Formulating Problems as Maximum Flow Problems

We completed our discussion of edge disjoint paths in directed graphs by showing that for any digraph G, the maximm number of edge disjoint s-t paths of G is equal to the minimum size of a set of edges whose removal separates s from t. This is (7.45) of KT. We then discussed the analagous result for undirected graphs. This is section 7.6 of KT.

We saw how to formulate Airline Scheduling as a Bipartite matching problen. We used the graph G defined in Section 7.9 of KT, except that we did not add the edges st as we discussed using exactly k planes, not at most k planes. We saw that we needed to find k edge disjoint paths using all edges of G of the form $u_i v_i$. We saw that of there were f flights this meant we used $f - k$ edges of the form $v_i u_j$ where flight j is reachable from flight i. We saw that these edges formed a matching, and more strongly that it was easy to construct the desired schedule if there was a matching of size $f - k$ using such edges. So to solve our schedule problem we needed to find a maximum matching in the graph formed by such edges. Since this graph has $O(f)$ edges and $O(f^2)$ edges this takes $O(f^3)$ time.

We then described how to formulate Image Segmentation as a Maximum Flow Problem. This is Sections 7.10 of KT.