}
       |
S2:  mov 4,R2         {(R0,?),(R1,3),(R2,?),(R3,?)}
       |
S3:  add R1,R2,R3     {(R0,?),(R1,3),(R2,4),(R3,?)}
       |
S4:  mov R3,R0        {(R0,?),(R1,3),(R2,4),(R3,7)}
       |
S5:  return           {(R0,7),(R1,3),(R2,4),(R3,7)}
```

A basic static analysis of JOOS and other object-oriented languages is *type inference*.

Given an expression, what are the possible classes of the objects to which it may evaluate?

The exact answer is undecidable, so we must conservatively approximate:

- we will accept a set that is too large;
- we want it as small as possible; and
- a trivial answer includes all classes.

This analysis is interprocedural and requires access to the whole program.

Possible uses of type inference:

- inline methods when there is only one possible receiver;
- eliminate run-time checks that can be decided statically;
- remove code that is never executed; and
- approximate the control flow graph to enable other static analyses.

In each case, smaller inferred sets will give better results.

The constraint technique:

- assign a variable $[\![\mathtt{E}]\!]$ to each occurrence of an expression `E`;
- assign a variable $[\![\mathtt{m}]\!]$ to each occurrence of a method `m`;
- the variables range over the set of all classes $C = \{\mathtt{C}_1, \mathtt{C}_2, \ldots, \mathtt{C}_n\}$;
- each parse tree node generates a local constraint on the variables; and
- the global minimal solution of these constraints is finally computed.

Again, we must compute a minimal fixed point in a finite lattice.

Each constraint models the flow of objects:

- the assignment "`i = E`" yields: $[\![\mathtt{E}]\!] \subseteq [\![\mathtt{i}]\!]$;
- the creation "`new C()`" yields: $\{\mathtt{C}\} \subseteq [\![\mathtt{new\ C()}]\!]$;
- the cast "`C(E)`" yields: $\{\mathtt{C}\} \subseteq [\![\mathtt{C(E)}]\!]$;
- the constant "`this`" yields: $\{\mathtt{C}\} \subseteq [\![\mathtt{this}]\!]$, where `C` is the surrounding class; and
- the statement "`return E`" yields: $[\![\mathtt{E}]\!] \subseteq [\![\mathtt{m}]\!]$, where `m` is the surrounding method.

The method invocation:

`E.m(E1,E2,...,Ek)`

yields the *conditional* constraints:

$$\mathtt{C}_i \in [\![\mathtt{E}]\!] \Rightarrow \begin{cases} [\![\mathtt{E}_1]\!] \subseteq [\![\mathtt{x}_1]\!] \\ [\![\mathtt{E}_2]\!] \subseteq [\![\mathtt{x}_2]\!] \\ \vdots \\ [\![\mathtt{E}_k]\!] \subseteq [\![\mathtt{x}_k]\!] \\ [\![\mathtt{m}]\!] \subseteq [\![\mathtt{E.m(E}_1\mathtt{,E}_2\mathtt{,...,E}_k\mathtt{)}]\!] \end{cases}$$

whenever the class $\mathtt{C}_i$ implements a method named `m` which accepts k arguments named $\mathtt{x}_1$, $\mathtt{x}_2$, ..., $\mathtt{x}_k$.

Since the constraint:

$$v \subseteq w$$

holds if and only if the equality:

$$w = v \cup w$$

does, we can rewrite a set of constraints into a function:

$$f : \mathcal{P}(C)^k \rightarrow \mathcal{P}(C)^k$$

such that fixed-points of f correspond to solutions to the constraints.

For the example constraints:

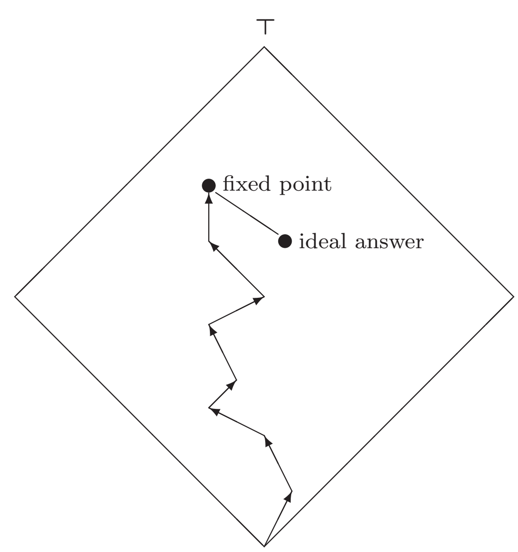
$$v_1 \subseteq v_2$$
$$\mathtt{C}_3 \in v_2 \Rightarrow v_3 \subseteq v_1$$
$$\{\mathrm{C}_7\} \subseteq v_3$$

we get the function:

$$f(X_1, X_2, X_3) = \begin{cases} (X_1 \cup X_3, X_1 \cup X_2, \{\mathrm{C}_7\} \cup X_3) & \text{if } \mathtt{C}_3 \in X_2 \\ (X_1, X_1 \cup X_2, \{\mathrm{C}_7\} \cup X_3) & \text{otherwise} \end{cases}$$

Solving the constraints:

- $\mathcal{P}(C)^k$ is a finite lattice;
- each function f is monotonic; and
- the least fixed point of f is the unique smallest solution of the constraints.



A tiny JOOS sketch:

```
public class A {
  public A() { super(); }
  public A id(A x) { return x; }
}

public class B extends A {
  public B() { super(); }
  public B me() { return (B)(new A()).id(this); }
}
```

The generated constraints are:

$[\![\texttt{x}]\!]_\texttt{A} \subseteq [\![\texttt{id}]\!]_\texttt{A}$
$[\![\texttt{x}]\!]_\texttt{B} \subseteq [\![\texttt{id}]\!]_\texttt{B}$
$[\![\texttt{(B)(new A()).id(this)}]\!] \subseteq [\![\texttt{me}]\!]$
$\{\texttt{B}\} \subseteq [\![\texttt{(B)(new A()).id(this)}]\!]$
$\{\texttt{B}\} \subseteq [\![\texttt{this}]\!]$
$\{\texttt{A}\} \subseteq [\![\texttt{new A()}]\!]$
$\texttt{A} \in [\![\texttt{new A()}]\!] \Rightarrow [\![\texttt{this}]\!] \subseteq [\![\texttt{x}]\!]_\texttt{A}$
$\texttt{A} \in [\![\texttt{new A()}]\!] \Rightarrow [\![\texttt{id}]\!]_\texttt{A} \subseteq [\![\texttt{(new A()).id(this)}]\!]$
$\texttt{B} \in [\![\texttt{new A()}]\!] \Rightarrow [\![\texttt{this}]\!] \subseteq [\![\texttt{x}]\!]_\texttt{B}$
$\texttt{B} \in [\![\texttt{new A()}]\!] \Rightarrow [\![\texttt{id}]\!]_\texttt{B} \subseteq [\![\texttt{(new A()).id(this)}]\!]$

The minimal solution is:

$[\![\texttt{new A()}]\!] = \{\texttt{A}\}$
$[\![\texttt{x}]\!]_\texttt{A} = [\![\texttt{id}]\!]_\texttt{A} = [\![\texttt{this}]\!] = [\![\texttt{(new A()).id(this)}]\!] = \{\texttt{B}\}$
$[\![\texttt{(B)(new A()).id(this)}]\!] = \{\texttt{B}\}$
$[\![\texttt{x}]\!]_\texttt{B} = [\![\texttt{id}]\!]_\texttt{B} = \{\}$

The generated code for the `me` method is:

```
.method public me()LB;
  .limit locals 1
  .limit stack 2
  new A
  dup
  invokenonvirtual A/<init>()V
  aload_0
  invokevirtual A/id(LA;)LA;
  checkcast B
  areturn
.end method
```

The information $[\![\texttt{new A().id(this)}]\!] = \{\texttt{B}\}$ eliminates the `checkcast` instruction.

That $[\![\texttt{new A()}]\!] = \{\texttt{A}\}$ is a singleton further allows inlining of the `id` method:

```
.method public me()LB;
  .limit locals 1
  .limit stack 1
  aload_0
  areturn
.end method
```

With type inference, many little methods become almost free.

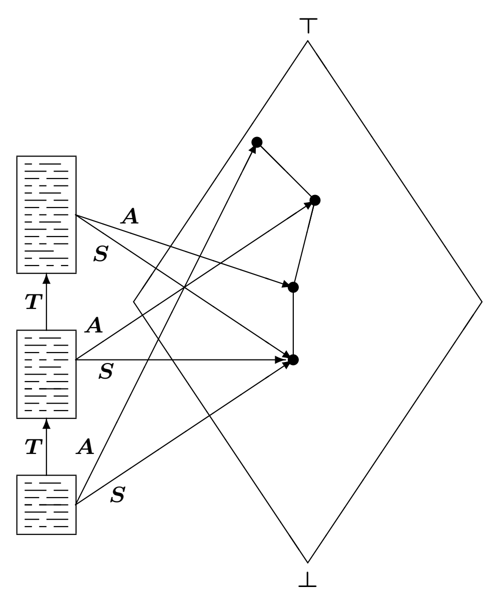
Improving analyses by transformations:

- let P be our set of programs;
- let $S : P \rightarrow D$ be an ideal static analysis (uncomputable); and
- let $T : P \rightarrow P$ be a program transformation that preserves the semantics.

Since S gives the ideal information, clearly $S(T(p)) = S(p)$ for all $p \in P$.

However, if $A : P \rightarrow D$ is a conservative approximation to S, then $A(T(p))$ may be different from $A(p)$, perhaps even better.

Transformations boost analyses:



The transformation T:

- may unfold the program to make it more explicit; or
- may itself be an optimization.