Assignment 1 – COMP 426: Automated Reasoning

Fall 2005
Due Sep 21th 2005

Exercise 1: Give proofs in natural deduction: **(50 pts)**

1. $(A \wedge (A \supset B)) \supset B$
2. $(A \vee (B \wedge C)) \supset ((A \vee B) \wedge (A \vee C))$
3. $\neg\neg\neg A \supset \neg A$
4. $((A \vee B) \wedge (A \vee C)) \supset (A \vee (B \wedge C))$
5. $A \wedge (B \vee C) \supset (A \wedge B) \vee (A \wedge C)$

Exercise 2: De Morgan's Law **(30 pts)**

In this exercise we try to prove the de Morgan's laws in constructive logic. One of the following conjectures only holds in classical logic. Give constructive proofs for all the conjectures which you believe are true in constructive logic and identify the one conjecture, which is not provable in constructive logic. (**Extra credit 10pts**: Provide a classical proof using the excluded middle for it).

1. $\neg(A \wedge B) \supset (\neg A \vee \neg B)$.
2. $\neg A \vee \neg B \supset (\neg(A \wedge B))$.
3. $\neg(A \vee B) \supset \neg A \wedge \neg B$.
4. $\neg A \wedge \neg B \supset \neg(A \vee B)$.

Exercise 3: Logical Equivalence **(20 pts)**

Logical equivalence, $A \equiv B$ is usually defined as $(A \supset B) \wedge (B \supset A)$. In this problem we explore the definition of equivalence using introduction and elimination rules. We have explored introduction and elimination rules for $\equiv$ in class.

$$\frac{A \equiv B \quad B}{A} \equiv E_L \qquad \frac{A \equiv B \quad A}{B} \equiv E_R \qquad \frac{\begin{matrix} \overline{A}^{\,u} & \overline{B}^{\,v} \\ \vdots & \vdots \\ B & A \end{matrix}}{A \equiv B} \equiv I^{u,v}$$

1. Display the local reductions that show the local soundness of the elimination rules.
2. Display the local expansion that show the local completeness of the elimination rules.