1. *Making change with a limited supply of coins.* Adapt the dynamic programming algorithm we saw in class for “making change” so it will work correctly even when the number of coins of a particular denomination is limited.

Comments: You can assume that only one denomination has a limited amount of coins, but your solution has to work whatever that denomination is. I don’t think this assumption really makes your life easier, so if you want to solve the general case, you are welcome to. (General case: your algorithm takes as input a set of denominations together with the number of coins available for each denomination. Simplification: only one of these values is less than $\infty$.)

2. Rewrite your solution to problem 1, using a recursive function and memoization.

3. Consider an alphabet $\Sigma = \{a, b, c\}$ over which is defined an arbitrary multiplication rule $\ast : \Sigma \times \Sigma \to \Sigma$ that needs be neither commutative nor associative. Thus, a chained multiplication $x_1 \ast x_2 \ast \ldots \ast x_n$ can have different values depending on how we decide to parenthesize it. Given a symbol $x \in \Sigma$, find an efficient algorithm that examines a multiplication $x_1 \ast x_2 \ast \ldots \ast x_n$ of $n$ characters of $\Sigma$ and decides whether or not it is possible to parenthesize it in such a way that the value of the resulting expression is $x$.

4. *Skylines with slanted rooftops.* Modify the skyline problem so that each building is a rectangle with an isosceles triangular roof on top. Each triangle should have for its base the top of the rectangle it sits on, and the two other sides should make 45-degree angles with the base. Design an algorithm to draw the skyline in this case. Compare its running time with that of the solution to the regular skyline problem.

5. Given $n$ points in the plane, find the pair of points such that the line segment connecting them has the maximal slope. The running time of your algorithm should be $O(n \log n)$ in the worst case.

6. *Join operation on red-black trees.* Exercise 13-2, on page 295 of CLRS.