Assignment 5 – COMP 426: Automated Reasoning

Fall 2007
Due Oct 19th 2007

Exercise 1: Syllogism. (15pts)
Prove that the following argument is valid.

No philosophers are Spartans.
Some Greeks are Spartans.

Some Greeks are not philosophers.

Let $P(x)$ denote "x is a philosopher." Let $S(x)$ denote "x is a Spartan". Let $G(x)$ denote "x is a Greek". The the argument is:

$$((\forall x \in \tau.P(x) \supset \neg(S(x))) \ \wedge \ (\exists x \in \tau.G(x) \wedge S(x))) \supset \exists x \in \tau.G(x) \wedge \neg(P(x)).$$

Exercise 2: Prove the following conjectures using natural deduction or indicate if a conjecture is not true. For the following conjectures, you can assume that x does not occur in A. (45 pts)

1. $(A \supset \forall x \in \tau.P(x)) \Rightarrow \exists x \in \tau.(A \supset P(x))$
2. $\exists x \in \tau.(A \supset P(x)) \Rightarrow A \supset \forall x \in \tau.P(x)$

3. $(\forall x \in \tau.P(x)) \supset A \Rightarrow \exists x \in \tau.(P(x) \supset A)$
4. $\exists x \in \tau.(P(x) \supset A) \Rightarrow (\forall x \in \tau.P(x)) \supset A$

5. $\neg(\forall x \in \tau.B(x)) \Rightarrow \exists x \in \tau.\neg B(x)$
6. $\exists x \in \tau.\neg B(x) \Rightarrow \neg(\forall x \in \tau.B(x))$

Exercise 3: Give proofs for the following propositions.(30pts)

1. $(A \wedge \exists x \in \tau.B(x)) \supset (\exists x \in \tau.A \wedge B(x))$
2. $(\exists x \in \tau.A \wedge B(x)) \supset (A \wedge \exists x \in \tau.B(x))$
3. $(A \vee \forall x \in \tau.B(x)) \supset (\forall x \in \tau.A \vee B(x))$

Exercise 4: Prove the following fact about natural numbers(10 pts)
$\forall x \in \mathsf{nat}.\forall y \in \mathsf{nat}.((\mathsf{s}\ x) = (\mathsf{s}\ y) \supset x = y) \wedge \neg((\mathsf{s}\ x) = \mathsf{0})$.