NAME: ________________________________STUDENT ID: ________________________

COMP 250 – Midterm
October 17th 2014, 18:10 – 19:55

- This exam has 7 questions.
- This is an open book and open notes exam. No electronic equipment is allowed.

Question 1 (15 points). Java programming

What will the following Java program print when executed?

```
class question1 {
      static public void questionA(int x) {
            x = x + 2;
      }

      static public int questionB(int x) {
            x = x + 3;
            return x;
      }

      static public void questionC(int array[]) {
            array[0] = array[0] + 4;
      }

      static public int questionD(int n) {
            if (n<=1) return 1;
            return questionD(n-1)+questionD(n-2);
      }

      public static void main(String args[]) {
            int x, y, z;
            int a[] = new int[10];
            x = 1;
            y = 1;
            a[0] = 1;
            questionA(x);
            y = questionB(y);
            questionC(a);
            z = questionD(6);
            System.out.println("x = " + x);
            System.out.println("y = " + y);
            System.out.println("a[0] = " + a[0]);
            System.out.println("z = " + z);
      }
}
```

Answer:

x =

y =

a[0] =

z =

Question 2 (20 points). Stacks and recursion

Professor Stackbottom proposes the following recursive algorithm that is using a stack as argument.

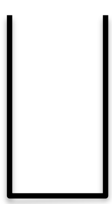
```
Algorithm mistery(Stack S)
Input: Stack S
Output: Modifies the stack S and returns a number

value = S.pop()
if (S is empty) then return value
else {
        result = mistery(S)
        S.push(value)
        return result
}
```

The objective of this question is to discover the purpose of this algorithm. We start by executing the following commands.

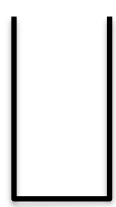
```
S = new Stack();
S.push('1');
S.push('2');
S.push('3');
```

a) (4 points) Draw the content of the stack at this point.

Stack S: 

b) (8 points) If we now execute
int x = mistery(S);

What is the value of x, and what is the content of the stack after the execution of the algorithm?

x = Stack S: 

c) (4 points) In *one sentence*, explain what is this algorithm doing when given a stack S as input.

d) (4 points) Using the big-Oh notation, give the running time of the mistery algorithm if it is executed on a stack of *n* elements. No justification is needed.

Question 3 (15 points). Proofs by induction

Prove by induction on n that for every integer $n \geq 0$ and any real number $a > 0$, we have

$$a^0 + a^1 + a^2 + \ldots + a^n = (a^{n+1} - 1) / (a - 1).$$

Base case:

Induction hypothesis:

Inductive step:

Question 4 (15 points). Recursive algorithms

Complete the pseudocode of the RecursiveSum algorithm below to obtain a recursive algorithm such that given a positive integer *n*, it prints all the ways of expressing *n* as sums of positive integers. For example, given *n*=4, the output should looks like this:

1+1+1+1=4
1+1+2=4
1+2+1=4
1+3=4
2+1+1=4
2+2=4
3+1=4
4=4

Note: This will be easier to do if we add, in addition to *n* itself, two additional arguments to the RecursiveSum algorithm:

- an array A large enough to store up to *n* elements, which will be used to accumulate partial sums through recursive calls.
- an integer soFar that keeps track of how many elements of A have been filled already.

Then, the result shown above would be obtained by calling RecursiveSum(A[ ], 0, 4).

Algorithm RecursiveSum(A[ ], soFar, *n*)
Inputs: A[] is an array of integers, where elements A[0,..., soFar-1] are already filled
n is an integer
Output: The algorithm prints out every possible ways to complete the partial sum already stored in A[0,…,soFar-1] so that the numbers add up to *n*.

sumSoFar = A[0] + A[1] + ... + A[soFar-1]

if (sumSoFar = *n*) **then** print A[0] "+" A[1] "+" ... "+" A[soFar-1] "=" *n*
else { /* WRITE YOUR PSEUDOCODE HERE */

}

Question 5 (10 points). Big-Oh notation

Prove, *using only the definition of the big-Oh notation*, that $\log(n^2 + 1) + n + 1$ is $O(n)$.

Question 6 (10 points). Solving recurrences

Using the substitution method, obtain an explicit formula for the following recurrence:

$$T(n) = \begin{cases} T(n-1) + 2n + 1 & \text{if } n > 0 \\ 0 & \text{if } n = 0 \end{cases}$$

Question 7 (15 points). Running time of algorithms

Give the worst-case running time of the following algorithms, using the simplest Θ() notation (big-Theta notation) possible. No justification needed.

<table>
<tr><th></th><th>Θ() Running time</th></tr>
<tr><td>**Algorithm1** (int n)
    i ← 2 * 2^n
    while (i > 1) **do {**
        i ← i / 2
    }</td><td></td></tr>
<tr><td>**Algorithm2** (int n)
    for i = 1 **to** n do {
        for j = 1 **to** 999 do {
            print “Bazinga!”
        }
    }</td><td></td></tr>
<tr><td>**Algorithm3**(A[ ], int n)
    for i = 0 **to** n-1 **do** { A[i]=i }
    merge(A, 0, n/2, n-1)
    pivot = partition(A, 0, n-1)

Note: merge and partition refer to the algorithms seen in class.</td><td></td></tr>
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