Hierarchical Reinforcement Learning

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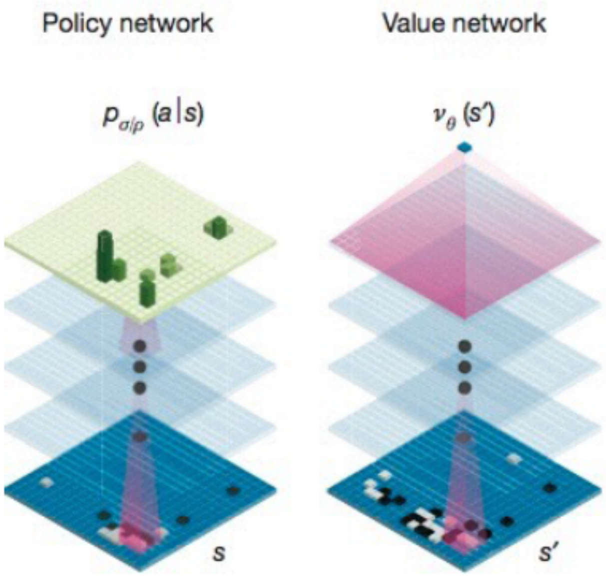
Knowledge in AlphaGo



- *Policy*: what to do (probability of action given current "state") - ie *procedural knowledge*
- *Value function*: estimation of expected long-term return - ie *predictive knowledge*

From Reinforcement Learning to AI

- Growing knowledge and abilities in an environment
- Learning efficiently from one stream of data
- Reasoning at multiple levels of abstraction
- Adapting quickly to new situations

Building Knowledge with Reinforcement Learning

- Focusing on two types of knowledge:
  - *Procedural knowledge*: skills, goal-driven behavior
  - *Predictive, empirical knowledge*: predicting effects of actions

- Knowledge must be:
  - *Expressive*: able to represent many things, including abstractions (objects, places, high-level strategies...)
  - *Learnable*: from data, ideally without supervision (for scalability)
  - *Composable*: suitable for fast planning by assembling existing pieces

Abstraction and generalization

- An *abstract representation* ignores low-level details of the problem, or modifies the problem representation altogether

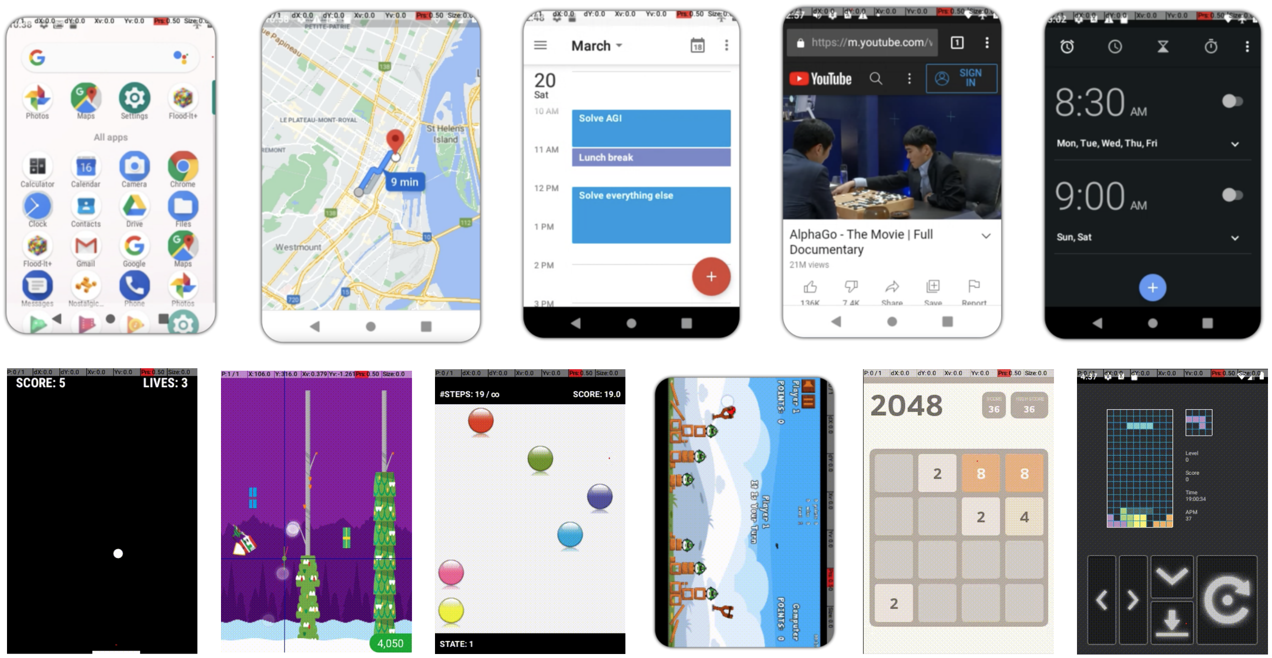
  Eg. addresses vs exact coordinates

- *Generalization* is the ability to take knowledge acquired in some circumstances and applying it in different circumstances

  Eg. Being good at some games helps us learn other games faster

- These two concepts are related but not identical: an abstract representation may helps us to generalize
- Generalization is often achieved in AI/ML by using function approximation (eg deep nets)
- In RL, we have an extra important dimension: *time/action* - can we build abstraction/generalization here too?

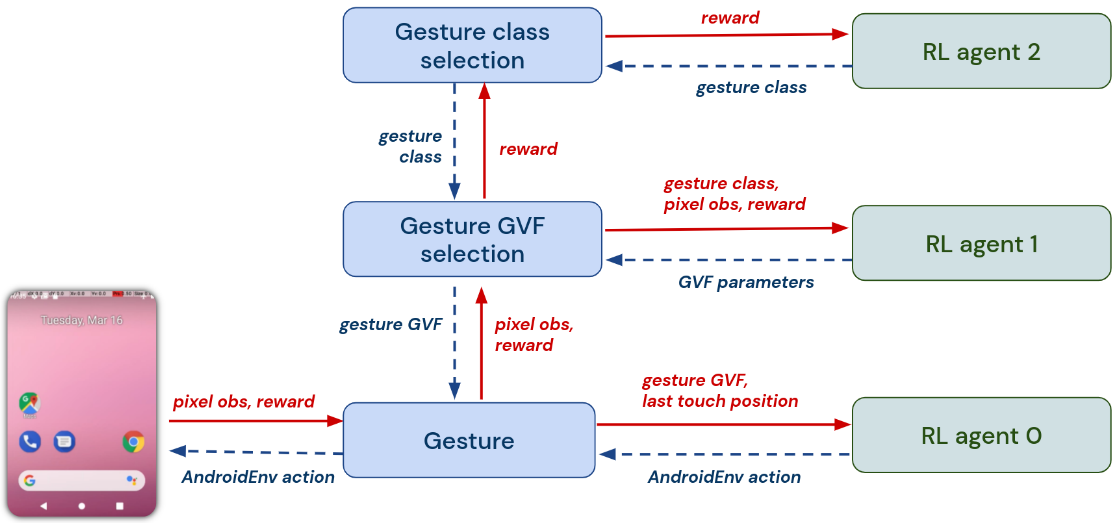
Motivating Example: Learning to Manipulate Complex Interfaces



- Agent interacting with a phone screen, learning how to control apps
- Native action space: touch anywhere on the screen

Toyama, Hamel, Gergely, Comanici et al (2021), https://arxiv.org/pdf/2105.13231.pdf

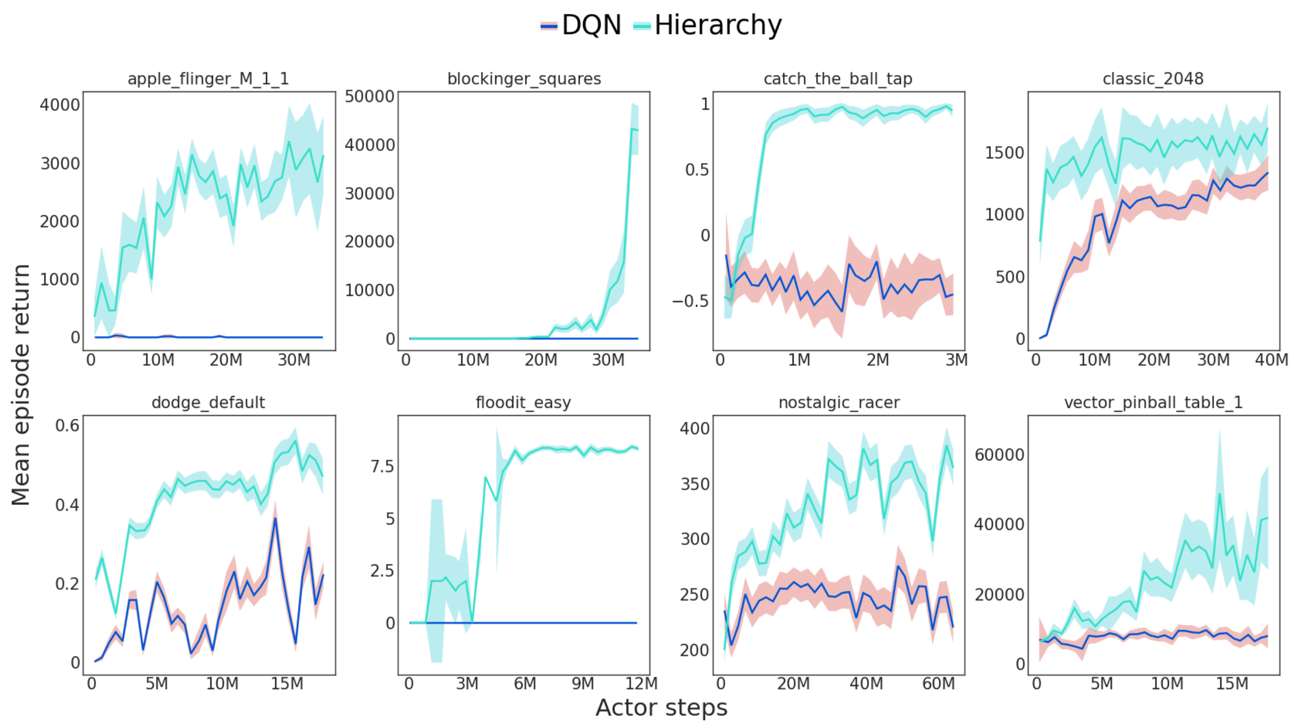
Example: Using Abstraction to Structure Learning



- Instead of primitive actions, learn and use gestures (tap, swipe, fling)
- Value functions predict reward associated with different gesture goals
- *Learning happens in parallel at all levels of abstraction*

Comanici, Glaese, Gergely, Toyama et al (2022) https://arxiv.org/pdf/2204.10374.pdf

Learning knowledge at multiple levels of abstraction drastically improves performance



Comanici, Glaese, Gergely, Toyama et al (2022) https://arxiv.org/pdf/2204.10374.pdf

What is temporal abstraction?

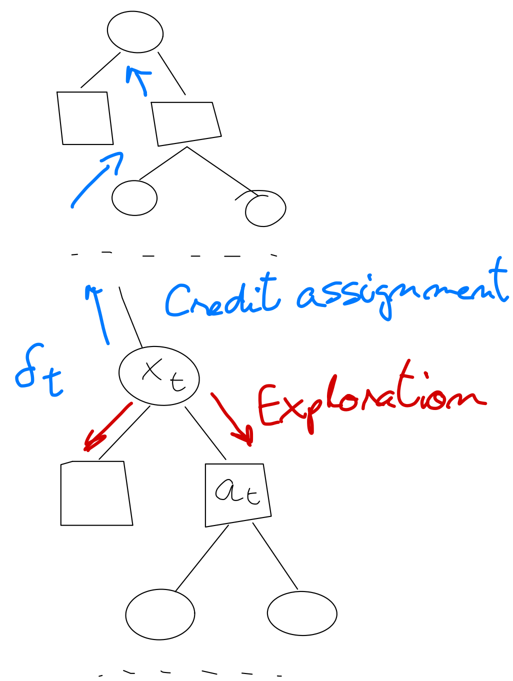
- Consider an activity such as cooking dinner

  - High-level steps: choose a recipe, make a grocery list, get groceries, cook,...
  - Medium-level steps: get a pot, put ingredients in the pot, stir until smooth, check the recipe ...
  - Low-level steps: wrist and arm movement while driving the car, stirring, ...
- All have to be seamlessly integrated!

Temporal abstraction in AI

- A cornerstone of AI planning since the 1970's:
  - Fikes et al. (1972), Newell (1972, Kuipers (1979), Korf (1985), Laird (1986), Iba (1989), Drescher (1991) etc.
- It has been shown to :
  - Generate shorter plans
  - Reduce the complexity of choosing actions
  - Provide robustness against model misspecification
  - Allows taking shortcuts in the environment
- In robotics and hybrid systems, the use of controllers provides similar benefits, and also improves interpretability and allows specifying prior knowledge

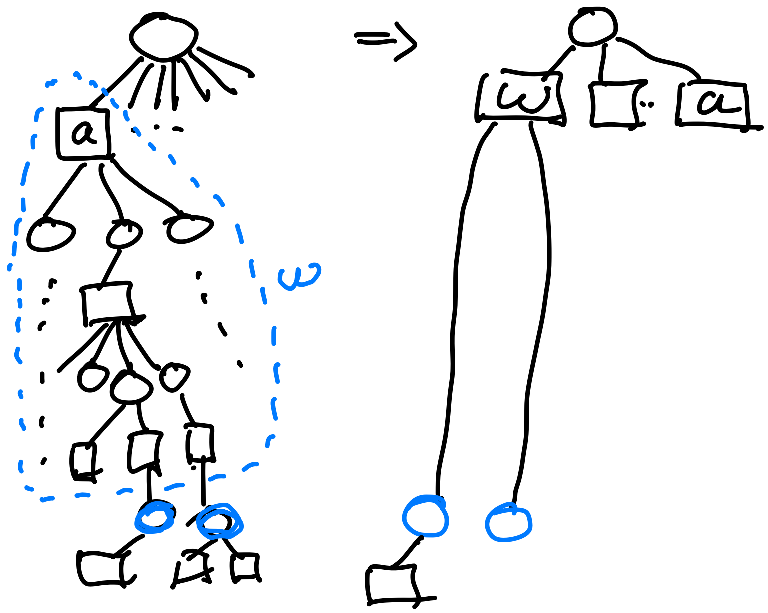
Recall: RL cartoon



Goals of temporal abstraction:

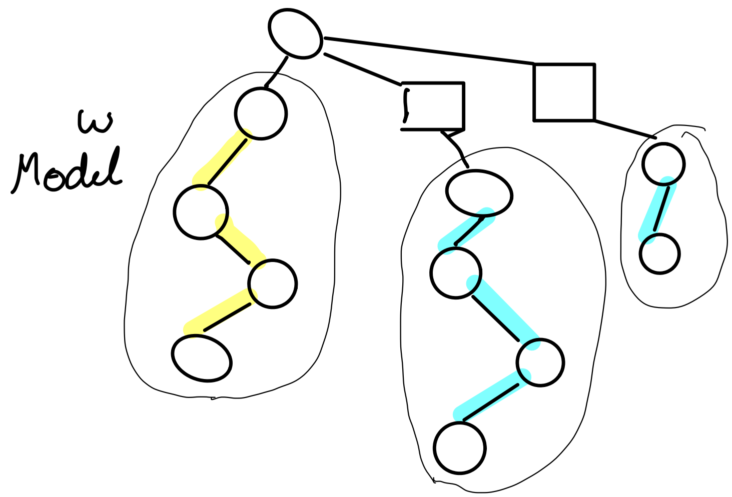
- Reduce tree width - helps exploration!
- Reduce tree depth - helps make planning/reasoning faster
- Generalize between different branches of the tree - improves learning

Options Intuition



- Package a whole sub-tree as an option ω
- Jumps are to the state *at the end* of the sub-tree
- Primitive actions are a special case (one-step tree)
- Two components: (sub)policy and model

Both abstraction and generalization!



Procedural, Temporally Abstract Knowledge: Options

- An *option* ω consists of 3 components
  - An *initiation set* $I_\omega \subseteq \mathcal{S}$ (aka precondition)
  - A *policy* $\pi_\omega : \mathcal{S} \times \mathcal{A} \to [0,1]$
    $\pi_\omega(a|s)$ is the probability of taking a in s when following option ω
  - A termination condition $\beta_\omega : \mathcal{S} \to [0,1]$:
    $\beta_\omega(s)$ is the probability of terminating the option ω upon entering s
- Eg., robot navigation: if there is no obstacle in front (I_ω), go forward (π_ω) until you get too close to another object (β_ω)
- Inspired from macro-actions / behaviors in robotics / hybrid planning and control

Cf. Sutton, Precup & Singh, 1999; Precup, 2000

Options as Behavioral Programs

- *Call-and-return execution*
  - When called, option ω is pushed onto the execution stack
  - During the option execution, the program looks at certain *variables (aka state)* and executes an *instruction (aka action)* until a termination condition is reached
  - The option can keep track of additional *local variables*, eg counting number of steps, saturation in certain features (e.g. Comanici, 2010)
  - *Options can invoke other options*
- *Interruption*
  - At each step, one can check if a better alternative has become available
  - If so, the option currently executing is *interrupted* (special form of concurrency)

Option models

- *Option model* has two parts:
  1. *Expected reward* $r_\omega(s)$: the expected return during ω's execution from state s
  2. *Transition model* $P_\omega(s'|s)$: specifies *where* the agent will end up after the option/program execution and *when* termination will happen
- Models are *predictions* about the future, conditioned on the option being executed
- Programming languages: preconditions (initiation set) and postconditions
- Models of options represent *(probabilistic) post-conditions*
- "Jumpy" planning is better for temporal credit assignment, accurate value estimation

What type of planning?

- *Models that are compositional can be used to plan through value iteration*
- *Sequencing*

$$\mathbf{r}_{\omega_1\omega_2} = \mathbf{r}_{\omega_1} + P_{\omega_1}\mathbf{r}_{\omega_2}$$
$$P_{\omega_1\omega_2} = P_{\omega_1}P_{\omega_2}$$

Cf. Sutton et al, 1999, Sorg & Singh, 2011

- *Stochastic choice*: can take expectations of reward and transition models
- These are sufficient conditions to allow Bellman equations to hold
- Silver & Ciosek (2012): re-write model in one matrix, compose models to construct programs
- Model-predictive control (receding horizon planning) is also possible

Option Models Provide Semantics

- Models of actions consist of immediate reward and transition probability to next state
- Models of options consist of reward until termination and (discounted) transition to termination state
- Models are *predictions about the future*

Illustration: Navigation

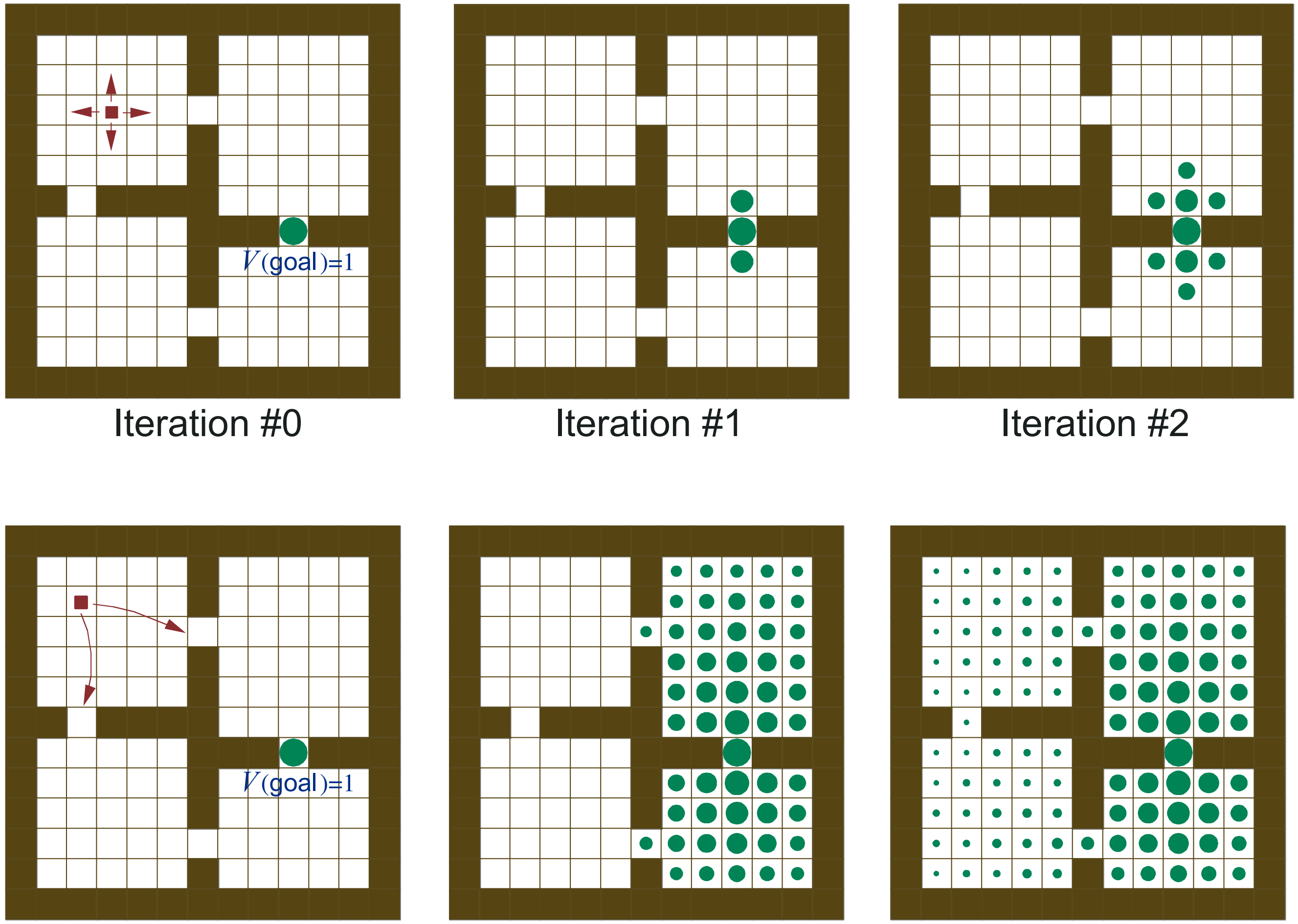
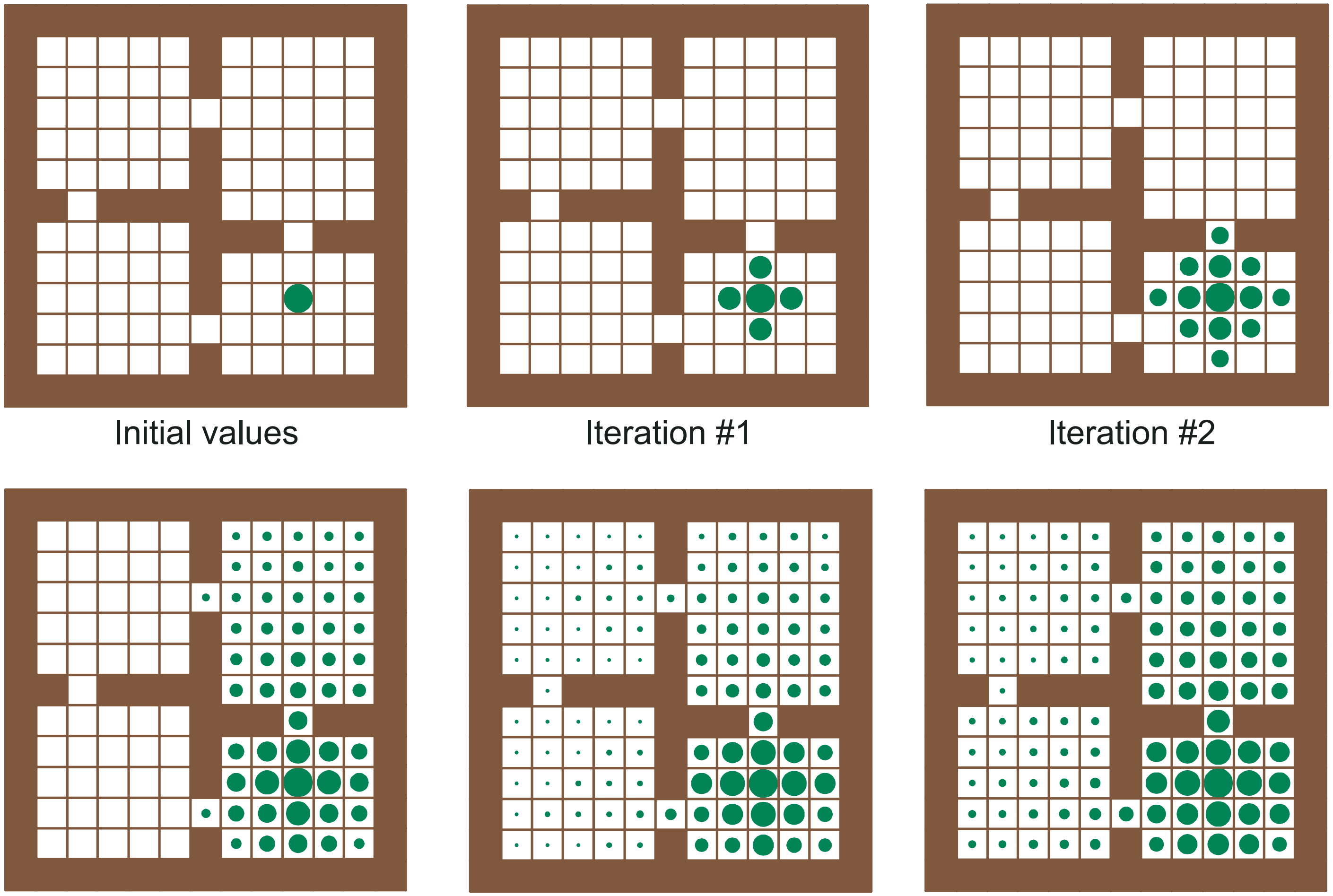
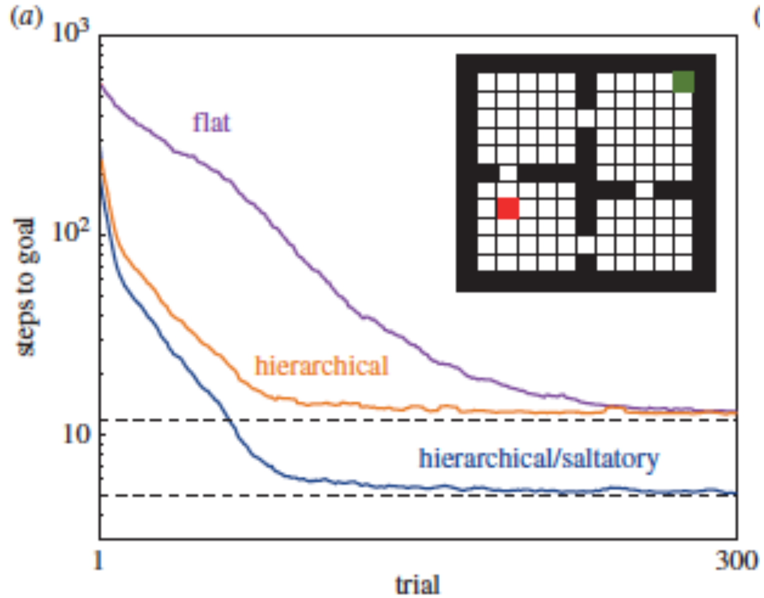


Illustration: Options and Primitives

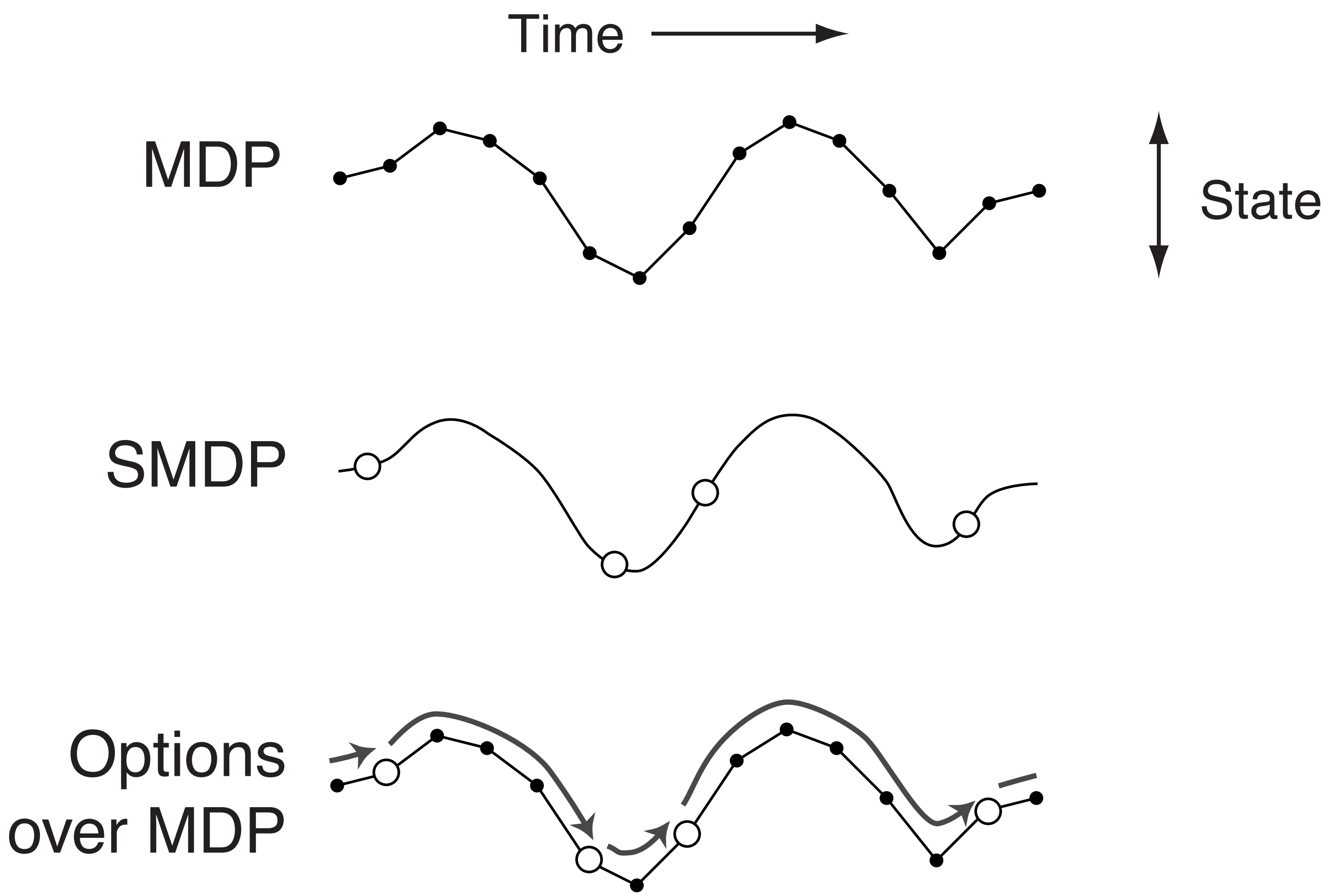


Initial values — Iteration #1 — Iteration #2 — Iteration #3 — Iteration #4 — Iteration #5

Benefits of options (cf Botvinick & Weinstein, 2014)



Decision-Making with Options



Learning and planning algorithms are the same at all levels of abstraction!

Option-value function

- The option-value function of a policy over options π_Ω is defined as:

$$q_{\pi_\Omega}(s,\omega) = \mathbf{E}_{\pi_\Omega}\Big[R_{t+1} + \gamma\beta_\omega(S_{t+1})q_{\pi_\Omega}(S_{t+1},\omega_{t+1}))$$
$$+ \gamma((1-\beta_\omega(S_{t+1}))q_{\pi_\Omega}(S_{t+1},\omega)|S_t = s\Big]$$

- One can use eg Q-learning, actor-critic,... to learn this!
- Note that if we learn/plan in an SMDP, the *contraction factor will be lower than γ*
- So fixing a set of options may allow solving the problem faster, but maybe in a slightly sub-optimal way
- Intuitively, models are more self-contained than option-value functions

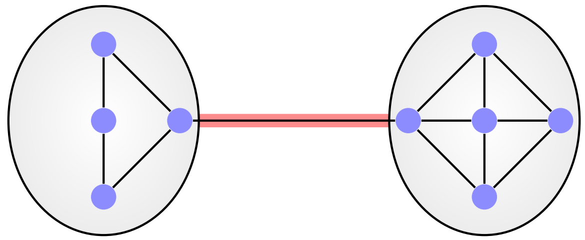
Advantages

- Easy to learn using temporal-difference-style methods, from a single stream of experience
- Planning with option models is done just like planning with primitives - *no explicit hierarchy*
- Result of planning with a set of options Ω is an option-value function, e.g. V_Ω, Q_Ω
- *But we can also use the underlying MDP structure to help in learning the options*

How Should Options Be Created?

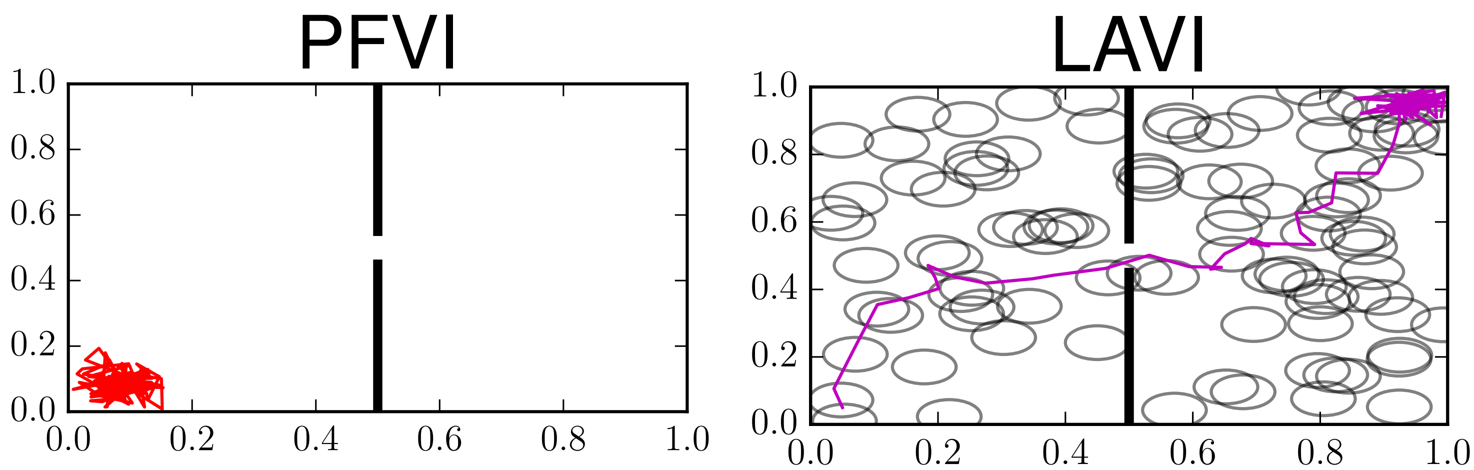
- Options can be given by a system designer (eg robotics)
- If subgoals / secondary reward structure is given, the option policy can be obtained, by solving a smaller planning or learning problem (cf. Precup, 2000)
  - Eg. acquiring certain objects in a game
  - Eg. Intrinsic motivation
- *What is a good set of subgoals / options?*
- This is a *representation discovery* problem
- Studied a lot over the last 20 years
- Bottleneck states and change point detection currently the most successful methods

Bottleneck States



- Perhaps the most explored idea in options construction
- A bottleneck allows "circulating" between many different states
- Lots of different approaches!
  - Frequency of states (McGovern et al, 2001, Stolle & Precup, 2002)
  - Graph partitioning / state graph analysis (Simsek et al, 2004, Menache et al, 2004, Bacon & Precup, 2013) / graph Laplacian (eg Klissarov and Machado, 2023)
  - Information-theoretic ideas (Peters et al., 2010)
- People seem quite good at generating these (cf. Botvinick, 2001, Solway et al, 2014)
- *Main drawback: expensive both in terms of sample size and computation*

Random Subgoals Also Help



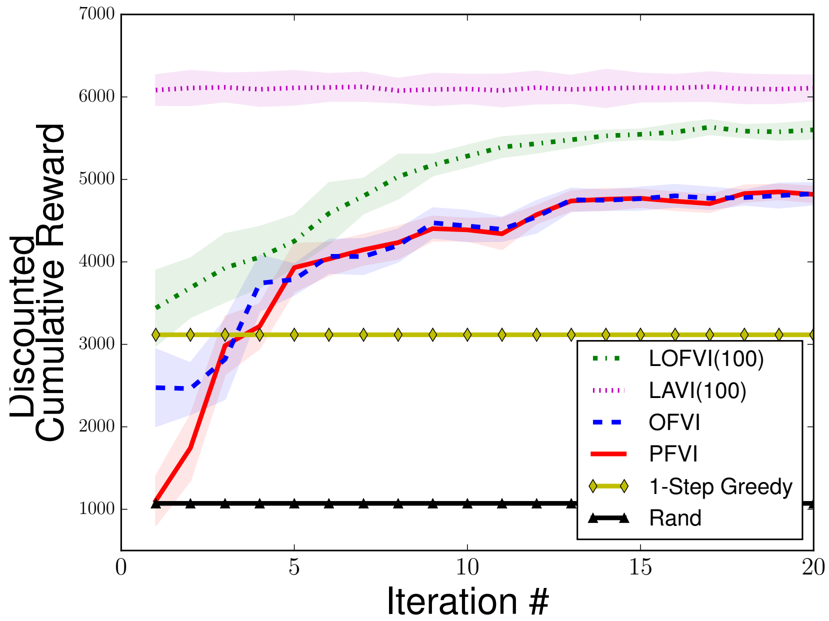
Cf. Mann, Mannor & Precup, 2015

Inventory management application

- Manage a warehouse that can stock 8 different commodities
- At most 500 items can be stored at any given time
- Demand is stochastic and depends on time of year
- Negative rewards are given for unfulfilled orders and for the cost of ordered items
- Hand-crafted options: order nothing until some threshold is crossed
- Primitive actions: specify amount of order for each item

Inventory management results

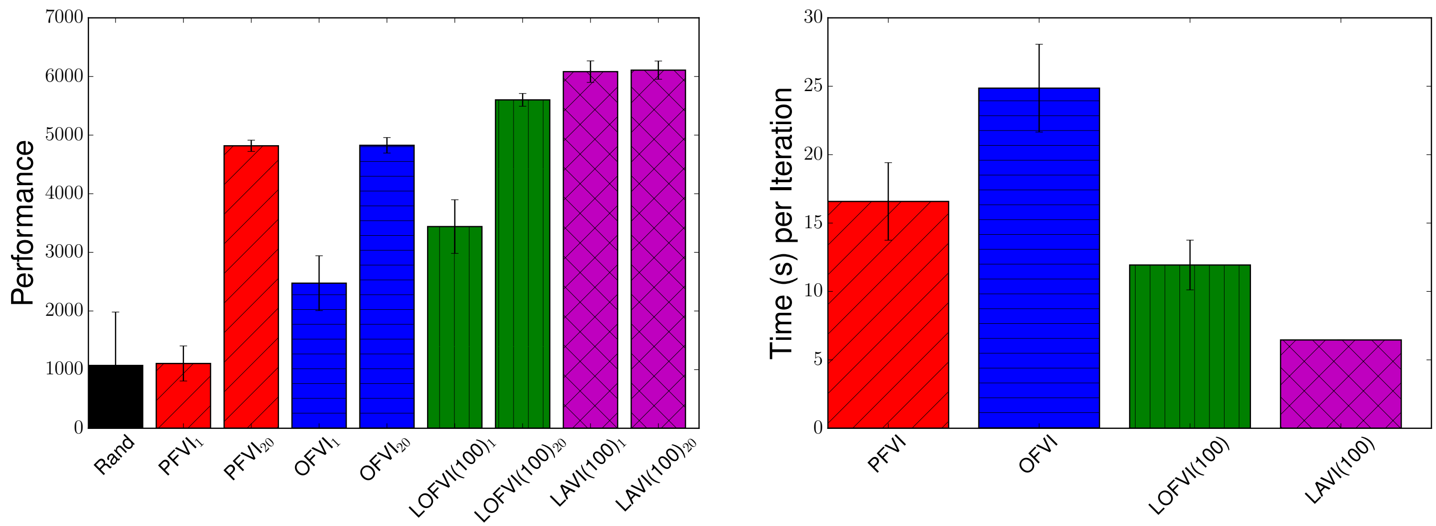
- Comparing a random policy and a 1-step greedy choice with using just primitives (PFVI) using primitives and hand-crafted options (OFVI), using "landmarks" (LOFVI) and using landmarks and only computing values for landmarks states (LAVI)



- *Randomly generated landmarks/subgoals perform much better*

Performance and time evaluation

- Performance of initial and final policy (left) and running time (right) averaged offer 20 independent runs

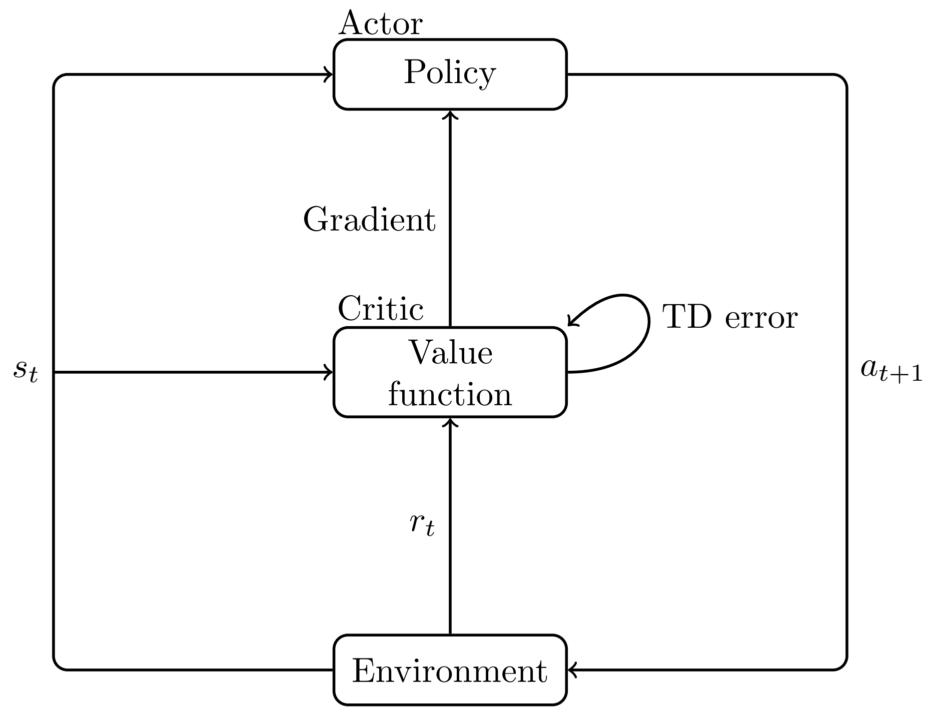


- Computing values only at landmark states yields a good policy almost immediately
- Handcrafted options are better than primitives in the beginning but slightly worse in the long run but *randomly generated landmarks are much better*

Option-Critic: Learn Options that Optimize Return

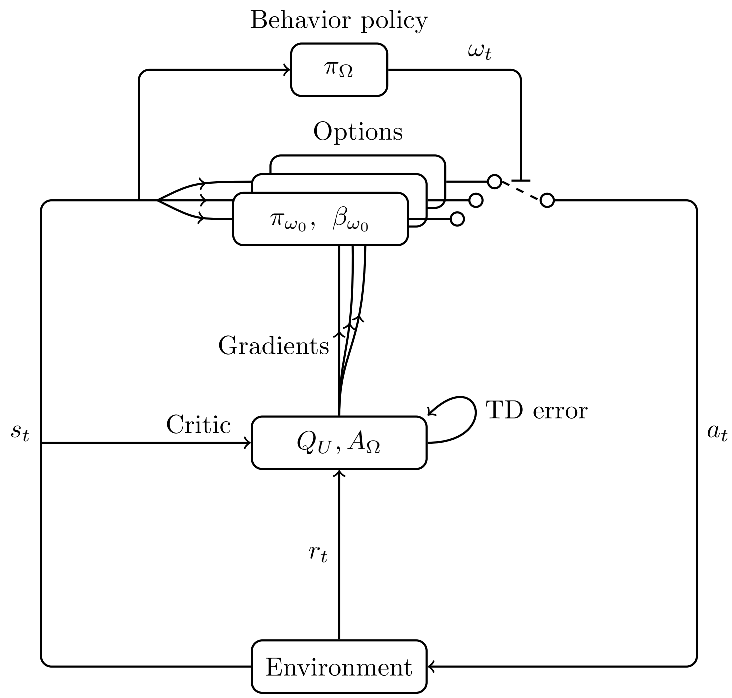
- Explicitly state an *optimization objective* and then solve it to find a set of options
- Handle both *discrete and continuous* set of state and actions
- Learning options should be *continual* (avoid combinatorially-flavored computations)
- Options should provide *improvement within one task* (or at least not cause slow-down...)

Actor-Critic Architecture



- Clear optimization objective: average or discounted return
- Continual learning
- Handles both discrete and continuous states and actions

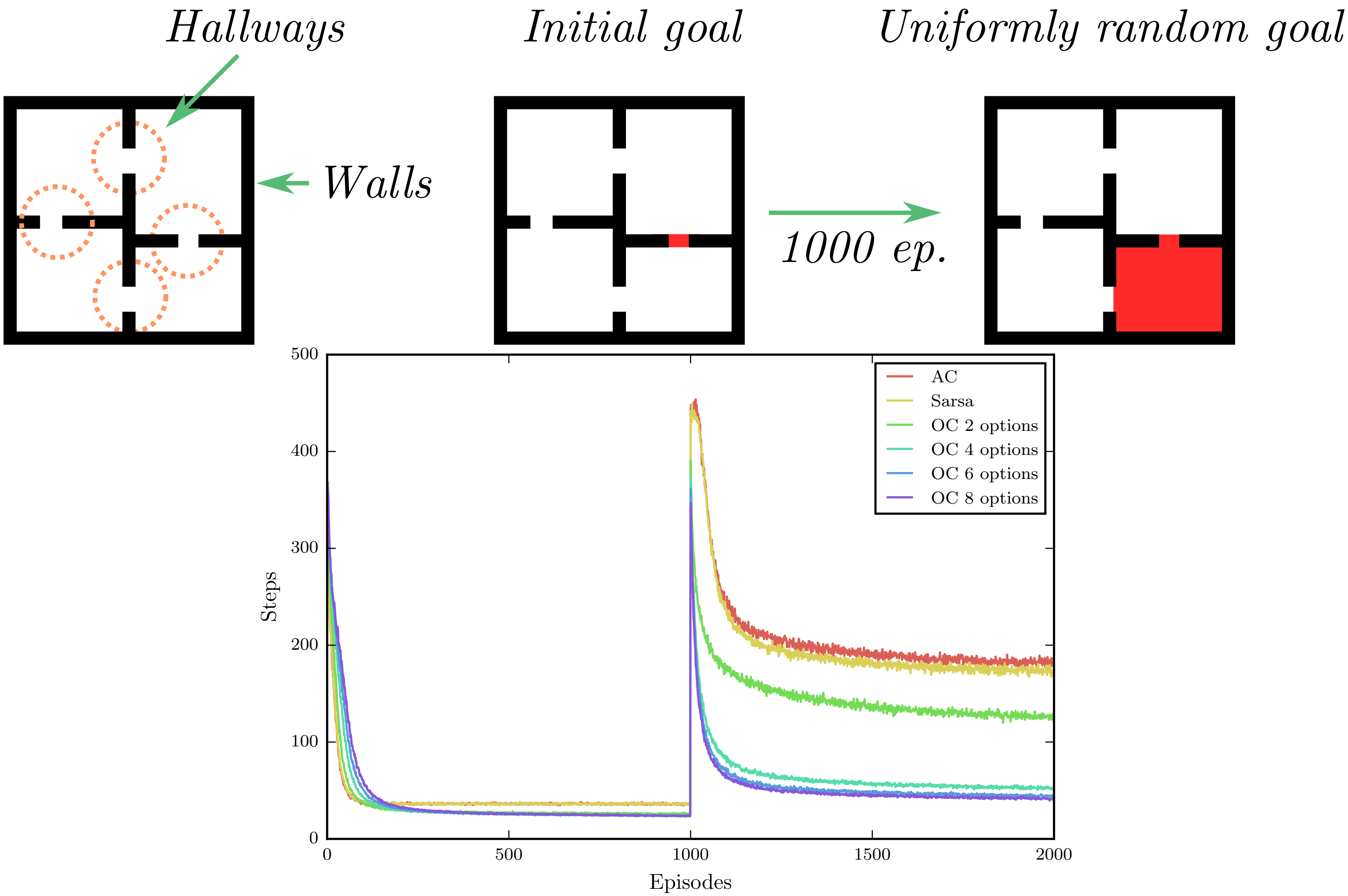
Option-Critic Architecture



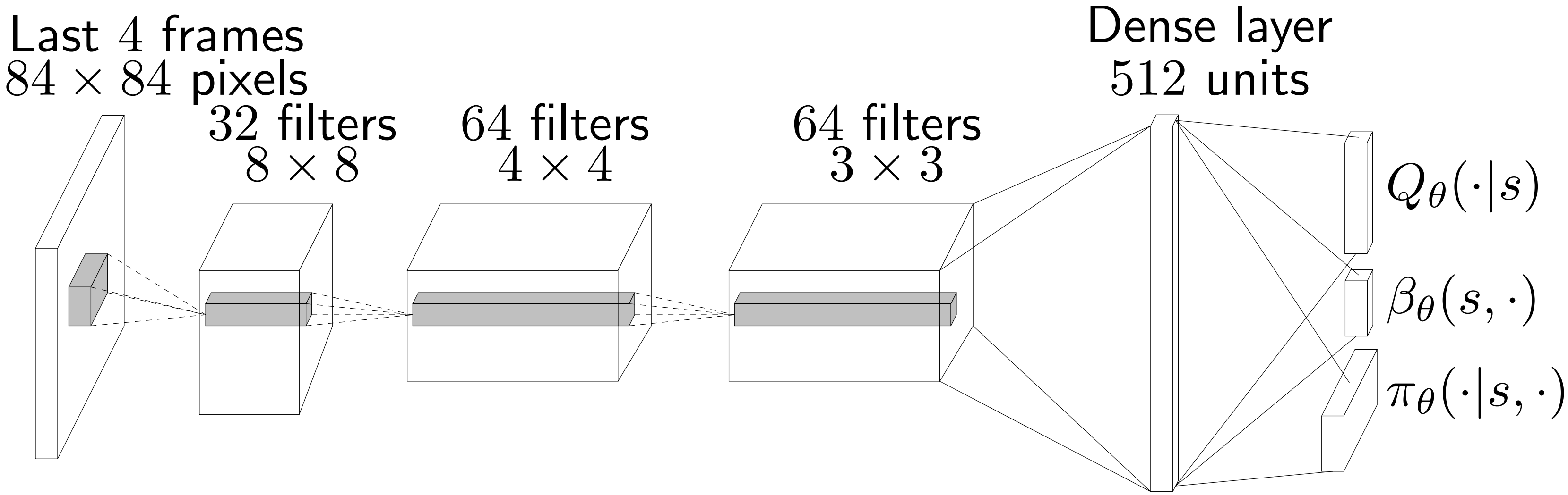
- Given a number of desired options, optimize internal policies and termination conditions using the cumulative reward signal

cf. Bacon et al, AAAI'2017

Results: Transfer in Rooms Domain

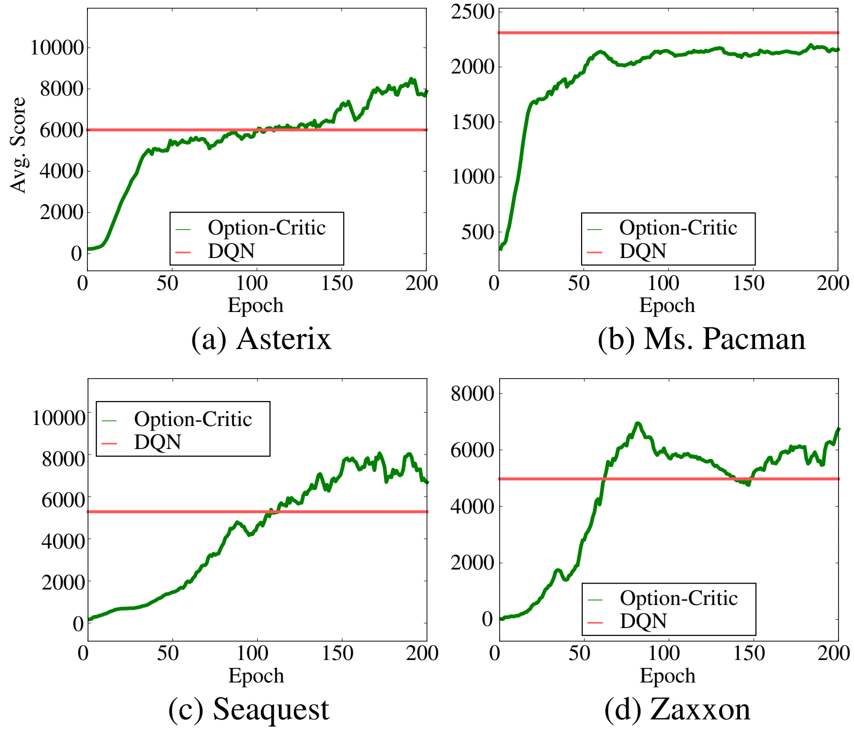


Option-Critic Architecture



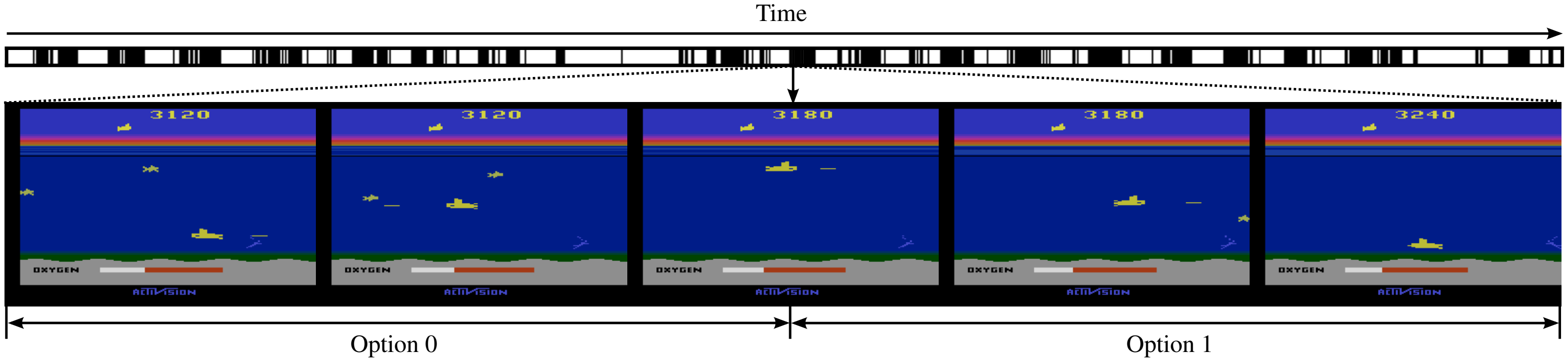
- Given a number of desired options, optimize internal policies and termination conditions using the reward signal
- DQN-style or advantage asynchronous option-critic (A2OC) (other choices possible)

Quantitative results in Atari games



- Performance matching or better than DQN *learning within a single task*
- Out of 8 games tested, option-critic does better that published results in 7, with A3C version superior to DQN - mainly due to exploration

Qualitative results in Atari games



- In Seaquest, separate options are learned to go up and down

Preserving Procedural Knowledge over Time

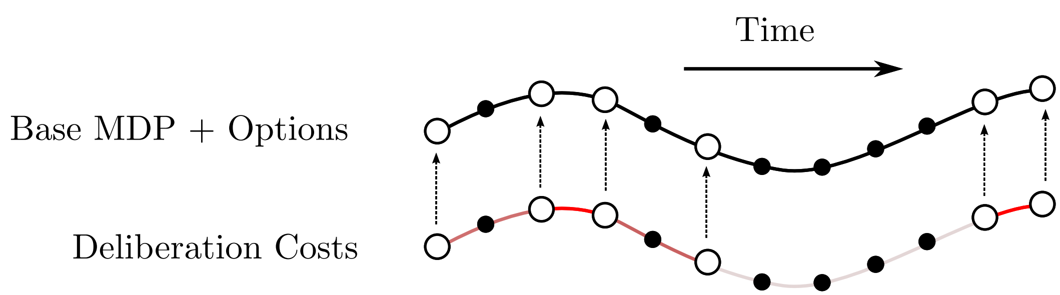
- Successful simultaneous learning of terminations and option policies
- But, as expected, *options shrink over time* unless additional regularization is imposed

  Cf. time-regularized options, Mann et al, (2014)
- Intuitively, using longer options increase the speed of learning and planning (but may lead to a worse result in call-and-return execution)
- Diverse options are useful for exploration in continual learning setting

Bounded Rationality as Regularization

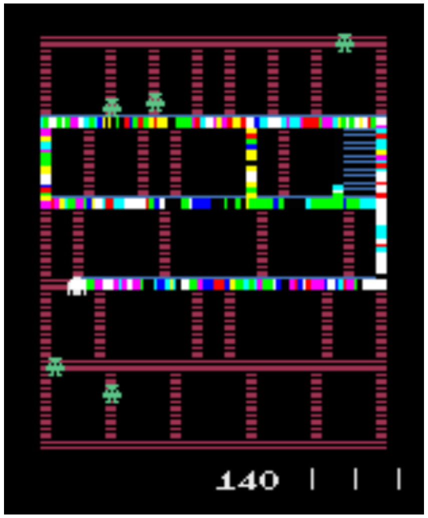
- Problem: optimizing return leads to option collapse (primitive actions are sufficient for optimal behaviour)
- Bounded rationality: reasoning about action choices is expensive (energy consumption and missed-opportunity cost)

  Eg Russell, 1995, Lieder & Griffiths, 2018
- Idea: switching options incurs an additional cost

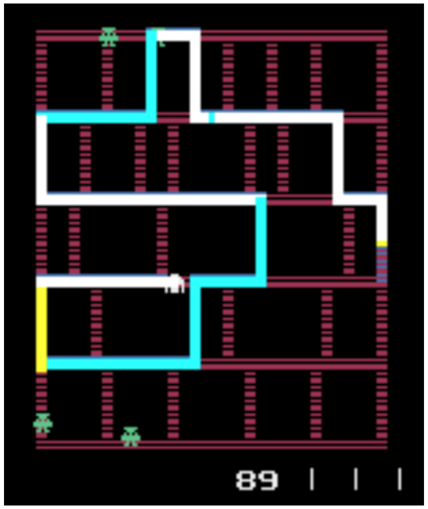


- Can be shown equivalent to requiring that *advantage exceeds a threshold* before switching

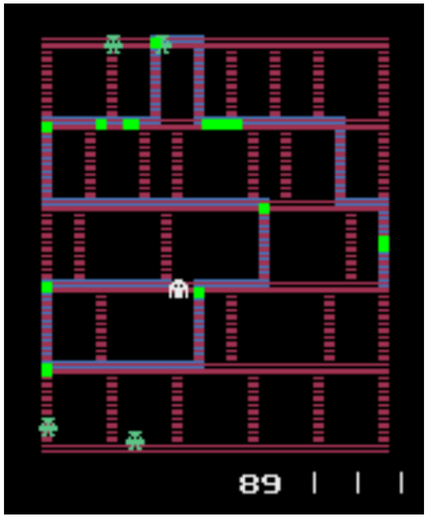
Illustration: Amidar



(a) Without a deliberation cost, options terminate instantly and are used in any scenario without specialization.



(b) Options are used for extended periods and in specific scenarios through a trajectory, when using a deliberation cost.



(c) Termination is sparse when using the deliberation cost. The agent terminates options at intersections requiring high level decisions.

- Deliberation costs prevent options from becoming too short
- Terminations are intuitive

Should All Option Components Optimize the Same Thing?

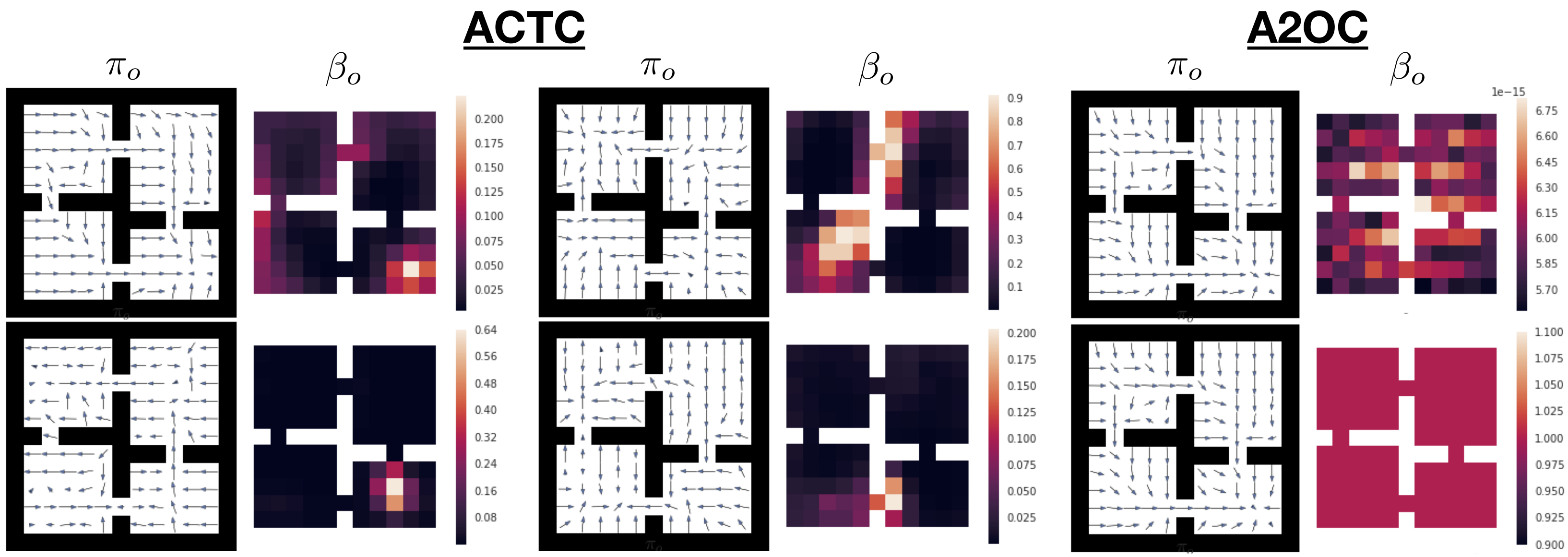
- Deliberation cost can be viewed as associated specifically with termination
- Rewards could be optimized mainly by the internal policy of the option
- Can we generalize this idea to other optimization criteria?

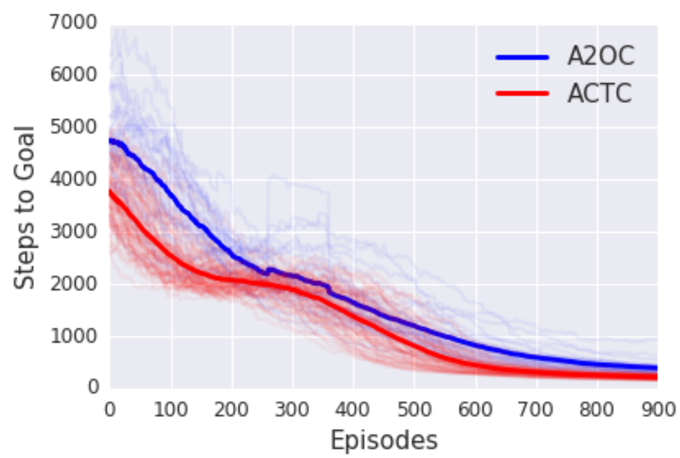
Termination-Critic

- *Optimize the termination condition independently of the policy inside the option*
- Option termination should focus on *predictability* ie finding "funnelling states"
- Interesting side effect: if each option ended at a funelling state, expectation and distribution model would be almost identical and the option would be almost deterministic
- Implementation: minimize the entropy of the option transition model P_ω

cf. Harutyunyan et al, AISTATS'2019

Illustration: Rooms environment





Why is temporal abstraction useful for complex RL tasks

- *Advantages to planning*
  - Need to generate shorter plans
  - Improves robustness to model errors
  - Might need to look at fewer states, since the abstract actions have pre-defined termination conditions
  - Discretize the action space in continuous problems
- *Advantages to learning*
  - Improves exploration (can travel in larger leaps)
  - Gives a natural way of using a single stream of data to learn many things (off-policy learning)
- *Advantages to interpretability*:
  - Focusing attention: Sub-plans ignore a lot of information
  - Improves readability of both models and resulting plans
  - Reduces the problem size