Hash tables

Dictionary ADT

- Reminder: A dictionary stores pairs
  (key, information)
- Operations:
  - find(key k)
  - insert(key k, info i)
  - remove(key k)
- Binary Search Trees implement all these operations in time O(h), where h is the height of the tree, which is O(log n) if we maintain the tree balanced
- We can sometimes do better...

Hash tables

- Suppose keys are integers between 0 and K-1
- Then, use an array A[0...K-1] containing elements of type "info" to store the dictionary:
  - insert(key k, info i): A[k] = i;
  - remove(key k): A[k] = null;
  - find(key k): return A[k]
- Running time: All operations are O(1)
- It's a miracle! Except that...

Problems with direct array implementation

- If K is large, the array will be very big
  - For McGill student ID, K = 1 000 000 000
- The amount of memory needed (K) is essentially independent of the number of items in the dictionary.
- Idea: compress the array...

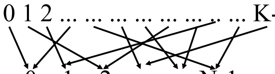
Hash functions

Idea: Map the K possible keys to N integers, with N being much smaller than K

Hash function f: [0...K-1] → [0...N-1]

Space of keys: 0 1 2 ... ... K-1

Hash function



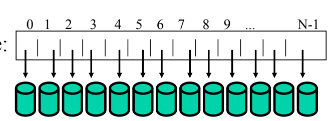
Hashed key 0 1 2 ... ... N-1

insert(key k, info i): A[f(k)] = i;
remove(key k): A[f(k)] = null;
find(key k): return A[f(k)];

Hash tables

- Collisions! Many keys map to the same index
- Solution: Each element of the array is itself a dictionary (called a bucket), implemented with linked-list, binary search tree, or a hash table...

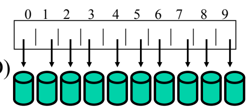
Hash table:



insert(key k, info i): A[f(k)].insert(k,i);
remove(key k, info i): A[f(k)].remove(k);
find(key k): return A[f(k)].find(k);

Hash tables - example

- Hashing student IDs:
  - K = 1,000,000,000
- N= 10
- Hash(ID) = lastDigit(ID)
- Bucket implemented as linked-lists



Hash tables - example

Insert(260053665,"Mathieu")

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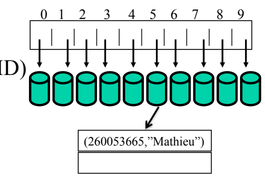

Hash tables - example

Insert(260625329,"John")

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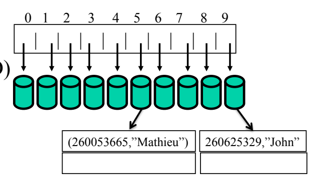

Hash tables - example

Insert(260313595, "Laura")

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- N= 10
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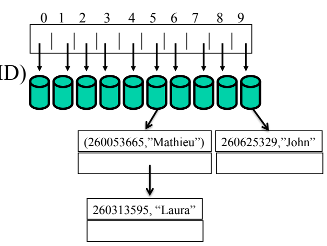
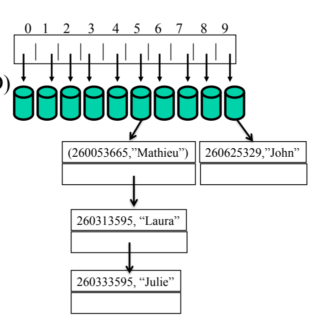


Hash tables - example

Insert(260435215,"Julie")

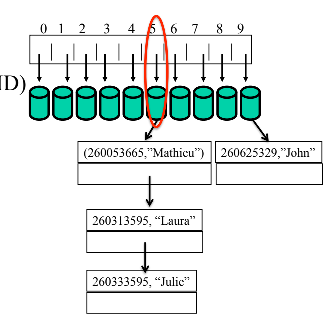
- Hashing student IDs:
  - K = 1,000,000,000
- N= 10
- Hash(ID) = lastDigit(ID)
- Bucket implemented as linked-lists



Hash tables - example

Find(260435215)

- Hashing student IDs:
  - K = 1,000,000,000
- N= 10
- Hash(ID) = lastDigit(ID)
- Bucket implemented as linked-lists



Hash tables - example

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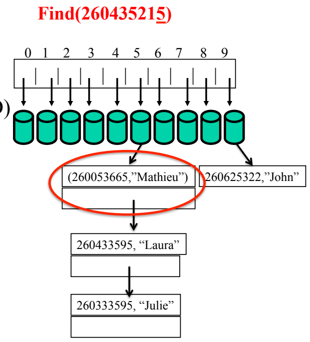

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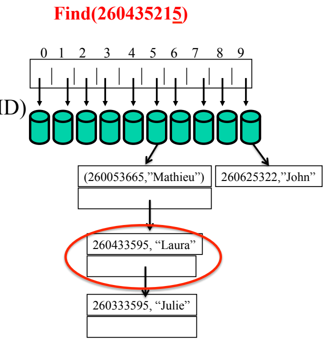

Analysis of Hashing with Chaining

- **Search time** = compute hash function + search the list.
- Time to compute hash function: O(1).
- Worst time for searching happens when all keys go in the same bucket. We need to scan the full list => O(n).
- **Search time** = O(1) + O(n) = O(n)
- **Insertion:** O(1) time.
- **Deletion:** O(1) + Search time.

Importance of good hash functions

- Worst case complexity for hash table containing n elements
  - if all keys end up in the same bucket and we use a linked-list to store buckets??
  - if keys are evenly spread among the N buckets??
- We want a hash function that spreads the keys evenly among the buckets.
- Example: N = 100, key = student ID #
  - f(key k) = $\lfloor k/10\ 000\ 000 \rfloor$ = first 2 digits
  - f(key k) = k mod 100 = last 2 digits
  - f(key k) = (sum of digits of k) mod 100

Good hash functions

- Choice of hash function depends on application
- In general, f(k) = k mod N is good choice when N is a prime number
- Example: For student Ids, choose N = 101
  - f(k) = k mod 101
- What if the key is not an integer (e.g. a String)?
  - map key to integer first with some function g(key)
  - use f() to map the integer to [0...N-1]

Hash functions on Strings

- We need a function g: String → Integers that minimizes collisions
  - Linear code:
    g(key k) = sum of ASCII values of each char.
    Problem:
  - Polynomial code: Choose a small prime number a
    If key $k = k_0 k_1 k_2 \ldots k_e$, choose
    $g(k) = k_0 + k_1\, a + k_2\, a^2 + \ldots + k_e\, a^e$