COMP/MATH 553 Algorithmic Game Theory
Lecture 2: Mechanism Design Basics

Sep 8, 2014

Yang Cai

An overview of the class

- *Broad View: Mechanism Design and Auctions*
- *First Price Auction*
- *Second Price/Vickrey Auction*
- *Case Study: Sponsored Search Auction*

[1] Broader View

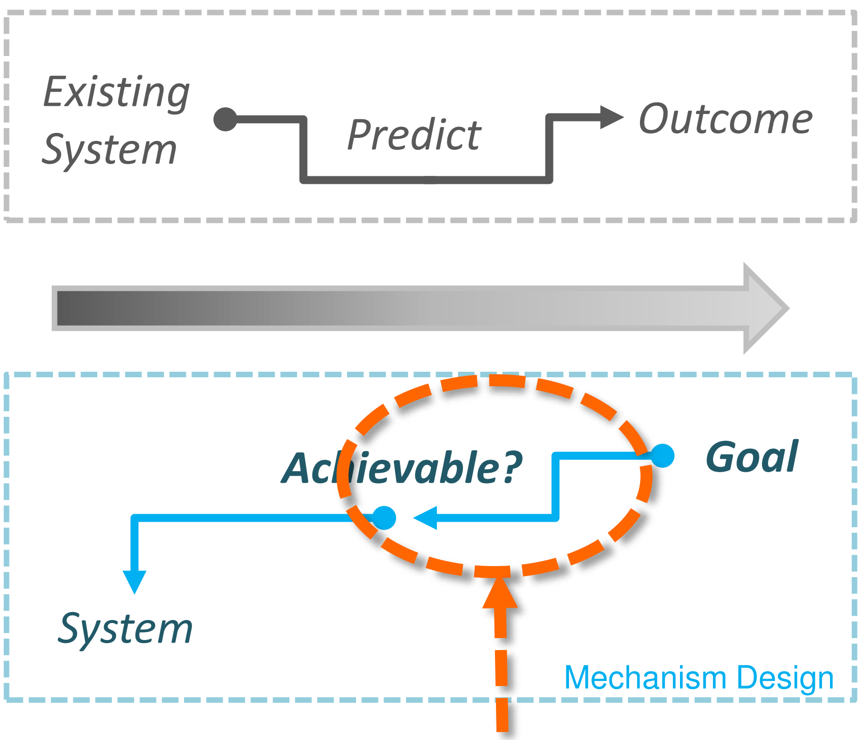
- ❑ Mechanism Design (MD)
- ❑ Auction

What is Mechanism Design?

It’s the Science of Rule Making.

What is Mechanism Design?

❖ **“Engineering” part of Game Theory/Economics**



❑ Most of Game Theory/Economics devoted to

- Understanding an **existing** game/economic system.
- Explain/predict the outcome.

❑ **Mechanism Design – reverse the direction**

- Identifies the desired outcome/goal first!
- Asks whether the goals are achievable?
- If so, how?

Auctions

Mechanism Design

Auctions

Elections, fair division, etc. (will cover if time permits)

Auction example 1 – **Online Marketplace**

ebay

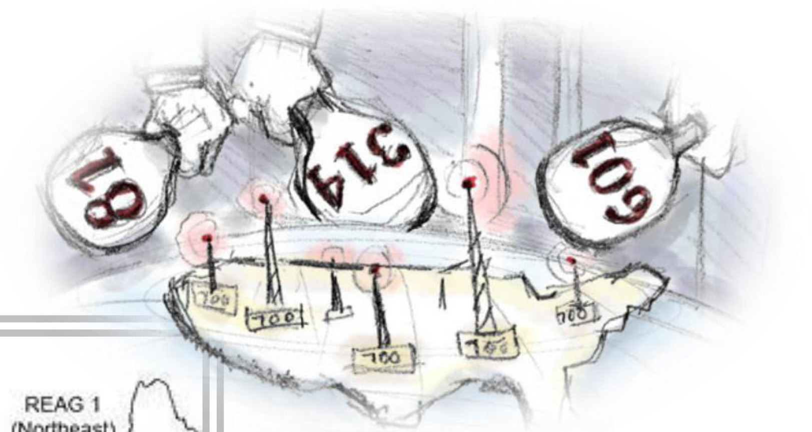
priceline.com
Name Your Own Price

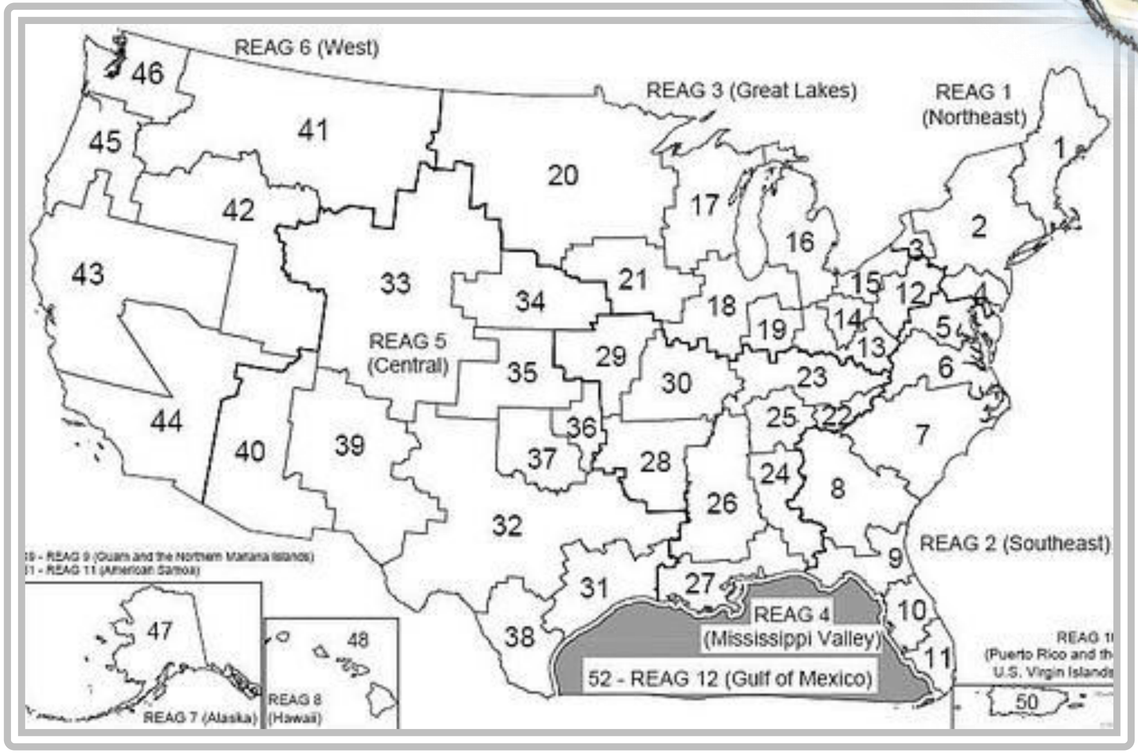
Hotwire.com
Fly. Sleep. Drive. Cheap.

skyauction.com
now you're going somewhere!

Auction example 2 – **Sponsored Search**

Auction example 3 – **Spectrum Auctions**





at&t

T · · Mobile ·

verizon wireless

Sprint

SINGLE ITEM AUCTION

Single-item Auctions: Set-up

Bidders

1 v_1

i v_i

n v_n

Auctioneer

Item

Bidders:

- have ***values*** on the item.
- These values are ***Private***.
- ***Quasilinear utility***:
  - $v_i - p$, if wins.
  - 0, if loses.

Auction Format: Sealed-Bid Auction

Sealed-Bid Auctions:

1. Each bidder ***i*** privately communicates a bid b_i to the auctioneer — in a sealed envelope, if you like.
2. The auctioneer decides ***who*** gets the good (if anyone).
3. The auctioneer decides on a ***selling price***.

Auction Format: Sealed-Bid Auction

Bidders

1  v_1

⋮

i  v_i

⋮

n  v_n

Auctioneer

Item

Sealed-Bid Auctions:

Goal: Maximize social welfare. (Give it to the bidder with the highest value)
Natural Choice: Give it to the bidder with the highest bid.
The only selection rule we use in this lecture.

2. The auctioneer decides ***who*** gets the good (if anyone).

3. The auctioneer decides on a ***selling price***.

Auction Format: Sealed-Bid Auction

Bidders

1  v_1

⋮

i  v_i

⋮

n  v_n

Auctioneer

Item

Sealed-Bid Auctions

1. Each bidder ***i*** p

   auctioneer — i

2. The auctioneer decides ***wh*** gets good (if anyone).
3. The auctioneer decides on a ***selling price***.

How about the selling price?

Auction Format: Sealed-Bid Auction

How about the selling price?

- Altruistic and charge nothing?
  - Name the largest number you can...
  - Fails terribly...

First Price Auction

- Pay you bid (First Price)?
  - Hard to reason about.
  - What did you guys bid?
  - For two bidders,each bidding half of her value is a Nash eq. Why?

First Price Auction Game played last time

- Assume your value v_i is sampled from U[0,1].
- You won't overbid, so you will discount your value. Your strategy is a number d_i in [0,1] which specifies how much you want to discount your value, e.g. $b_i = (1-d_i)\ v_i$
- Game 1: What will you do if you are playing with only one student (picked random) from the class?
- Game 2: Will you change your strategy if you are playing with two other students? If yes, what will it be?

First Price Auction

- Pay you bid (First Price)?
  - For two bidders,each bidding half of her value is a Nash eq. Why?
  - How about three bidders? n bidders?
    - Discounting a factor of 1/n is a Nash eq.

First Price Auction

- Pay you bid (First Price)?
  - What if the values are not drawn from U[0,1], but from some arbitrary distribution F?
    - $b_i(v) = E[\max_{j \neq i} v_j \mid v_j \leq v]$
  - What if different bidders have their values drawn from different distributions?
    - Eq. strategies could get really complicated...

First Price Auction

- Example [Kaplan and Zamir '11]: Bidder 1's value is drawn from U[0,5], bidder 2's value is drawn from U[6,7].

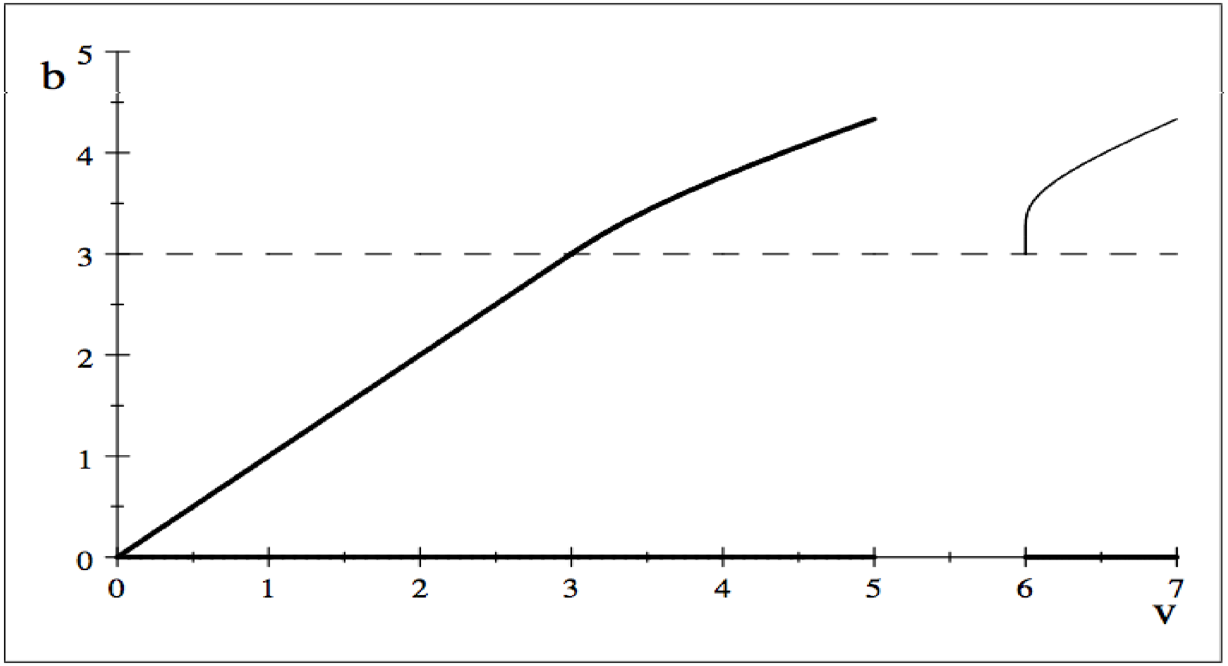


Figure 1: Equilibrium 1. The thicker line is buyer 1's bid function.

First Price Auction

❖ Example [Kaplan and Zamir '11]: Bidder 1's value is drawn from U[0,5], bidder 2's value is drawn from U[6,7].

- Nash eq. : bidder 1 bids 3 if his value is in [0,3], otherwise for b in (3, 13/3]:

$$v_1(b) = \frac{36}{(2b-6)\left(\frac{1}{5}\right)e^{\frac{9}{4}+\frac{6}{6-2b}} + 24 - 4b},$$

$$v_2(b) = 6 + \frac{36}{(2b-6)(20)e^{-\frac{9}{4}-\frac{6}{6-2b}} - 4b}.$$

First Price Auction

❖ Pay you bid (First Price)?

- Depends on the number of bidders.
- Depends on your information about other bidders.
- Optimal bidding strategy complicated!
- Nash eq. might not be reached.
- Winner might not value the item the most.

Second Price/Vickrey Auction

- Another idea
  - Charge the winner the second highest bid.
  - Seems arbitrary...
  - But actually used in Ebay.

Second-Price/Vickrey Auction

Lemma 1: In a second-price auction, every bidder has a ***dominant strategy***: set its bid b_i equal to its private valuation v_i. That is, this strategy maximizes the utility of bidder i, no matter what the other bidders do.

- Super easy to participate in. (unlike first price)
- Proof: See the board.

Second-Price/Vickrey Auction

Lemma 2: In a second-price auction, every ***truthful*** bidder is guaranteed non-negative utility.

Proof: See the board.

Second Price/Vickrey Auction

[Vickrey '61  **]** The Vickrey auction is has three quite different and desirable properties:

(1) **[strong incentive guarantees]** It is dominant-strategy incentive-compatible (DSIC), i.e., Lemma 1 and 2 hold.

(2) **[strong performance guarantees]** If bidders report truthfully, then the auction ***maximizes the social welfare*** $\Sigma_i v_i x_i$, where x_i is 1 if i wins and 0 if i loses.

(3) **[computational efficiency]** The auction can be implemented in **polynomial** (indeed linear) time.

What's next?

These three properties are criteria for a good auction:

- ❖ More complicated allocation problem
- ❖ Optimize Revenue

Case Study:

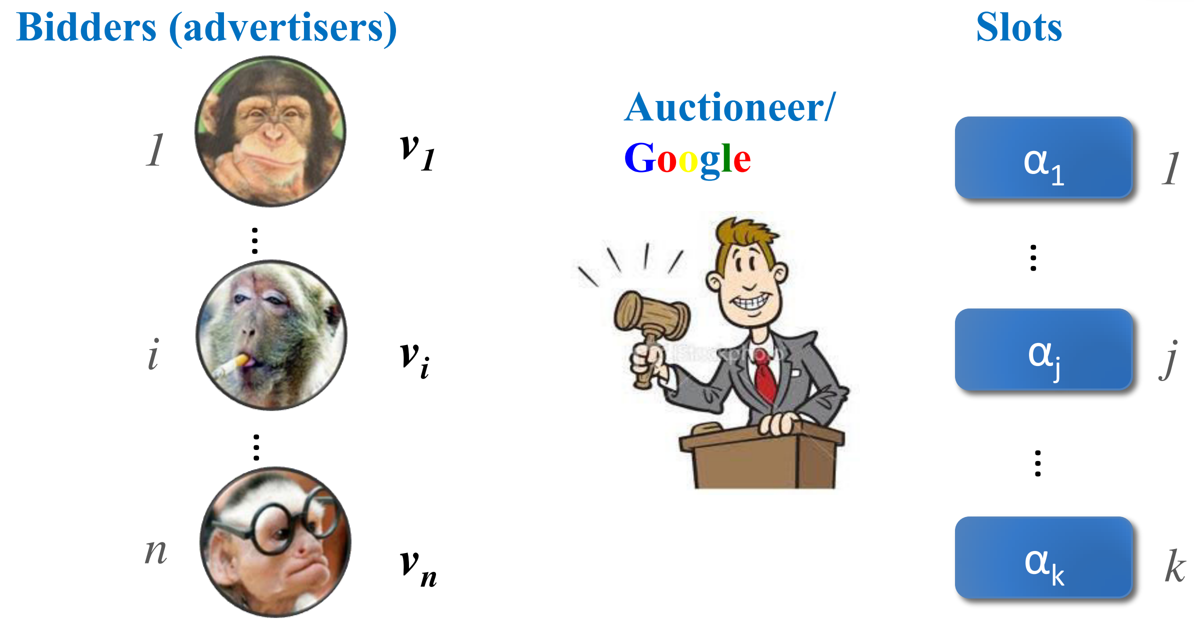
Sponsored Search Auction

Sponsored Search Auction

In 2012, sponsored search auction generates 43.6 billion dollars for Google, which is 95% of its total revenue.

In the meantime, the market grows by 20% per year.

Sponsored Search **Auctions: Set-up**



- k slots for sale.
- Slot j has click-through-rate (CTR) α_j.
- Bidder i's value for slot j is $\alpha_j v_i$.
- Two complications:
  - Multiple items
  - Items are non identical

Sponsored Search Auction: Goal

(1)DSIC. That is, truthful bidding should be a dominant strategy, and never leads to negative utility.

(2) Social welfare maximization. That is, the assignment of bidders to slots should maximize $\Sigma v_i x_i$,
where x_i now denotes the CTR of the slot to which i is assigned (or 0 if i is not assigned to a slot). Each slot can only be assigned to one bidder, and each bidder gets only one slot.

(3) Polynomial running time. Remember zillions of these auctions need to be run every day!