Teleporting an Unknown Quantum State via Dual Classical and EPR Channels

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Abstract

An unknown quantum state $|\phi\rangle$ can be disassembled into, then later reconstructed from, purely classical information and purely nonclassical EPR correlations. To do so the sender, "Alice," and the receiver, "Bob," must prearrange the sharing of an EPR-correlated pair of particles. Alice makes a joint measurement on her EPR particle and the unknown quantum system, and sends Bob the classical result of this measurement. Knowing this, Bob can convert the state of his EPR particle into an exact replica of the unknown state $|\phi\rangle$ which Alice destroyed.

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The existence of long range correlations between Einstein-Podolsky-Rosen (EPR) [1] pairs of particles raises the question of their use for information transfer. Einstein himself used the word "telepathically" in this context [2]. It is known that *instantaneous* information transfer is definitely impossible [3]. Here, we show that EPR correlations can nevertheless assist in the "teleportation" of an intact quantum state from one place to another, by a sender who knows neither the state to be teleported nor the location of the intended receiver.

Suppose one observer, whom we shall call "Alice," has been given a quantum system such as a photon or spin-$\frac{1}{2}$ particle, prepared in a state $|\phi\rangle$ unknown to her, and she wishes to communicate to another observer, "Bob", sufficient information about the quantum system for him to make an accurate copy of it. Knowing the state vector $|\phi\rangle$ itself would be sufficient information, but in general there is no way to learn it. Only if Alice knows beforehand that $|\phi\rangle$ belongs to a given orthonormal set can she make a measurement whose result will allow her to make an accurate copy of $|\phi\rangle$. Conversely, if the possibilities for $|\phi\rangle$ include two or more non-orthogonal states, then no measurement will yield sufficient information to prepare a perfectly accurate copy.

A trivial way for Alice to provide Bob with all the information in $|\phi\rangle$ would be to send the particle itself. If she wants to avoid transferring the original particle, she can make it interact unitarily with another system, or "ancilla," initially in a known state $|a_0\rangle$, in such a way that after the interaction the original particle is left in a standard state $|\phi_0\rangle$ and the ancilla is in an unknown state $|a\rangle$ containing complete information about $|\phi\rangle$. If Alice now sends Bob the ancilla (perhaps technically easier than sending the original particle), Bob can reverse her actions to prepare a replica of her original state $|\phi\rangle$. This "spin-exchange measurement" [4] illustrates an essential feature of quantum information: it can be swapped from one system to another, but it cannot be duplicated or "cloned" [5]. In this regard it is quite unlike classical information, which can be duplicated at will. The most tangible manifestation of the non-classicality of quantum information is the violation of Bell's inequalities [6] observed [7] in experiments on EPR states. Other manifestations include the possibility of quantum cryptography [8], quantum parallel computation [9], and the superiority of interactive measurements for extracting information from a pair of identically prepared particles [10].

The spin-exchange method of sending full information to Bob still lumps classical and nonclassical information together in a single transmission. Below, we show how Alice can divide the full information encoded in $|\phi\rangle$ into two parts, one purely classical and the other purely nonclassical, and send them to Bob through two different channels. Having received these two transmissions, Bob can construct an accurate replica of $|\phi\rangle$. Of course Alice's original $|\phi\rangle$ is destroyed in the process, as it must be to obey the no-cloning theorem. We call the process we are about to describe "teleportation," a term from science fiction meaning to make a person or object disappear while an exact replica appears somewhere else. It must be emphasized that our teleportation, unlike some science fiction versions, defies no physical laws. In particular it cannot take place instantaneously or over a space-like interval, because it requires, among other things, sending a classical message from Alice to Bob. The net result of teleportation is completely prosaic: the removal of $|\phi\rangle$ from Alice's hands and its appearance in Bob's hands a suitable time later. The only remarkable feature is that in the interim, the information in $|\phi\rangle$ has been cleanly separated into classical and nonclassical parts. First we shall show how to teleport the quantum state $|\phi\rangle$ of a spin-$\frac{1}{2}$ particle. Later we discuss teleportation of more complicated states.

The nonclassical part is transmitted first. To do so, two spin-$\frac{1}{2}$ particles are prepared in an EPR singlet state

$$|\Psi_{23}^{(-)}\rangle = \sqrt{\tfrac{1}{2}}\left(|{\uparrow_2}\rangle|{\downarrow_3}\rangle - |{\downarrow_2}\rangle|{\uparrow_3}\rangle\right). \tag{1}$$

The subscripts 2 and 3 label the particles in this EPR pair. Alice's original particle, whose unknown state $|\phi\rangle$ she seeks to teleport to Bob, will be designated by a subscript 1 when necessary. These three particles may be of different kinds, e.g. one or more may be photons, the polarization degree of freedom having the same algebra as a spin.

One EPR particle (particle 2) is given to Alice, while the other (particle 3) is given to Bob. Although this establishes the possibility of nonclassical correlations between Alice and Bob, the EPR pair at this stage contains no information about $|\phi\rangle$. Indeed the entire system, comprising Alice's unknown particle 1 and the EPR pair, is in a pure product state, $|\phi_1\rangle\,|\Psi_{23}^{(-)}\rangle$, involving neither classical correlation nor quantum entanglement between the unknown particle and the EPR pair. Therefore no measurement on either member of the EPR pair, or both together, can yield any information about $|\phi\rangle$. An entanglement between these two subsystems is brought about in the next step.

To couple the first particle with the EPR pair, Alice performs a complete measurement of the von Neumann type on the joint system consisting of particle 1 and particle 2 (her EPR particle). This measurement is performed in the Bell operator basis [11] consisting of $|\Psi_{12}^{(-)}\rangle$ and

$$|\Psi_{12}^{(+)}\rangle = \sqrt{\tfrac{1}{2}}\left(|{\uparrow_1}\rangle|{\downarrow_2}\rangle + |{\downarrow_1}\rangle|{\uparrow_2}\rangle\right) \quad \textit{and} \quad |\Phi_{12}^{(\pm)}\rangle = \sqrt{\tfrac{1}{2}}\left(|{\uparrow_1}\rangle|{\uparrow_2}\rangle \pm |{\downarrow_1}\rangle|{\downarrow_2}\rangle\right). \tag{2}$$

Note that these four states are a complete orthonormal basis for particles 1 and 2.

It is convenient to write the unknown state of the first particle as

$$|\phi_1\rangle = a\,|{\uparrow_1}\rangle + b\,|{\downarrow_1}\rangle, \tag{3}$$

with $|a|^2 + |b|^2 = 1$. The complete state of the three particles before Alice's measurement is thus

$$|\Psi_{123}\rangle = \frac{a}{\sqrt{2}}\left(|{\uparrow_1}\rangle|{\uparrow_2}\rangle|{\downarrow_3}\rangle - |{\uparrow_1}\rangle|{\downarrow_2}\rangle|{\uparrow_3}\rangle\right) + \frac{b}{\sqrt{2}}\left(|{\downarrow_1}\rangle|{\uparrow_2}\rangle|{\downarrow_3}\rangle - |{\downarrow_1}\rangle|{\downarrow_2}\rangle|{\uparrow_3}\rangle\right). \tag{4}$$

In this equation, each direct product $| \ _1\rangle | \ _2\rangle$ can be expressed in terms of the Bell operator basis vectors $|\Phi_{12}^{(\pm)}\rangle$ and $|\Psi_{12}^{(\pm)}\rangle$, and we obtain

$$|\Psi_{123}\rangle = \tfrac{1}{2}\Big[|\Psi_{12}^{(-)}\rangle\Big(-a|\uparrow_3\rangle - b|\downarrow_3\rangle\Big) + |\Psi_{12}^{(+)}\rangle\Big(-a|\uparrow_3\rangle + b|\downarrow_3\rangle\Big) +$$
$$|\Phi_{12}^{(-)}\rangle\Big(a|\downarrow_3\rangle + b|\uparrow_3\rangle\Big) + |\Phi_{12}^{(+)}\rangle\Big(a|\downarrow_3\rangle - b|\uparrow_3\rangle\Big)\Big]. \tag{5}$$

It follows that, regardless of the unknown state $|\phi_1\rangle$, the four measurement outcomes are equally likely, each occurring with probability 1/4. Furthermore, after Alice's measurement, Bob's particle 3 will have been projected into one of the four pure states superposed in Eq. (5), according to the measurement outcome. These are, respectively,

$$-|\phi_3\rangle \equiv -\begin{pmatrix} a \\ b \end{pmatrix}, \quad \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}|\phi_3\rangle, \quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}|\phi_3\rangle, \quad \textit{and} \quad \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}|\phi_3\rangle. \tag{6}$$

Each of these possible resultant states for Bob's EPR particle is related in a simple way to the original state $|\phi\rangle$ which Alice sought to teleport. In the case of the first (singlet) outcome, Bob's state is the same except for an irrelevant phase factor, so Bob need do nothing further to produce a replica of Alice's spin. In the three other cases, Bob must apply one of the unitary operators in Eq. (6), corresponding respectively to 180 degree rotations around the z, x, and y axes, in order to convert his EPR particle into a replica of Alice's original state $|\phi\rangle$. (If $|\phi\rangle$ represents a photon polarization state, a suitable combination of half-wave plates will perform these unitary operations.) Thus an accurate teleportation can be achieved in all cases by having Alice tell Bob the classical outcome of her measurement, after which Bob applies the required rotation to transform the state of his particle into a replica of $|\phi\rangle$. Alice, on the other hand, is left with particles 1 and 2 in one of the states $|\Psi_{12}^{(\pm)}\rangle$ or $|\Phi_{12}^{(\pm)}\rangle$, without any trace of the original state $|\phi\rangle$.

Unlike the quantum correlation of Bob's EPR particle 3 to Alice's particle 2, the result of Alice's measurement is purely classical information, which can be transmitted, copied, and stored at will in any suitable physical medium. In particular, this information need not be destroyed or canceled to bring the teleportation process to a successful conclusion: the teleportation of $|\phi\rangle$ from Alice to Bob has the side effect of producing two bits of random classical information, uncorrelated to $|\phi\rangle$, which are left behind at the end of the process.

Since teleportation is a *linear* operation applied to the quantum state $|\phi\rangle$, it will work not only with pure states, but also with mixed or entangled states. For example, let Alice's original particle 1 be itself part of an EPR singlet with another particle, labelled 0, which may be far away from both Alice and Bob. Then, after teleportation, particles 0 and 3 would be left in a singlet state, even though they had originally belonged to separate EPR pairs.

All of what we have said above can be generalized to systems having $N > 2$ orthogonal states. In place of an EPR spin pair in the singlet state, Alice would use a pair of N-state particles in a completely entangled state. For definiteness let us write this entangled state as $\sum_j |j\rangle \otimes |j\rangle/\sqrt{N}$, where $j = 0, 1, \ldots, N-1$ labels the N elements of an orthonormal basis for each of the N-state systems. As before, Alice performs a joint measurement on particles 1 and 2. One such measurement that has the desired effect is the one whose eigenstates are $|\psi_{nm}\rangle$, defined by

$$|\psi_{nm}\rangle = \sum_j e^{2\pi i j n/N} \, |j\rangle \otimes |(j+m) \, mod \, N\rangle \, / \, \sqrt{N}. \tag{7}$$

Once Bob learns from Alice that she has obtained the result nm, he performs on his previously entangled particle (particle 3) the unitary transformation

$$U_{nm} = \sum_k e^{2\pi i k n/N} \, |k\rangle \, \langle (k+m) \, mod \, N| \, . \tag{8}$$

This transformation brings Bob's particle to the original state of Alice's particle 1, and the teleportation is complete.

The classical message plays an essential role in teleportation. To see why, suppose that Bob is impatient, and tries to complete the teleportation by guessing Alice's classical message before it arrives. Then Alice's expected $|\phi\rangle$ will be reconstructed (in the spin-$\frac{1}{2}$ case) as a random mixture of the four states of Eq. (6). For any $|\phi\rangle$, this is a maximally mixed state, giving no information about the input state $|\phi\rangle$. It could not be otherwise, because any correlation between the input and the guessed output could be used to send a superluminal signal.

One may still inquire whether accurate teleportation of a two-state particle requires a full two bits of classical information. Could it be done, for example, using only two or three distinct classical messages instead of four, or four messages of unequal probability? Later we show that a full two bits of classical channel capacity are necessary. Accurate teleportation using a classical channel of any lesser capacity would allow Bob to send superluminal messages through the teleported particle, by guessing the classical message before it arrived (cf Fig. 2).

Conversely one may enquire whether other states besides an EPR singlet can be used as the nonclassical channel of the teleportation process. Clearly any direct product state of particles 2 and 3 is useless, because for such states manipulation of particle 2 has no effect on what can be predicted about particle 3. Consider now a nonfactorable state $|\Upsilon_{23}\rangle$. It can readily be seen that after Alice's measurement, Bob's particle 3 will be related to $|\phi_1\rangle$ by four fixed unitary operations if and only if $|\Upsilon_{23}\rangle$ has the form

$$\sqrt{\tfrac{1}{2}}\Big(|u_2\rangle \, |p_3\rangle + |v_2\rangle \, |q_3\rangle\Big), \tag{9}$$

where $\{|u\rangle, |v\rangle\}$ and $\{|p\rangle, |q\rangle\}$ are any two pairs of orthonormal states. These are maximally entangled states [11], having maximally random marginal statistics for measurements on either particle separately. States which are less entangled reduce the fidelity of teleportation, and/or the range of states $|\phi\rangle$ that can be accurately teleported. The states in Eq. (9) are also precisely those obtainable from the EPR singlet by a local one-particle unitary operation [12]. Their use for the nonclassical channel is entirely equivalent to that of the singlet (1). Maximal entanglement is necessary and sufficient for faithful teleportation.

Although it is currently infeasible to store separated EPR particles for more than a brief time, if it becomes feasible to do so, quantum teleportation could be quite useful. Alice and Bob would only need a stockpile of EPR pairs (whose reliability can be tested by violations of Bell's inequality [7]) and a channel capable of carrying robust classical messages. Alice could then teleport quantum states to Bob over arbitrarily great distances, without worrying about the effects of attenuation and noise on, say, a single photon sent through a long optical fiber. As an application of teleportation, consider the problem investigated by Peres and Wootters [10], in which Bob already has another copy of $|\phi\rangle$. If he acquires Alice's copy, he can measure both together, thereby determining the state $|\phi\rangle$ more accurately than can be done by making a separate measurement on each one. Finally, teleportation has the advantage of still being possible in situations where Alice and Bob, after sharing their EPR pairs, have wandered about independently and no longer know each others' locations. Alice cannot reliably send Bob the original quantum particle, or a spin-exchanged version of it, if she does not know where he is; but she can still teleport the quantum state to him, by broadcasting the classical information to all places where he might be.

Teleportation resembles another recent scheme for using EPR correlations to help transmit useful information. In "4-way coding" [12] modulation of one member of an EPR pair serves to reliably encode a 2-bit message in the joint state of the complete pair. Teleportation and 4-way coding can be seen as variations on the same underlying process, illustrated by the spacetime diagrams in Fig. 1. Note that *closed loops* are involved for both processes. Trying to draw similar "Feynman diagrams" with tree structure, rather than loops, would lead to physically impossible processes.

On the other hand, more complicated closed-loop diagrams are possible, such as Fig. 2, obtained by substituting Fig. 1a into the wavy line of Fig. 1b. This represents a 4-way coding scheme in which the modulated EPR particle is teleported instead of being transmitted directly. Two incoming classical bits on the lower left are reproduced reliably on the upper right, with the assistance of two shared EPR pairs and two other classical bits, uncorrelated with the external bits, in an internal channel from A' to B'. This diagram is of interest because it can be used to show that a full two bits of classical channel capacity are necessary for accurate teleportation of a two-state particle (cf caption).

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FIG. 1. Spacetime diagrams for (a) quantum teleportation, and (b) 4-way coding [12]. As usual, time increases from bottom to top. The solid lines represent a classical pair of bits, the dashed lines an EPR pair of particles (which may be of different types), and the wavy line a quantum particle in an unknown state $|\phi\rangle$. Alice (A) performs a quantum measurement, and Bob (B) a unitary operation.

FIG. 2. Spacetime diagram of more complex 4-way coding scheme in which the modulated EPR particle (wavy line) is teleported rather than being transmitted directly. This diagram can be used to prove that a classical channel of 2 bits capacity is necessary for teleportation. To do so, assume on the contrary that the teleportation from A' to B' uses an internal classical channel of capacity $C < 2$ bits, but is still able to transmit the wavy particle's state accurately from A' to B', and therefore still transmit the external two bit message accurately from B to A. The assumed lower capacity $C < 2$ of the internal channel means that if B' were to guess the internal classical message superluminally instead of waiting for it to arrive, his probability 2^{-C} of guessing correctly would exceed 1/4, resulting in a probability greater than 1/4 for successful superluminal transmission of the external two-bit message from B to A. This in turn entails the existence of two distinct external two-bit messages, r and s, such that $P(r|s)$, the probability of superluminally receiving r if s was sent, is less than 1/4, while $P(r|r)$, the probability of superluminally receiving r if r was sent, is greater than 1/4. By redundant coding, even this statistical difference between r and s could be used to send reliable superluminal messages; therefore reliable teleportation of a 2-state particle cannot be achieved with a classical channel of less than 2 bits capacity. By the same argument, reliable teleportation of an N-state particle requires a classical channel of $2\log_2(N)$ bits capacity