An overview of today’s class

- Myerson’s Lemma (cont’d)
- Application of Myerson’s Lemma
- Revelation Principle
- Intro to Revenue Maximization

(a) An allocation rule $x$ is implementable if and only if it is monotone.

(b) If $x$ is monotone, then there is a unique payment rule such that the sealed-bid mechanism $(x, p)$ is DSIC [assuming the normalization that $b_i = 0$ implies $p_i(b) = 0$].

(c) The payment rule in (b) is given by an explicit formula.
Application of Myerson’s Lemma
Single-item Auctions: Set-up

Allocation Rule: give the item to the highest bidder.

Payment Rule: ?

Allocation Rule: give the item to the highest bidder.
Sponsored Search Auctions: Set-up

Bidders (advertisers)

\[ \begin{align*}
& 1 \quad v_1 \\
& \vdots \\
& i \quad v_i \\
& \vdots \\
& n \quad v_n \\
\end{align*} \]

Slots

\[ \begin{align*}
& \alpha_1 \\
& \vdots \\
& \alpha_j \\
& \vdots \\
& \alpha_k \\
\end{align*} \]

- Allocation Rule: allocate the slots greedily based on the bidders’ bids.

- Payment Rule: ?
Revelation Principle
Q: Why DSIC?

- It’s easy for the bidders to play.
- Designer can predict the outcome with weak assumption on bidders’ behavior.
- But sometimes first price auctions can be useful in practice.

- Can non-DSIC mechanisms accomplish things that DSIC mechanisms can’t?
Two assumptions about DSIC

- Assumption (1): Every participant in the mechanism has a dominant strategy, no matter what its private valuation is.

- Assumption (2): This dominant strategy is direct revelation, where the participant truthfully reports all of its private information to the mechanism.

- There are mechanisms that satisfy (1) but not (2).
  - Run Vickrey on bids × 2...
Assumption (1): Every participant in the mechanism has a dominant strategy, no matter what its private valuation is.

- Can relax (1)? but need stronger assumptions on the bidders’ behavior, e.g. Nash eq. or Bayes-Nash eq.

- Relaxing (1) can give stronger results in certain settings.

- DSIC is enough for most of the simple settings in this class.

- Incomparable: Performance or Robustness?
Assumption 2: This dominant strategy is direct revelation, where the participant truthfully reports all of its private information to the mechanism.

- Comes for “free”.
- Proof: Simulation.
Theorem (Revelation Principle): For every mechanism $M$ in which every participant has a dominant strategy (no matter what its private information), there is an equivalent direct-revelation DSIC mechanism $M'$. 
Revelation Principle

- Same principle can be extended to other solution concept, e.g. Bayes Nash Eq.

- The requirement of truthfulness is not what makes mechanism design hard...

- It’s hard to find a desired outcome in a certain type of Equilibrium.

- Changing the type of equilibrium leads to different theory of mechanism design.
REVENUE-OPTIMAL AUCTION
Why did we start with Welfare?

Obviously a fundamental objective, and has broad real world applications. (government, highly competitive markets)

For welfare, you have DSIC achieving the optimal welfare as if you know the values (single item, sponsored search, and even arbitrary settings (will cover in the future))

Not true for many other objectives.
The only DSIC auctions are the “posted prices”.

If the seller posts a price of $r$, then the revenue is either $r$ (if $v \geq r$), or $0$ (if $v < r$).

If we know $v$, we will set $r = v$. But $v$ is private...

Fundamental issue is that, for revenue, different auctions do better on different inputs.

Requires a model to reason about tradeoffs between different inputs.
Bayesian Analysis/Average Case

Classical Model: pose a distribution over the inputs, and compare the expected performance.

- A single-dimensional environment.
- The private valuation \( v_i \) of participant \( i \) is assumed to be drawn from a distribution \( F_i \) with density function \( f_i \) with support contained in \([0, v_{max}]\).
  - We assume that the distributions \( F_1, \ldots, F_n \) are independent (not necessarily identical).
  - In practice, these distributions are typically derived from data, such as bids in past auctions.
- The distributions \( F_1, \ldots, F_n \) are known in advance to the mechanism designer. The realizations \( v_1, \ldots, v_n \) of bidders’ valuations are private, as usual.
Solution for One Bidder + One Item

- Expected revenue of a posted price $r$ is $r (1 - F(r))$

- When $F$ is the uniform dist. on $[0,1]$, optimal choice of $r$ is $1/2$ achieving revenue $1/4$.

- The optimal posted price is also called the *monopoly price*. 
Two Bidders + One Item

- Two bidders’ values are drawn i.i.d. from $U[0,1]$.

- Revenue of Vickrey’s Auction is the expectation of the min of the two random variables $= 1/3$.

- What else can you do? Can try reserve price.

- Vickrey with reserve at $1/2$ gives revenue $5/12 > 1/3$.

- Can we do better?
Revenue-Optimal Auctions

- [Myerson '81

- Single-dimensional settings
- Simple Revenue-Optimal auction

- [Myerson '81]