COMP 426 Automated Reasoning
Homework 4, Exercise 3: Solutions
Prepared by Maja Frydrychowicz
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Exercise 3: Data-type `exp` of arithmetic expressions.

Part 1

Formation rule:

$$\frac{}{\mathbf{exp}\ \mathit{type}}\ \mathbf{exp}F$$

Introduction rules:

$$\frac{\Gamma \vdash n \in \mathbf{nat}}{\Gamma \vdash \texttt{Numb}\ n \in \mathbf{exp}}\ \mathbf{exp}I_N$$

$$\frac{\Gamma \vdash e_1 \in \mathbf{exp} \quad \Gamma \vdash e_2 \in \mathbf{exp}}{\Gamma \vdash \texttt{Pxy}\ e_1\ e_2 \in \mathbf{exp}}\ \mathbf{exp}I_P$$

$$\frac{\Gamma \vdash e_1 \in \mathbf{exp} \quad \Gamma \vdash e_2 \in \mathbf{exp}}{\Gamma \vdash \texttt{Txy}\ e_1\ e_2 \in \mathbf{exp}}\ \mathbf{exp}I_T$$

Elimination Rule:

$$\frac{\Gamma \vdash e \in \mathbf{exp} \quad \Gamma, n \in \mathbf{nat} \vdash e_0 \in \tau \quad \Gamma, x \in \mathbf{exp}, y \in \mathbf{exp}, f(x) \in \tau, f(y) \in \tau \vdash e_1 \in \tau \quad \Gamma, x \in \mathbf{exp}, y \in \mathbf{exp}, f(x) \in \tau, f(y) \in \tau \vdash e_2 \in \tau}{\Gamma \vdash \mathbf{rec}\ e\ \mathbf{of}\ f(\texttt{Numb}\ n) \Rightarrow e_0 \mid f(\texttt{Pxy}\ x\ y) \Rightarrow e_1 \mid f(\texttt{Txy}\ x\ y) \Rightarrow e_2 \in \tau}\ \mathbf{exp}E^{f,e}$$

Part 2

We use the definitions of *plus* and *times* listed in the Constructive Logic notes.

count Specification:

$count\ (\texttt{Numb}\ n) = s(0)$
$count(\texttt{Txy}\ e_1\ e_2) = count(e_1) + count(e_2)$
$count(\texttt{Pxy}\ e_1\ e_2) = count(e_1) + count(e_2)$

count Implementation:

$$\lambda e.(\mathbf{rec}\ e\ \mathbf{of}\ f(\texttt{Numb}\ n \Rightarrow s(0) \mid f(\texttt{Pxy}\ x\ y) \Rightarrow plus\ f(x)\ f(y) \mid f(\texttt{Txy}\ x\ y) \Rightarrow plus\ f(x)\ f(y))$$

eval Specification:

$eval\ (\texttt{Numb}\ n) = n$
$eval(\texttt{Txy}\ e_1\ e_2) = eval(e_1) * eval(e_2)$
$eval(\texttt{Pxy}\ e_1\ e_2) = eval(e_1) + eval(e_2)$

eval Implementation:

$$\lambda e.(\mathbf{rec}\ e\ \mathbf{of}\ f(\texttt{Numb}\ n \Rightarrow n \mid f(\texttt{Pxy}\ x\ y) \Rightarrow plus\ f(x)\ f(y) \mid f(\texttt{Txy}\ x\ y) \Rightarrow times\ f(x)\ f(y))$$