COMP/MATH 553 Algorithmic Game Theory
Lecture 2: Mechanism Design Basics

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An overview of the class

Broad View: Mechanism Design and Auctions

First Price Auction

Second Price/Vickrey Auction

Case Study: Sponsored Search Auction
[1] Broader View

- Mechanism Design (MD)
- Auction
What is Mechanism Design?

It’s the Science of Rule Making.
What is Mechanism Design?

- “Engineering” part of Game Theory/Economics

Most of Game Theory/Economics devoted to

- Understanding an existing game/economic system.
- Explain/predict the outcome.

Mechanism Design — reverse the direction

- Identifies the desired outcome/goal first!
- Asks whether the goals are achievable?
- If so, how?
Auctions

Elections, fair division, etc. (will cover if time permits)

Mechanism Design
Auction example 1 – Online Marketplace

MD Auction

[1] Broader View
Auction example 2 – Sponsored Search

Your ads appear beside related search results... People click your ads... And connect to your business

Your ad here
See your ad on Google and our partner sites.
www.our-company-site.com
Auction example 3 – Spectrum Auctions

[Map of the United States with regions labeled]

- at&t
- T-Mobile
- verizonwireless
- Sprint
SINGLE ITEM AUCTION
Bidders: 

- have values on the item.
- These values are Private.
- **Quasilinear utility:**
  - $v_i - p$, if wins.
  - 0, if loses.
Sealed-Bid Auctions:

1. Each bidder $i$ privately communicates a bid $b_i$ to the auctioneer — in a sealed envelope, if you like.

2. The auctioneer decides who gets the good (if anyone).

3. The auctioneer decides on a selling price.
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Goal: Maximize social welfare. (Give it to the bidder with the highest value)
Natural Choice: Give it to the bidder with the highest bid.
The only selection rule we use in this lecture.

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How about the selling price?
Auction Format: Sealed-Bid Auction

How about the **selling price**?

- Altruistic and charge nothing?
  - Name the largest number you can...
  - Fails terribly...
Pay you bid (First Price)?

- Hard to reason about.

- What did you guys bid?

- For two bidders, each bidding half of her value is a Nash eq. Why?
First Price Auction Game played last time

• Assume your value $v_i$ is sampled from $U[0,1]$.

• You won’t overbid, so you will discount your value. Your strategy is a number $d_i$ in $[0,1]$ which specifies how much you want to discount your value, e.g. $b_i = (1-d_i) v_i$

• Game 1: What will you do if you are playing with only one student (picked random) from the class?

• Game 2: Will you change your strategy if you are playing with two other students? If yes, what will it be?
Pay you bid (First Price)?

- For two bidders, each bidding half of her value is a Nash eq. Why?

- How about three bidders? n bidders?
  - Discounting a factor of $1/n$ is a Nash eq.
Pay you bid (First Price)?

- What if the values are not drawn from $U[0,1]$, but from some arbitrary distribution $F$?
  
  $b_i(v) = E[\max_{j \neq i} v_j \mid v_j \leq v]$

- What if different bidders have their values drawn from different distributions?
  
  Eq. strategies could get really complicated...
Example [Kaplan and Zamir ’11]: Bidder 1’s value is drawn from $U[0,5]$, bidder 2’s value is drawn from $U[6,7]$.

**Figure 1:** Equilibrium 1. The thicker line is buyer 1’s bid function.
Example [Kaplan and Zamir ’11]: Bidder 1’s value is drawn from \(U[0,5]\), bidder 2’s value is drawn from \(U[6,7]\).

- Nash eq.: bidder 1 bids 3 if his value is in \([0,3]\), otherwise for \(b\) in \((3, 13/3]\):

\[
v_1(b) = \frac{36}{(2b - 6) \left(\frac{1}{5}\right) e^{\frac{9}{4}} + \frac{6}{6-2b} + 24 - 4b},
\]

\[
v_2(b) = 6 + \frac{36}{(2b - 6)(20) e^{-\frac{9}{4}} - \frac{6}{6-2b} - 4b}.
\]
Pay you bid (First Price)?

- Depends on the number of bidders.
- Depends on your information about other bidders.
- Optimal bidding strategy complicated!
- Nash eq. might not be reached.
- Winner might not value the item the most.
Another idea

- Charge the winner the second highest bid.

- Seems arbitrary...

- But actually used in Ebay.
Lemma 1: In a second-price auction, every bidder has a dominant strategy: set its bid $b_i$ equal to its private valuation $v_i$. That is, this strategy maximizes the utility of bidder $i$, no matter what the other bidders do.

- Super easy to participate in. (unlike first price)
- Proof: See the board.
Lemma 2: In a second-price auction, every *truthful* bidder is guaranteed non-negative utility.

Proof: See the board.
The Vickrey auction is has three quite different and desirable properties:

1. **[strong incentive guarantees]** It is dominant-strategy incentive-compatible (DSIC), i.e., Lemma 1 and 2 hold.

2. **[strong performance guarantees]** If bidders report truthfully, then the auction *maximizes the social welfare* $\sum_i v_i x_i$, where $x_i$ is 1 if $i$ wins and 0 if $i$ loses.

3. **[computational efficiency]** The auction can be implemented in polynomial (indeed linear) time.
These three properties are criteria for a good auction:

- More complicated allocation problem
- Optimize Revenue
Case Study:  
Sponsored Search  
Auction
Sponsored Search Auction

Your ads appear beside related search results...

People click your ads...

...And connect to your business

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See your ad on Google and our partner sites.

www.our-company-site.com
In 2012, sponsored search auction generates 43.6 billion dollars for Google, which is 95% of its total revenue.

In the meantime, the market grows by 20% per year.
- \( k \) slots for sale.
- Slot \( j \) has click-through-rate (CTR) \( \alpha_j \).
- Bidder \( i \)'s value for slot \( j \) is \( \alpha_j v_i \).
- Two complications:
  - Multiple items
  - Items are non identical
(1) DSIC. That is, truthful bidding should be a dominant strategy, and never leads to negative utility.

(2) Social welfare maximization. That is, the assignment of bidders to slots should maximize \( \sum v_i x_i \), where \( x_i \) now denotes the CTR of the slot to which \( i \) is assigned (or 0 if \( i \) is not assigned to a slot). Each slot can only be assigned to one bidder, and each bidder gets only one slot.

(3) Polynomial running time. Remember zillions of these auctions need to be run every day!