Tabled higher-order logic programming

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Outline

- Logical frameworks and certified code
- Tabled higher-order logic programming
  - Basic idea and challenges
  - Experiments and Evaluation
  - Improving efficiency
- Conclusion and future work

Deductive systems and logical frameworks

Deductive systems are plentiful computer science.

- Axioms and inference rules
- Examples: operational semantics, type system, logic, etc.

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Logical framework: meta-language for deductive systems

- High-level specifications (e.g. type system)
- Execution via logic programming interpretation (e.g. type checker)
- Meta-reasoning via theorem prover combining induction and logic programming search (e.g. type preservation)

Declarative description of subtyping

types τ ::= zero | pos | nat | bit | $\tau_1 \Rightarrow \tau_2$ | $\ldots$

Example: $6 = \epsilon 110$ and $\epsilon 110 \in$ nat

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Example: $6 = \epsilon 110$ and $\epsilon 110 \in$ nat

$$\frac{}{\text{zero} \preceq \text{nat}}\ \text{zn} \qquad \frac{}{\text{pos} \preceq \text{nat}}\ \text{pn} \qquad \frac{}{\text{nat} \preceq \text{bit}}\ \text{nb}$$

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$$\frac{}{\tau \preceq \tau}\ \text{refl} \qquad \frac{\tau_1 \preceq \tau \qquad \tau \preceq \tau_2}{\tau_1 \preceq \tau_2}\ \text{tr}$$

Typing rules for Mini-ML

expressions e ::= $\epsilon \mid e\,0 \mid e\,1 \mid$ fun $x.e \mid$ app $e_1\ e_2$

$$\frac{\Gamma \vdash e : \tau' \qquad \tau' \preceq \tau}{\Gamma \vdash e : \tau}\ \text{tp-sub}$$

$$\frac{\Gamma, x{:}\tau_1 \vdash e : \tau_2}{\Gamma \vdash \text{fun}\ x.e : \tau_1 \Rightarrow \tau_2}\ \text{tp-fun}$$

Implementation of subtyping

```
zn:  sub zero nat.
pn:  sub pos nat.
nb:  sub nat bit.
refl:  sub T T.
tr:  sub T S
     <- sub T R
     <- sub R S.
```

Implementation of subtyping

```
zn:  sub zero nat.
pn:  sub pos nat.        ?- sub zero bit.
nb:  sub nat bit.
refl:  sub T T.
tr:  sub T S
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Implementation of subtyping

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zn:   sub zero nat.
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tr:   sub T S
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```

```
?- sub zero bit.
```

yes

Proof: `(tr nb zn)`

Implementation of typing rules

```
tp_sub:  of E T
      <- of E T'
      <- sub T' T.

tp_fun:  of (fun λx.E x) (T1 => T2)
      <-(Πx:exp.of x T1 -> of (E x) T2).
```

"forall `x:exp`, assume `of x T1`
and show `of (E x) T2`"

Higher-order logic programming

- Higher-order data-types:
  - λ-abstraction
  - dependent types
- Dynamic program clauses
- Explicit proof objects

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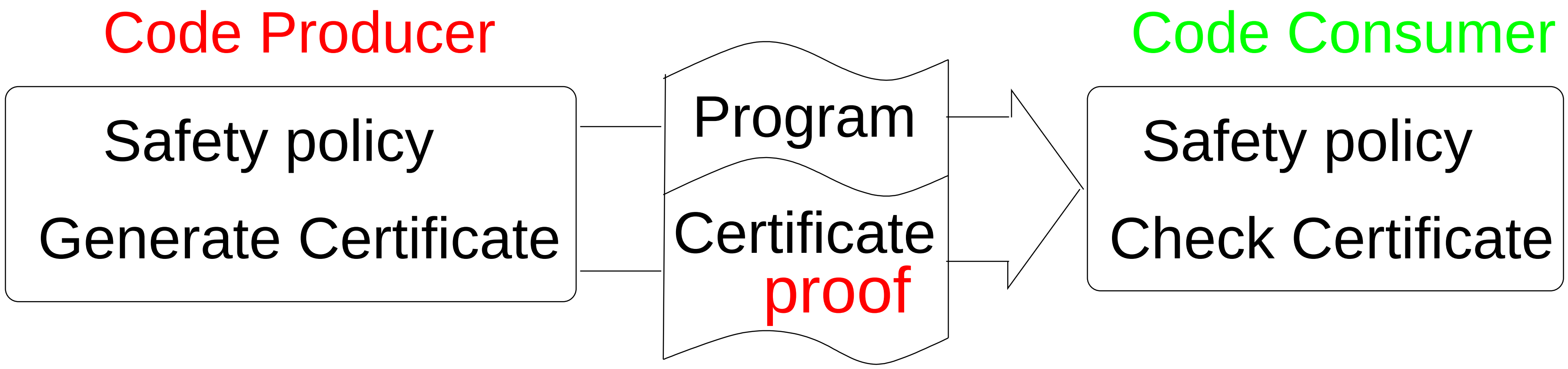
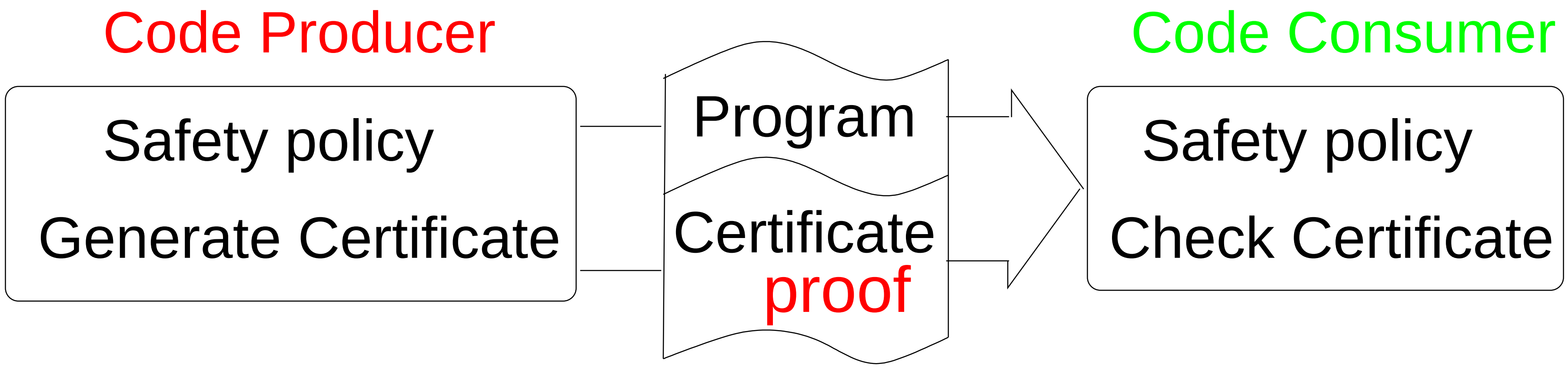
Different approaches: λProlog, Isabelle, Twelf

Application: certified code



- Foundational proof-carrying code : [Appel, Felty 00]
- Temporal-logic proof carrying code [Bernard,Lee02]
- Foundational typed assembly language : [Crary 03]
- Proof-carrying authentication: [Felten, Appel 99]

Application: certified code



Large-scale applications

- Typical code size: 70,000 – 100,000 lines
  includes data-type definitions and proofs
- Higher-order logic program: 5,000 lines
- Over 600 – 700 clauses

Some limitations in practice

- Straightforward specifications are not executable.
- Redundancy severely hampers performance.
- Meta-reasoning capabilities limited in practice.

Overcome some of these limitations using
tabelling and other optimizations!

This thesis

Tabled higher-order logic programming allows us to

- efficiently execute logical systems
  (interpreter using tabled search)
- automate the reasoning with and about them.
  (meta-theorem prover using tabled search)

This is a significant step towards
applying logical frameworks in practice.

Contributions

Tabled higher-order logic programming

- Characterization based on uniform proofs (ICLP'02)
- Implementation of a tabled interpreter
- Case studies (parsing, refinement types, rewriting)(LFM'02)

Efficient data-structures and algorithms

- Foundation for meta-variables (LFM'03)
- Optimizing higher-order unification (CADE'03)
- Higher-order term indexing (ICLP'03)

Meta-reasoning based on tabled search

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The idea

"...it is very common for the proofs to have repeated sub-proofs that should be hoisted out and proved only once ..." [Necula,Lee97]

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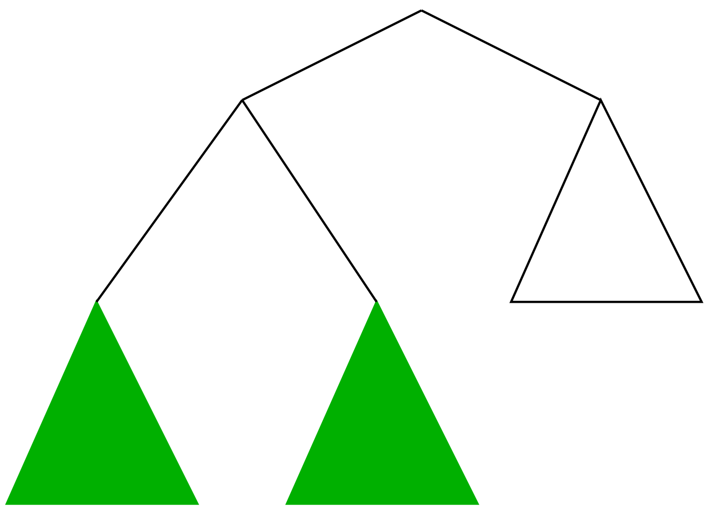

Redundant computation

The idea

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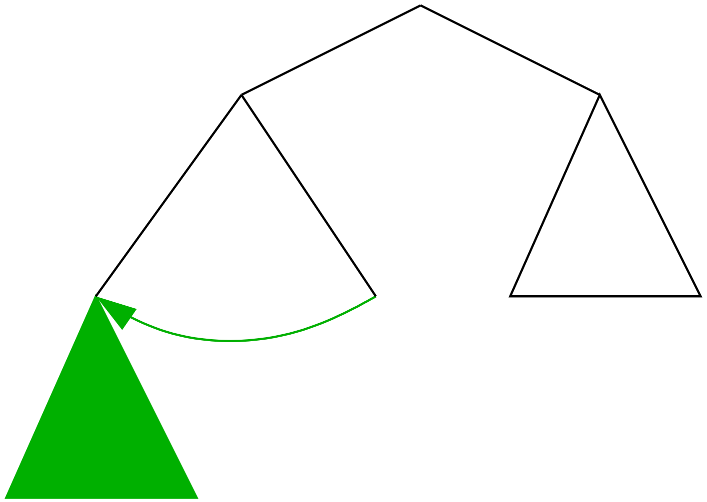

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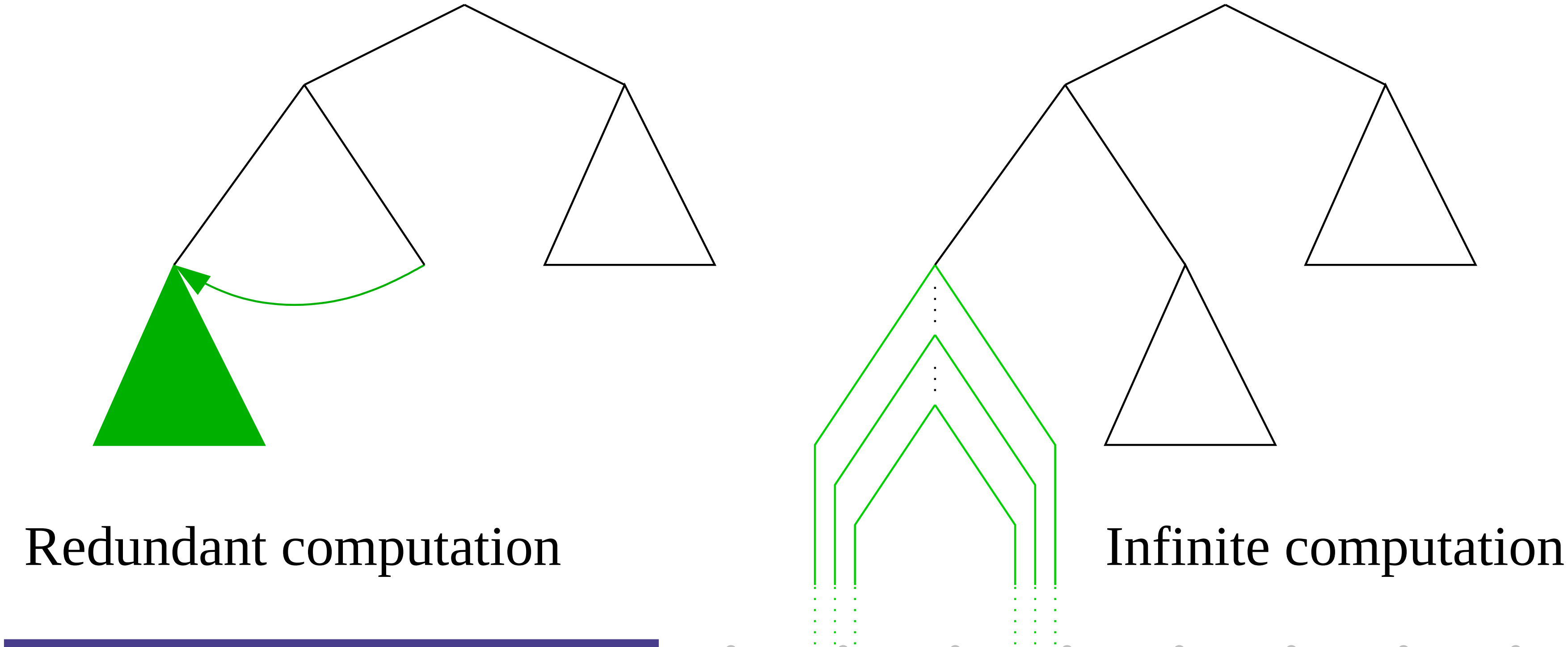


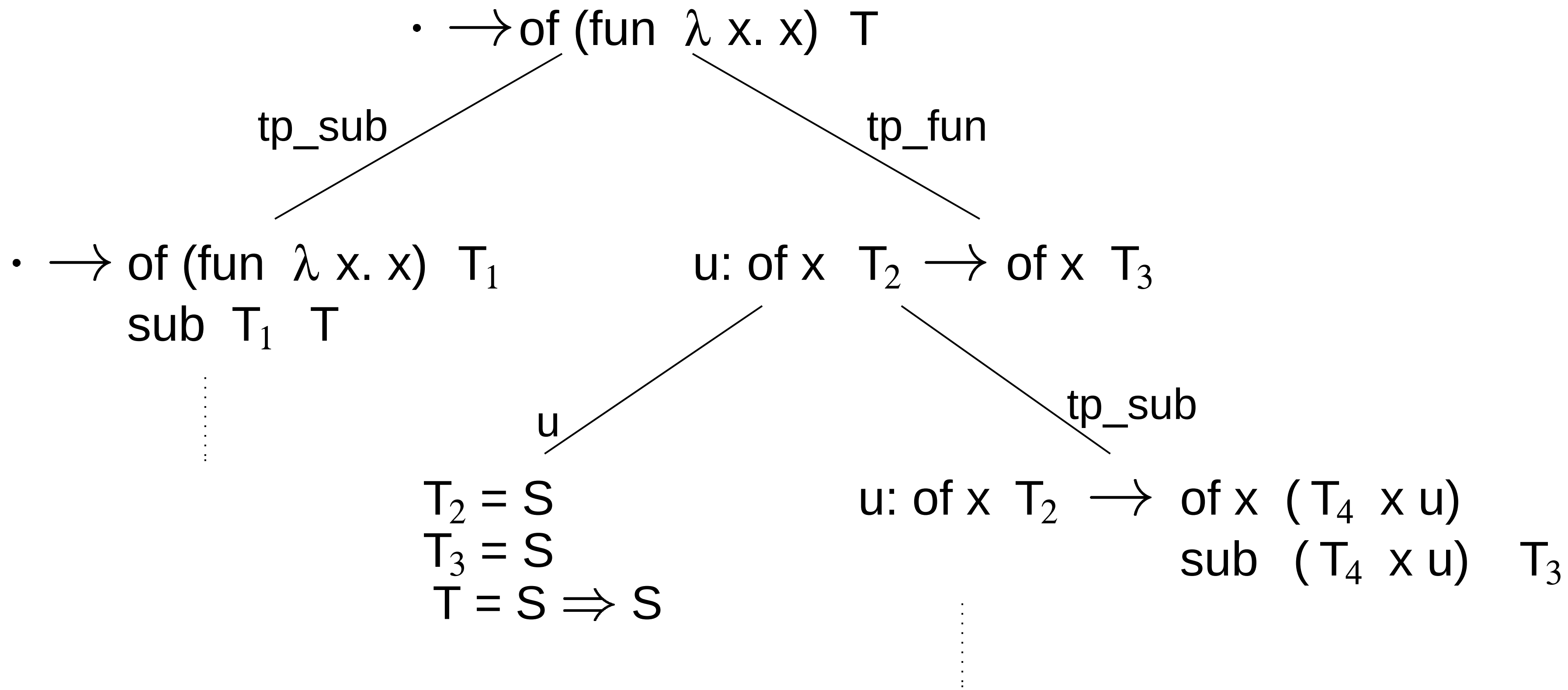
Recall...subtyping

```
tp_sub:  of E T
      <- of E T'
      <- sub T' T.

tp_fun:  of (fun λx.E x) (T1 => T2)
      <-(Πx:exp.of x T1 -> of (E x) T2).
```

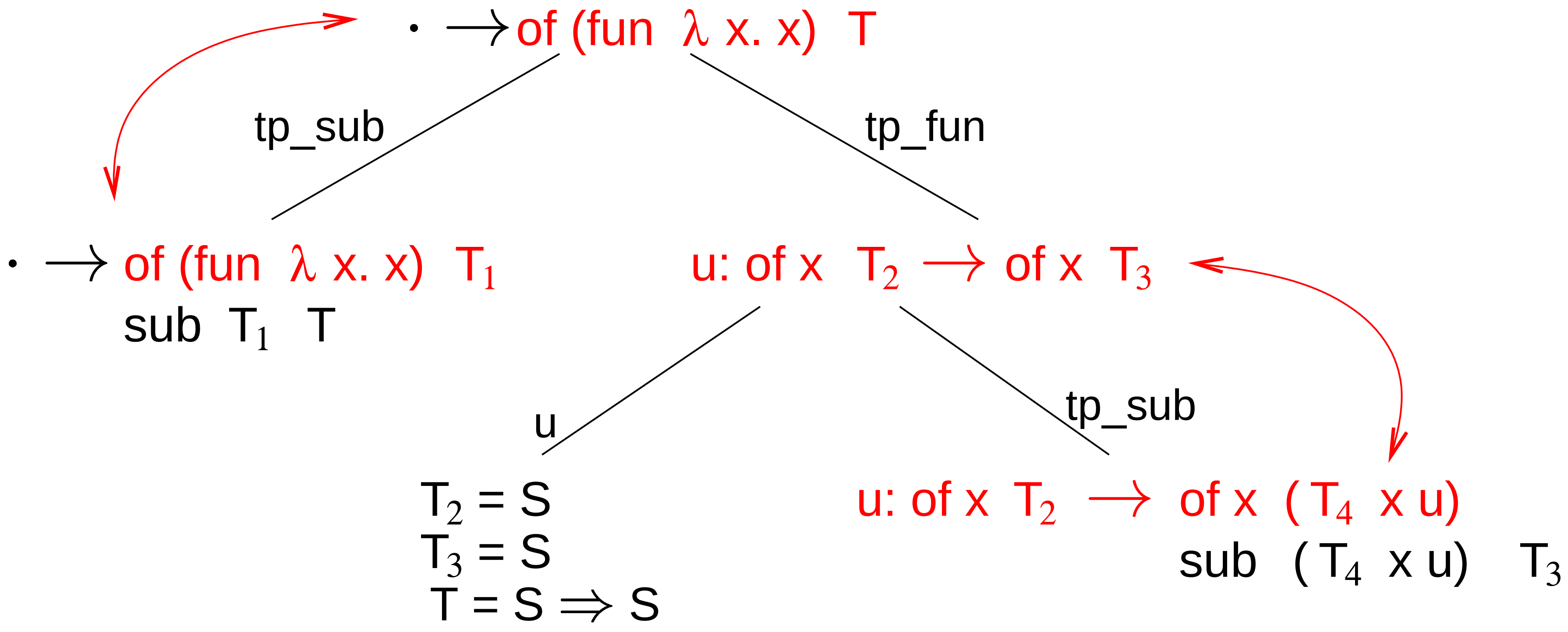
"forall `x:exp`, assume `of x T1`
and show `of (E x) T2`"

Proof tree



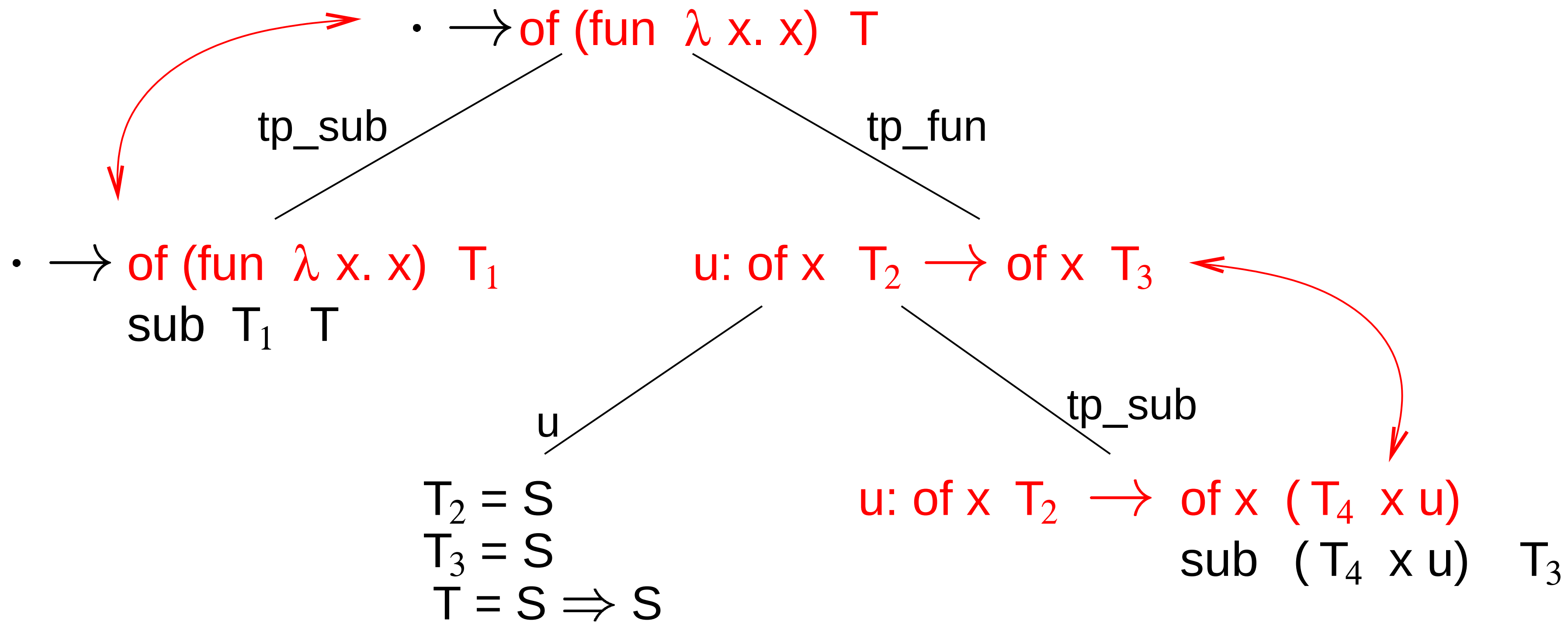
· ⟶ of (fun λ x. x) T
tp_sub
tp_fun
· ⟶ of (fun λ x. x) T_1
sub T_1 T
u: of x T_2 ⟶ of x T_3
u
tp_sub
T_2 = S
T_3 = S
T = S ⇒ S
u: of x T_2 ⟶ of x (T_4 x u)
sub (T_4 x u) T_3

Proof tree



Loop detection

Proof tree



Loop detection

How can we detect loops?

Loops modulo strengthening

- Dependencies among terms

$$\text{u:of x } T_2 \longrightarrow \text{of x } (T_4 \text{ x u})$$

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strengthen $\quad \text{u:of x } T_2 \rightarrow \text{of x } T_4$

Loops modulo strengthening

- Dependencies among terms

  u:of x $T_2 \longrightarrow$ of x (T_4 x u)

  strengthen u:of x $T_2 \longrightarrow$ of x T_4

- Dependencies among propositions

  u:of x $T_2 \longrightarrow$ sub (T_4 x u) T_3

Loops modulo strengthening

- Dependencies among terms

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strengthen $\text{u:of x } T_2 \rightarrow \text{of x } T_4$

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Loops modulo strengthening

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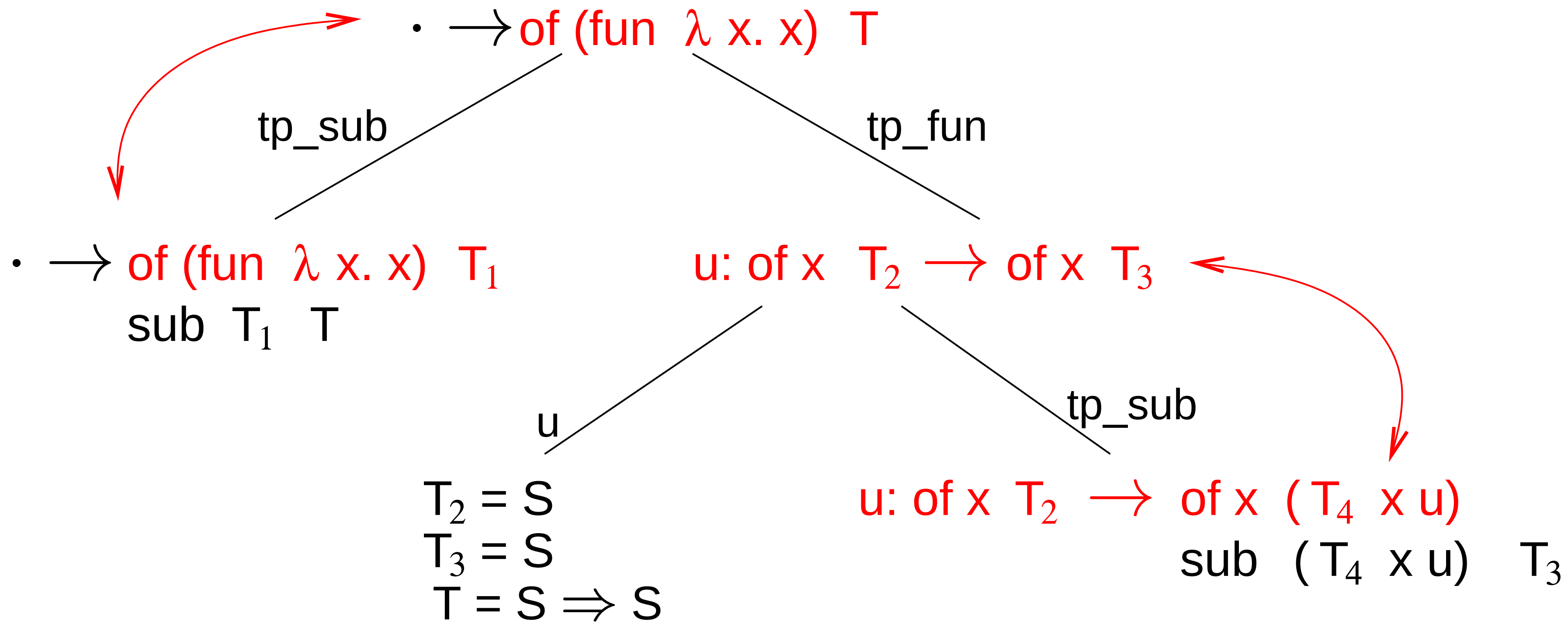
strengthen: $\quad \cdot \to \text{sub } T_4 \; T_3$

- Subordination analysis [Virga99]

Proof tree (cont.)



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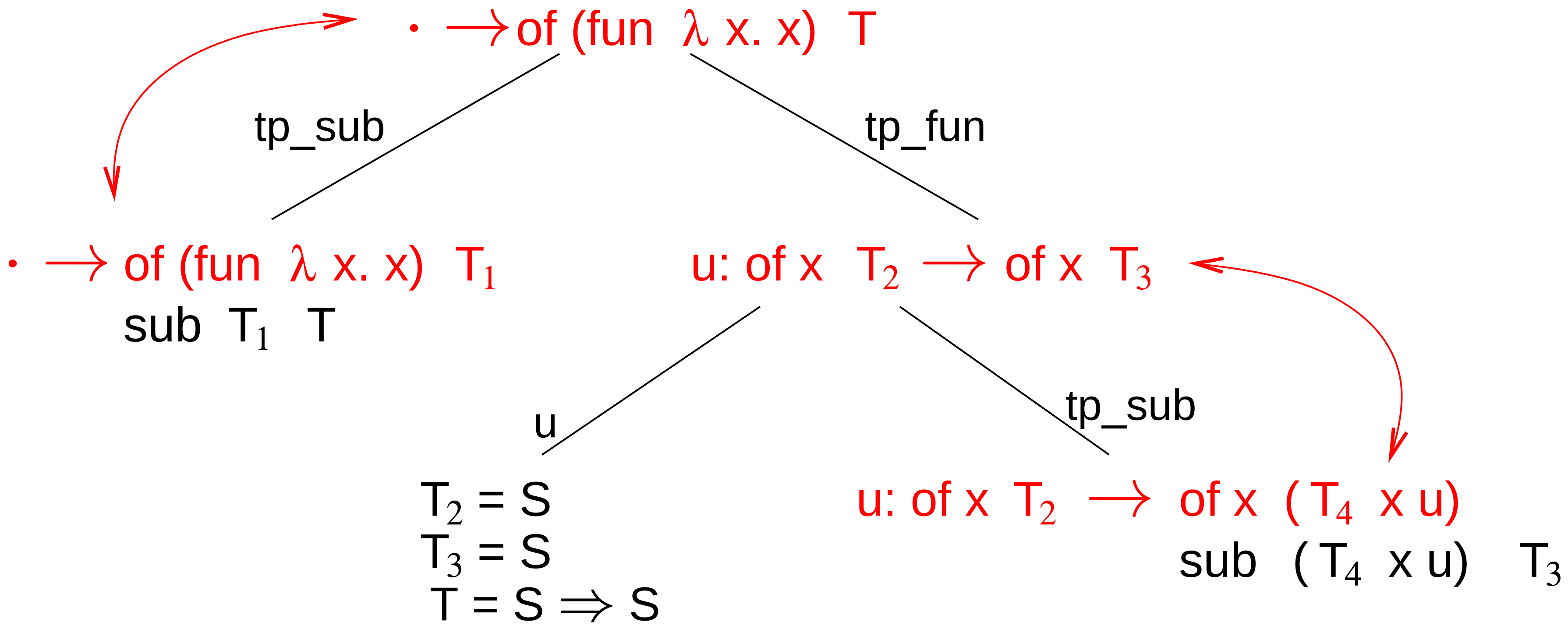
How can we detect loops?

Proof tree (cont.)



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How can we detect loops? Subordination

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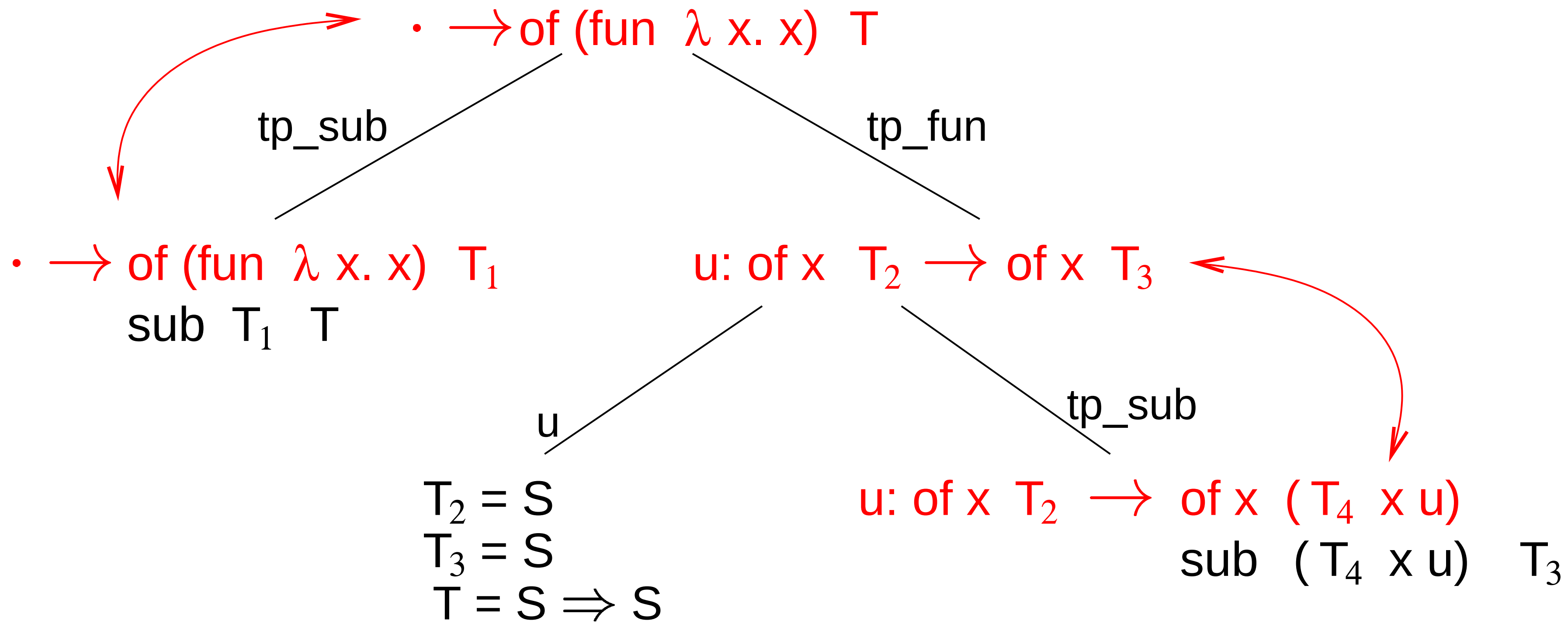
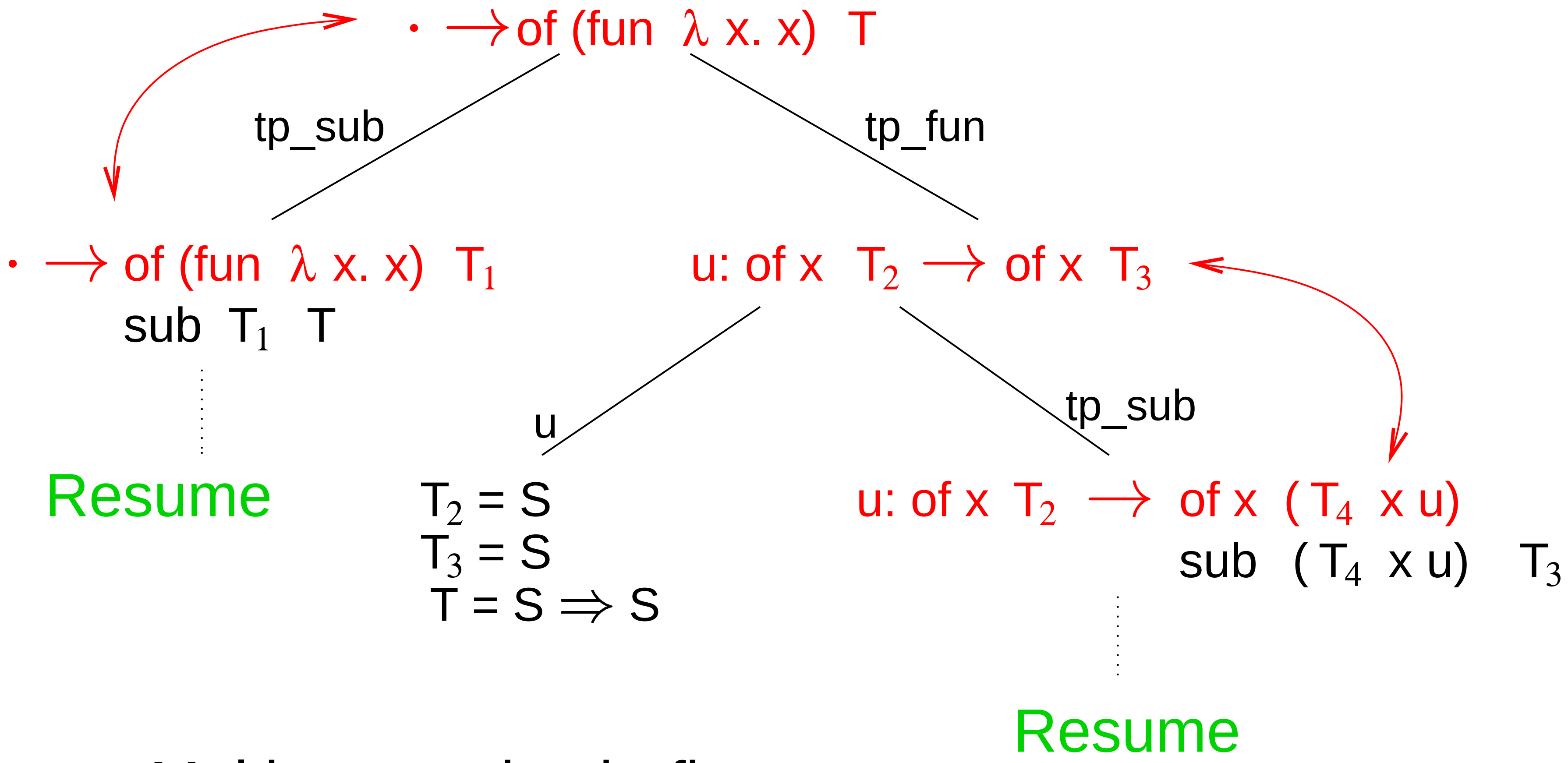


Loop detection

How can we detect loops? Subordination

How can we still produce all answers?

Proof tree (cont.)



Multi−stage depth−first strategy
adapted from [Tamaki, Sato89]

Memoization based proof search

- Proof search using a memo-table
- Store intermediate goals and re-use results
- May need to use subordination!
- Eliminate redundant computation
- Eliminate infinite paths
- More specifications are executable!

Memo-table

- Table entry: ($\Gamma \rightarrow a$, $\mathcal{A}$)
  - Γ : context of assumptions (i.e. u:of x T_2)
  - a : atomic goal (i.e. of (fun λx. x) T, of x T_3)
  - $\mathcal{A}$: list of answer substitutions for all existential variables in Γ and a

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  - $\mathcal{A}$: list of answer substitutions for all existential variables in Γ and a

Goal			Answer
$\cdot$	$\to$	of (fun λx.x) T	T = S $\Rightarrow$ S
u:of x T_2	$\to$	of x T_3	T_2 = S, T_3 = S

Properties

- Selective memoization
- Finds all answers to a query
- Terminates for programs over a finite domain

Theoretical foundation

Conservative extension of LF [Harper *et. al.* 93] with meta-variables

- Foundation for proof search and for other optimization (e.g. higher-order unification, higher-order term indexing)
- Type-checking remains decidable.
- Canonical forms exist.
- Proofs follow [Harper,Pfenning03]

Tabled proof search

Uniform proofs as a foundation for logic programming [Miller *et.al* 91]

Soundness Any uniform proof *with answer substitution* has a uniform proof.

Completeness Any uniform proof has a uniform proof *with answer substitution*.

Soundness of tabled higher-order logic programming : Any *tabled* uniform proof with an answer substitution has a uniform proof with the same answer substitution.

Related work

- Related Work: XSB system [Warren et al. 99]
  Very successful for first-order logic programming

- Applicable to other higher-order systems:
  - λProlog[Nadathur,Miller88]
  - Linear logic programming [Hodas et al. 94][Cervesato96]

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Experiments

- Parsing of formulas (adapted from [Warren99])
  - Left and right recursion
  - Not executable with depth-first search
  - Memoization vs iterative deepening
- Refinement type checking [Davies, Pfenning00]
  - Decidable
  - Memoization vs depth-first search

Parser for formulas

#tok	memo	iterative deepening
20	0.13 sec	0.98 sec
58	2.61 sec	∞
117	10.44 sec	∞
235	75.57 sec	∞

∞ = process does not terminate

Intel Pentium 1.6GHz, RAM 256MB,

SML New Jersey 110, Twelf 1.4

Refinement type-checking

	example	memo	depth-first
First answer	sub		0.15 sec
	mult		0.15 sec
	square		0.16 sec
Not provable	mult		13.50 sec
	plus		∞
	square		∞
All answers	sub		5.59 sec
	mult		∞
	square		∞

Refinement type-checking

	example	memo	depth-first
First answer	sub	3.19 sec	0.15 sec
	mult	7.78 sec	0.15 sec
	square	9.02 sec	0.16 sec
Not provable	mult	2.38 sec	13.50 sec
	plus	6.48 sec	∞
	square	9.29 sec	∞
All answers	sub	6.88 sec	5.59 sec
	mult	9.06 sec	∞
	square	10.30 sec	∞

Evaluation

- Benefits:
  - Superior to iterative deepening
  - Meaningful failure: decision procedure
  - Consistent performance
  - Quick failure
  - Small proof size
- Drawbacks:
  - Overhead of storing and retrieving information
  - Multi-stage strategy delays the reuse of answers

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Efficiently accessing the memo-table

“...an automated reasoning program’s rate of drawing conclusions falls off sharply both with time and with an increase in the size of the database of retained information.” [Wos92]

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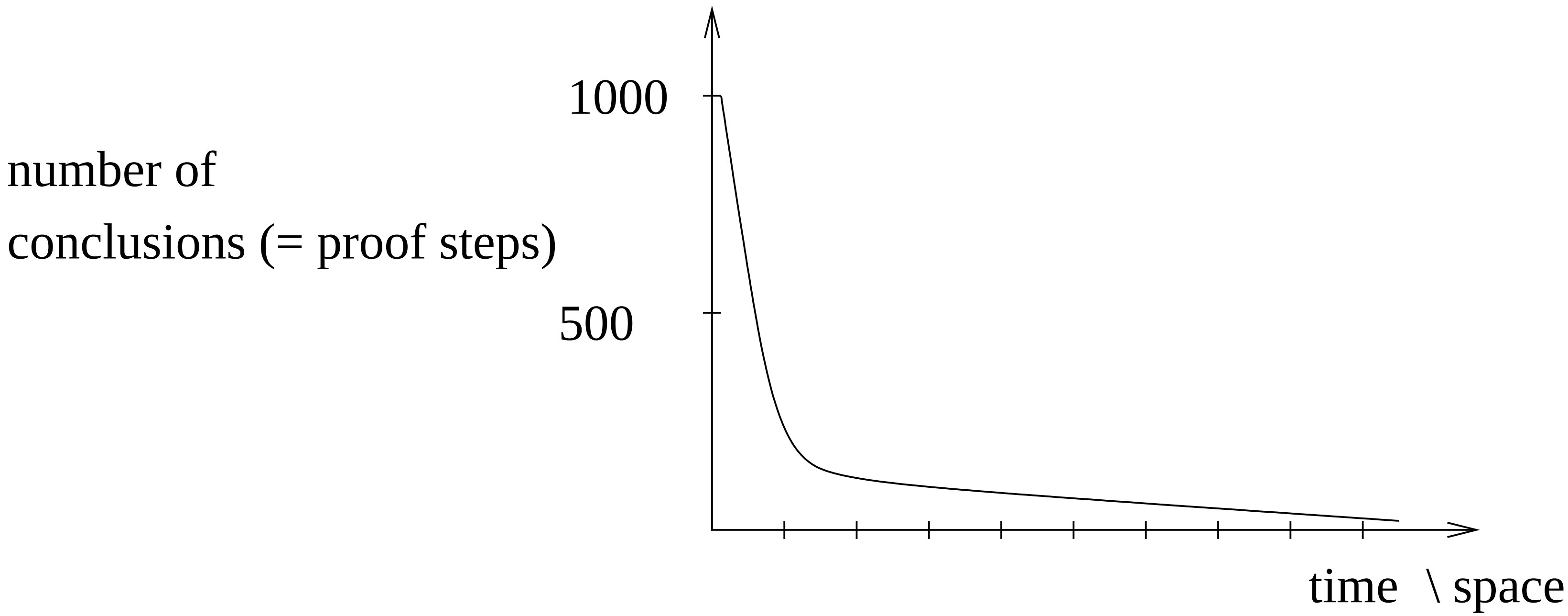

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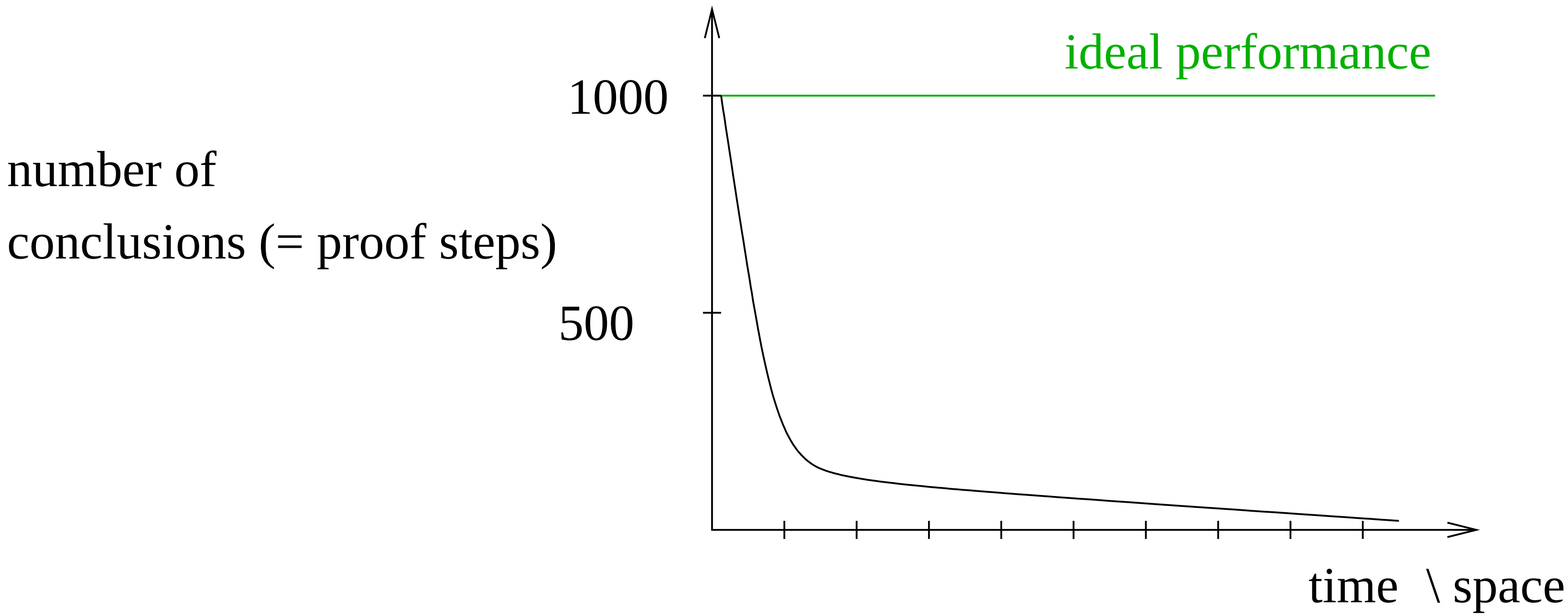

Indexing

Set of terms

(1) pred (h (h b)) (g b) (f λx. E x)

(2) pred (h (h a)) (g b) (f λx. E x)

(3) pred (h (g a)) (g b) a

Query:

pred (h (h b)) (g b) a

How can we efficiently store and retrieve data?

Indexing

Set of terms

(1) pred (h (h b)) (g b) (f λx. E x)

(2) pred (h (h a)) (g b) (f λx. E x)

(3) pred (h (g a)) (g b) a

Query:

pred (h (h b)) (g b) a

How can we efficiently store and retrieve data?

- Share term structure
- Share common operations

Common sub-expression

Set of terms

(1) pred (h (h b)) (g b) (f λx. E x)

(2) pred (h (h a)) (g b) (f λx. E x)

(3) pred (h (g a)) (g b) a

Query:

pred (h (h b)) (g b) a

- Factor out common sub-expressions!

pred (h (h a)) (g b) (f λ x. E x)

pred (h (g a)) (g b) a

pred (h *1) (g b) *2

Common sub-expression

Set of terms

(1) pred (h (h b)) (g b) (f λx. E x)

(2) pred (h (h a)) (g b) (f λx. E x)

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Query:

pred (h (h b)) (g b) a

- Factor out common sub-expressions!

  pred (h (h a)) (g b) (f λ x. E x)

  pred (h (g a)) (g b) a

  pred (h *1) (g b) *2

- In general the most specific common generalization (msg) does not exist!

MSG of higher-order patterns

Set of terms

(1) pred (h (h b)) (g b) (f λx. E x)

(2) pred (h (h a)) (g b) (f λx. E x)

(3) pred (h (g a)) (g b) a

Query:

pred (h (h b)) (g b) a

- Most specific generalization exists for higher-order patterns.
- Not all terms fall within this class.
- Is this efficient?

Our approach

Set of terms

(1) pred (h (h b)) (g b) (f λx. E x)

(2) pred (h (h a)) (g b) (f λx. E x)

(3) pred (h (g a)) (g b) a

Query:

pred (h (h b)) (g b) a

- Further restrict higher-order patterns!
  (Linear higher-order patterns)
  - Every meta-variable occurs only once.
  - Every meta-variable is fully applied.
- Translate terms into linear higher-order patterns and residual equations (variable definitions)

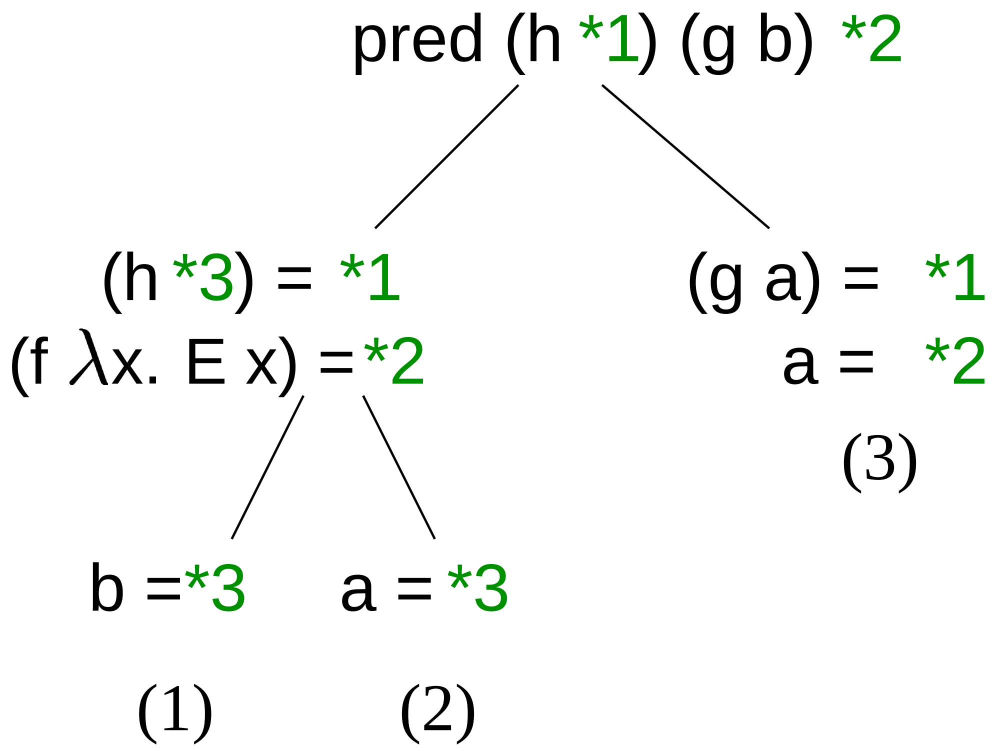
Higher-order substitution trees

Set of terms

(1) pred (h (h b)) (g b) (f λx. E x)

(2) pred (h (h a)) (g b) (f λx. E x)

(3) pred (h (g a)) (g b) a



Compose substitutions!

Parser for formulas

	iterative	memoization		
#tok	deepening	noindex	index	speed-up
20	0.98 sec	0.13 sec	0.07 sec	85%
58	∞	2.61 sec	1.25 sec	108%
117	∞	10.44 sec	5.12 sec	103%
235	∞	75.57 sec	26.08 sec	190%

∞ = process does not terminate

Intel Pentium 1.6GHz, RAM 256MB,

SML New Jersey 110, Twelf 1.4.

Refinement type-checking

	example	noindex	index	speed-up	orig
First	sub	3.19 sec	0.46 sec	593%	
answer	mult	7.78 sec	0.89 sec	774%	
	square	9.02 sec	0.98 sec	820%	
Not	mult	2.38 sec	0.38 sec	526%	
provable	plus	6.48 sec	0.85 sec	662%	
	square	9.29 sec	1.09 sec	752%	
All	sub	6.88 sec	0.71 sec	869%	
answers	mult	9.06 sec	0.98 sec	824%	
	square	10.30 sec	1.08 sec	854%	

Refinement type-checking

	example	noindex	index	speed-up	orig
First	sub	3.19 sec	0.46 sec	593%	0.15 sec
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Contribution and related work

- Contribution:
  - Higher-order term indexing (key: linearization, η-longform)
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  - Higher-order term indexing (key: linearization, η-longform)
  - Indexing substantially improves performance between 85% and 820%
- Related Work:
  - Substitution trees for first-order terms [Graf95]
  - (Higher-order) automata-driven indexing [Necula,Rahul01] imperfect filter, calls full higher-order unification to check candidates

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Summary

This talk

- Tabled higher-order logic programming
- Higher-order indexing

In the thesis

- More theory
- Optimizing higher-order unification
- Meta-theorem proving based on tabled higher-order logic programming

Conclusion

- This opens many new opportunities
  - to experiment and develop large-scale systems.
    for example: proof-carrying code
  - to explore the full potential of logical frameworks
    new applications: authentication, security
- Efficient proof search techniques are critical
  - to sustain performance.
  - to reduce response time to the developer.

Future work

- Narrowing the performance gap further
  - Improving tabling (e.g. subsumption, different scheduling strategies)
  - Eliminating redundancy in the representation of clauses, goals and proofs: approximate typing [Necula,Lee98]
  - Mode, determinism, termination analysis [Schrijvers et al. 02]
  - Ordered resolution [Bachmair, Ganzinger 01]
  - ...

Theory

- Foundation for meta-variables
  - Abstract over meta-variables ($\Pi^{\Box} u{::}\Psi{\vdash}A.$)
  - First-class variable definitions ($\Pi^{\Box} u = M{::}\Psi{\vdash}A$)
  - Representing and type-checking dag-style objects
- Meta-theorem proving
  - Automating complete induction
  - Further work on redundancy elimination

Applications

Proof-carrying code

- How can we transmit small proofs?[Necula,Rahul 01], (collaboration with Crary and Sarkar)
- How can we check them efficiently? [Stump, Dill 02]
- How can we automate some of the meta-proofs? [Crary,Sarkar03]

Applications

Proof-carrying code

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Proof-carrying authorization [Bauer et al. 02]

Bob proves that he is authorized to access Alice's web-page.

- How can we efficiently generate proofs?
- How can we cache and re-use proof attempts?

Finally ...

The End.

Finally ...

The End.

if you want to find out more:

http://www.cs.mcgill.ca/~bpientka