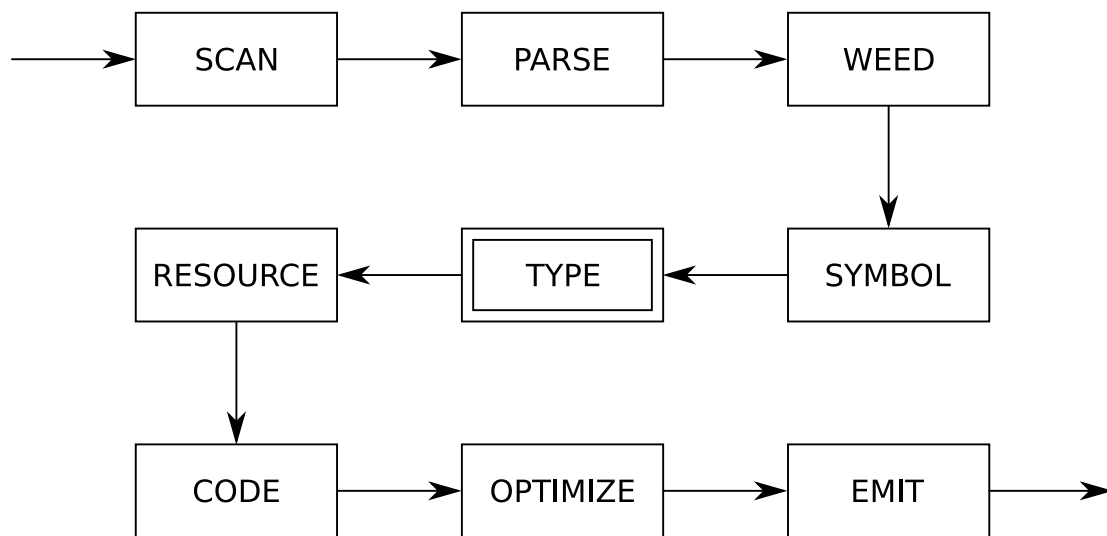


Type checking



The *type checker* has several tasks:

- determine the types of all expressions;
- check that values and variables are used correctly; and
- resolve certain ambiguities by transforming the program.

Some languages have no type checker.

A *type* describes possible values.

The JOOS types are:

- `void`: the empty type;
- `int`: the integers;
- `char`: the characters;
- `boolean`: `true` and `false`; and
- `C`: objects of class `C` or any subclass.

Plus an artificial type:

- `polynull`

which is the type of the polymorphic `null` constant.

A type annotation:

```
int x;
```

```
Cons y;
```

specifies an *invariant* about the run-time behavior:

- `x` will always contain an integer value; and
- `y` will always contain `null` or an object of type `Cons` or any subclass.

Usual type annotations are not very expressive as invariants.

You can have types without annotations, through type inference (e.g. in ML).

Types can be arbitrarily complex in theory.

A program is *type correct* if the type annotations are valid invariants.

Type correctness is undecidable:

```
int x;  
int j;  
  
x = 0;  
scanf("%i",&j);  
TM(j);  
x = true;
```

where $TM(j)$ simulates the j 'th Turing machine on empty input.

The program is type correct if and only if $TM(j)$ does not halt on empty input.

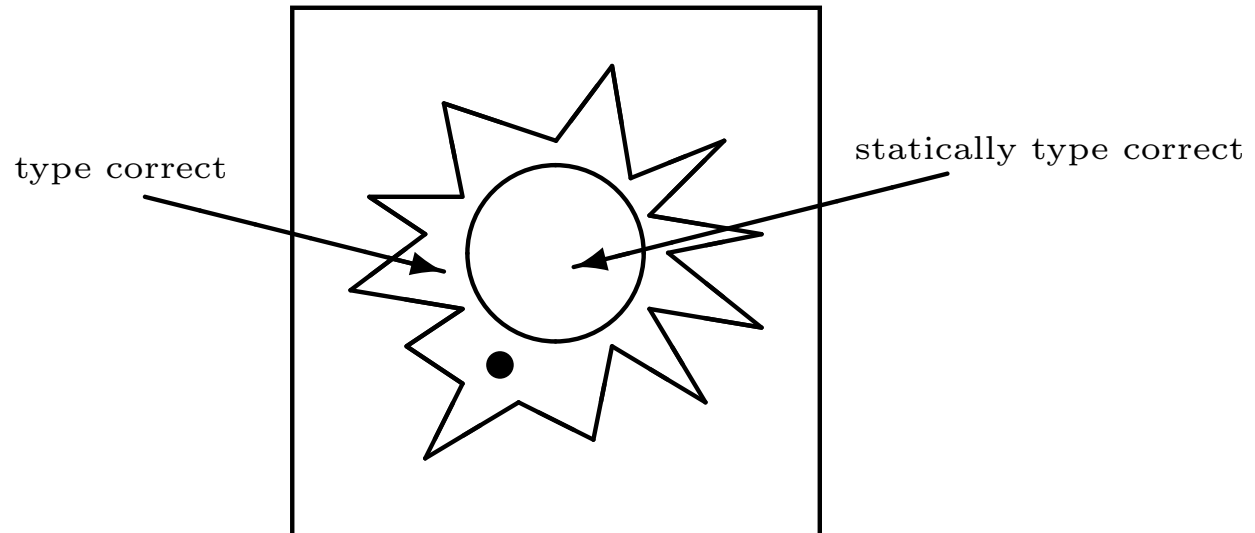
A program is *statically* type correct if it satisfies some type rules.

The type rules are chosen to be:

- simple to understand;
- efficient to decide; and
- conservative with respect to type correctness.

Type rules are rarely canonical.

Static type systems are necessarily flawed:



There is always *slack*, i.e. programs that are unfairly rejected by the type checker. Some are even quite useful.

Can you think of such a program?

Type rules may be specified:

- in ordinary prose:

The argument to the `sqrt` function must be of type `int`; the result is of type `real`.

- as constraints on type variables:

$$\text{sqrt}(x) : \llbracket \text{sqrt}(x) \rrbracket = \text{real} \wedge \llbracket x \rrbracket = \text{int}$$

- as logical rules:

$$\frac{\mathcal{S} \vdash x : \text{int}}{\mathcal{S} \vdash \text{sqrt}(x) : \text{real}}$$

There are always three kinds:

1. declarations: introduction of variables;
2. propagations: expression type determines enclosing expression type; and
3. restrictions: expression type constrained by usage context

The judgement for statements:

$$L, C, M, V \vdash S$$

means that S is statically type correct with:

- class library L ;
- current class C ;
- current method M ; and
- variables V .

The judgement for expressions:

$$L, C, M, V \vdash E : \tau$$

means that E is statically type correct and has type τ .

The tuple L, C, M, V is an abstraction of the symbol table.

Type rules for statement sequence:

$$\frac{L, C, M, V \vdash S_1 \quad L, C, M, V \vdash S_2}{L, C, M, V \vdash S_1 S_2}$$

$$\frac{L, C, M, V[x \mapsto \tau] \vdash S}{L, C, M, V \vdash \tau \ x; S}$$

$V[x \mapsto \tau]$ just says x maps to τ within V .

Corresponding JOOS source:

```

case sequenceK:
    typeImplementationSTATEMENT(s->val.sequenceS.first,
                                class,returntype);
    typeImplementationSTATEMENT(s->val.sequenceS.second,
                                class,returntype);
    break;
.
.
.

case localK:
    break;

```

Type rules for return statements:

$$\frac{\begin{array}{c} \text{type}(L, C, M) = \text{void} \\ L, C, M, V \vdash \text{return} \\ L, C, M, V \vdash E : \tau \quad \text{type}(L, C, M) = \sigma \quad \sigma := \tau \end{array}}{L, C, M, V \vdash \text{return } E}$$

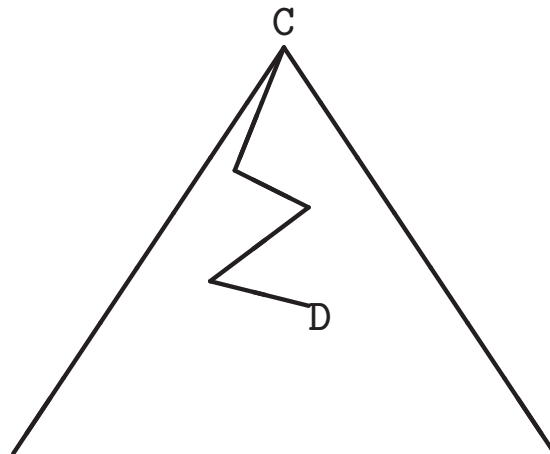
$\sigma := \tau$ just says something of type σ can be assigned something of type τ .

Corresponding JOOS source:

```
case returnK:
  if (s->val.returnS!=NULL) {
    typeImplementationEXP(s->val.returnS,class);
  }
  if (returntype->kind==voidK && s->val.returnS!=NULL) {
    reportError("return value not allowed",s->lineno);
  }
  if (returntype->kind!=voidK && s->val.returnS==NULL) {
    reportError("return value expected",s->lineno);
  }
  if (returntype->kind!=voidK && s->val.returnS!=NULL) {
    if (!assignTYPE(returntype,s->val.returnS->type)) {
      reportError("illegal type of expression",
        s->lineno);
    }
  }
}
break;
```

Assignment compatibility:

- `int:=int;`
- `int:=char;`
- `char:=char;`
- `boolean:=boolean;`
- `C:=polynull;` and
- `C:=D`, if $D \leq C$.



Corresponding JOOS source:

```
int assignTYPE(TYPE *s, TYPE *t)
{ if (s->kind==refK && t->kind==polynullK) return 1;
  if (s->kind==intK && t->kind==charK) return 1;
  if (s->kind!=t->kind) return 0;
  if (s->kind==refK) return subclass(t->class,s->class);
  return 1;
}
```


Type rule for variables:

$$\frac{V(x) = \tau}{L, C, M, V \vdash x : \tau}$$

Corresponding JOOS source:

```
case idK:
    e->type = typeVar(e->val.idE.idsym);
    break;
```

Type rule for assignment:

$$\frac{L, C, M, V \vdash x : \tau \quad L, C, M, V \vdash E : \sigma \quad \tau := \sigma}{L, C, M, V \vdash x = E : \tau}$$

Corresponding JOOS source:

```
case assignK:
    e->type = typeVar(e->val.assignE.leftsym);
    typeImplementationEXP(e->val.assignE.right, class);
    if (!assignTYPE(e->type, e->val.assignE.right->type)) {
        reportError("illegal assignment", e->lineno);
    }
    break;
```

Type rule for minus:

$$\frac{L,C,M,V \vdash E_1 : \text{int} \quad L,C,M,V \vdash E_2 : \text{int}}{L,C,M,V \vdash E_1 - E_2 : \text{int}}$$

Corresponding JOOS source:

```
case minusK:
    typeImplementationEXP(e->val.minusE.left,class);
    typeImplementationEXP(e->val.minusE.right,class);
    checkINT(e->val.minusE.left->type,e->lineno);
    checkINT(e->val.minusE.right->type,e->lineno);
    e->type = intTYPE;
    break;
```

Implicit integer cast:

$$\frac{L,C,M,V \vdash E : \text{char}}{L,C,M,V \vdash E : \text{int}}$$

Corresponding JOOS source:

```
int checkINT(TYPE *t, int lineno)
{ if (t->kind!=intK && t->kind!=charK) {
    reportError("int type expected",lineno);
    return 0;
}
return 1;
}
```

Type rule for equality:

$$\frac{\begin{array}{l} L, C, M, V \vdash E_1 : \tau_1 \\ L, C, M, V \vdash E_2 : \tau_2 \\ \tau_1 := \tau_2 \vee \tau_2 := \tau_1 \end{array}}{L, C, M, V \vdash E_1 == E_2 : \text{boolean}}$$

Corresponding JOOS source:

```
case eqK:
  typeImplementationEXP(e->val.eqE.left,class);
  typeImplementationEXP(e->val.eqE.right,class);
  if (!assignTYPE(e->val.eqE.left->type,
                  e->val.eqE.right->type) &&
      !assignTYPE(e->val.eqE.right->type,
                  e->val.eqE.left->type)) {
    reportError("arguments for == have wrong types",
                e->lineno);
  }
  e->type = boolTYPE;
  break;
```


Type rule for `this`:

$$L, C, M, V \vdash \text{this} : C$$

Corresponding JOOS source:

```
case thisK:
  if (class==NULL) {
    reportError("'this' not allowed here",e->lineno);
  }
  e->type = classTYPE(class);
  break;
```

Type rule for cast:

$$\frac{L, C, M, V \vdash E : \tau \quad \tau \leq C \vee C \leq \tau}{L, C, M, V \vdash (C)E : C}$$

Corresponding JOOS source:

```

case castK:
  typeImplementationEXP(e->val.castE.right, class);
  e->type = makeTYPEextref(e->val.castE.left,
                          e->val.castE.class);
  if (e->val.castE.right->type->kind!=refK &&
      e->val.castE.right->type->kind!=polynullK) {
    reportError("class reference expected", e->lineno);
  } else {
    if (e->val.castE.right->type->kind==refK &&
        !subClass(e->val.castE.class,
                  e->val.castE.right->type->class) &&
        !subClass(e->val.castE.right->type->class,
                  e->val.castE.class)) {
      reportError("cast will always fail", e->lineno);
    }
  }
}
break;

```

Type rule for instanceof:

$$\frac{L, C, M, V \vdash E : \tau \quad \tau \leq C \vee C \leq \tau}{L, C, M, V \vdash E \text{ instanceof } C : \text{boolean}}$$

Corresponding JOOS source:

```

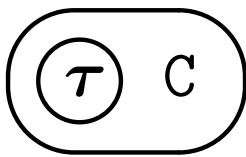
case instanceofK:
    typeImplementationEXP(e->val.instanceofE.left,class);
    if (e->val.instanceofE.left->type->kind!=refK) {
        reportError("class reference expected",e->lineno);
    }
    if (!subClass(e->val.instanceofE.left->type->class,
        e->val.instanceofE.class) &&
        !subClass(e->val.instanceofE.class,
            e->val.instanceofE.left->type->class)) {
        reportError("instanceof will always fail",e->lineno);
    }
    e->type = boolTYPE;
    break;

```

Why the predicate:

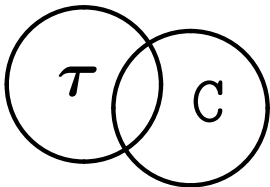
$$\tau \leq C \vee C \leq \tau$$

for “(C) *E*” and “*E* instanceof C”?



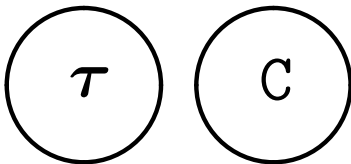
succeeds

$$\tau \leq C$$



really useful

$$C \leq \tau$$



fails

$$\tau \not\leq C \wedge C \not\leq \tau$$

Circle denotes type and all its subtypes. For instance, the following would fail to type check, as no subtype of `List` can ever be a subtype of the final (!) class `String`:

```
List l;
if(l instanceof String) ...
```

Type rule for method invocation:

$$L, C, M, V \vdash E : \sigma \wedge \sigma \in L$$

$$\exists \rho : \sigma \leq \rho \wedge m \in \text{methods}(\rho)$$

$$\neg \text{static}(m)$$

$$L, C, M, V \vdash E_i : \sigma_i$$

$$\text{argtype}(L, \rho, m, i) := \gamma_i \wedge \gamma_i := \sigma_i$$

$$\text{type}(L, \rho, m) = \tau$$

$$L, C, M, V \vdash E.m(E_1, \dots, E_n) : \tau$$

Corresponding JOOS source:

```
case invokeK:
  t = typeImplementationRECEIVER(
    e->val.invokeE.receiver,class);
  typeImplementationARGUMENT(e->val.invokeE.args,class);
  if (t->kind!=refK) {
    reportError("receiver must be an object",e->lineno);
    e->type = polynullTYPE;
  } else {
    s = lookupHierarchy(e->val.invokeE.name,t->class);
    if (s==NULL || s->kind!=methodSym) {
      reportStrError("no such method called %s",
        e->val.invokeE.name,e->lineno);
      e->type = polynullTYPE;
    } else {
      e->val.invokeE.method = s->val.methodS;
      if (s->val.methodS.modifier==modSTATIC) {
        reportStrError(
          "static method %s may not be invoked",
          e->val.invokeE.name,e->lineno);
      }
      typeImplementationFORMALARGUMENT(
        s->val.methodS->formals,
        e->val.invokeE.args,e->lineno);
      e->type = s->val.methodS->returntype;
    }
  }
  break;
```

Type rule for constructor invocation:

$$\begin{array}{c}
 L, C, M, V \vdash E_i : \sigma_i \\
 \exists \vec{\tau} : \text{constructor}(L, C, \vec{\tau}) \wedge \\
 \quad \vec{\tau} := \vec{\sigma} \wedge \\
 \quad (\forall \vec{\gamma} : \text{constructor}(L, C, \vec{\gamma}) \wedge \vec{\gamma} := \vec{\sigma} \\
 \quad \quad \Downarrow \\
 \quad \quad \vec{\gamma} := \vec{\tau} \\
 \quad) \\
 \hline
 L, C, M, V \vdash \text{new } C(E_1, \dots, E_n) : C
 \end{array}$$

Corresponding JOOS source:

```

case newK:
  if (e->val.newE.class->modifier==modABSTRACT) {
    reportStrError("illegal abstract constructor %s",
                  e->val.newE.class->name,
                  e->lineno);
  }
  typeImplementationARGUMENT(e->val.newE.args, this);
  e->val.newE.constructor =
    selectCONSTRUCTOR(e->val.newE.class->constructors,
                      e->val.newE.args,
                      e->lineno);
  e->type = classTYPE(e->val.newE.class);
  break;

```

Different kinds of type rules are:

- *axioms*:

$$L, C, M, V \vdash \text{this} : C$$

- *predicates*:

$$\tau \leq C \vee C \leq \tau$$

- *inferences*:

$$\frac{L, C, M, V \vdash E_1 : \text{int} \quad L, C, M, V \vdash E_2 : \text{int}}{L, C, M, V \vdash E_1 - E_2 : \text{int}}$$

A *type proof* is a tree in which:

- nodes are inferences; and
- leaves are axioms or true predicates.

A program is statically type correct
iff
it is the root of some type proof.

A type proof is just a trace of a successful run of the type checker.

An example type proof:

$$\begin{array}{c}
 \frac{V[x \mapsto A][y \mapsto B](y) = B}{\mathcal{S} \vdash y : B} \quad \frac{V[x \mapsto A][y \mapsto B](x) = A}{\mathcal{S} \vdash x : A} \quad A \leq B \vee B \leq A}{\mathcal{S} \vdash (B)x : B} \quad B := B \\
 \hline
 L, C, M, V[x \mapsto A][y \mapsto B] \vdash y = (B)x : B \\
 \hline
 L, C, M, V[x \mapsto A][y \mapsto B] \vdash y = (B)x; \\
 \hline
 L, C, M, V[x \mapsto A] \vdash B \ y; \ y = (B)x; \\
 \hline
 L, C, M, V \vdash A \ x; \ B \ y; \ y = (B)x;
 \end{array}$$

where $\mathcal{S} = L, C, M, V[x \mapsto A][y \mapsto B]$ and we assume that $B \leq A$.

Type rules for plus:

$$\frac{L, C, M, V \vdash E_1 : \text{int} \quad L, C, M, V \vdash E_2 : \text{int}}{L, C, M, V \vdash E_1 + E_2 : \text{int}}$$

$$\frac{L, C, M, V \vdash E_1 : \text{String} \quad L, C, M, V \vdash E_2 : \tau}{L, C, M, V \vdash E_1 + E_2 : \text{String}}$$

$$\frac{L, C, M, V \vdash E_1 : \tau \quad L, C, M, V \vdash E_2 : \text{String}}{L, C, M, V \vdash E_1 + E_2 : \text{String}}$$

The operator $+$ is *overloaded*.

Corresponding JOOS source:

```
case plusK:
    typeImplementationEXP(e->val.plusE.left,class);
    typeImplementationEXP(e->val.plusE.right,class);
    e->type = typePlus(e->val.plusE.left,
                      e->val.plusE.right,e->lineno);
    break;
.
.
.

TYPE *typePlus(EXP *left, EXP *right, int lineno)
{ if (equalTYPE(left->type,intTYPE) &&
      equalTYPE(right->type,intTYPE)) {
    return intTYPE;
}
if (!equalTYPE(left->type,stringTYPE) &&
    !equalTYPE(right->type,stringTYPE)) {
    reportError("arguments for + have wrong types",
                lineno);
}
left->tostring = 1;
right->tostring = 1;
return stringTYPE;
}
```

A *coercion* is a conversion function that is inserted automatically by the compiler.

The code:

```
"abc" + 17 + x
```

is transformed into:

```
"abc" + (new Integer(17).toString()) + x.toString()
```

What effect would a rule like:

$$\frac{L, C, M, V \vdash E_1 : \tau \quad L, C, M, V \vdash E_2 : \sigma}{L, C, M, V \vdash E_1 + E_2 : \text{String}}$$

have on the type system if it were included?

The testing strategy for the type checker involves a further extension of the pretty printer, where the type of every expression is printed explicitly.

These types are then compared to a corresponding manual construction for a sufficient collection of programs.

Furthermore, every error message should be provoked by some test program.