COMP 204

Algorithm design: Linear and Binary Search

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based on material from Yue Li, Christopher J.F. Cameron and Carlos G. Oliver

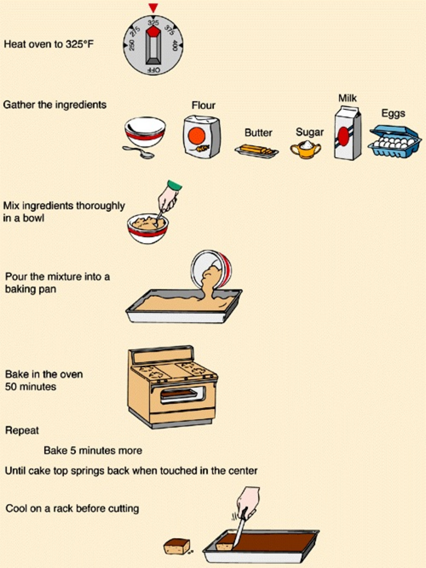
Algorithms

An **algorithm** is a predetermined series of instructions for carrying out a task in a finite number of steps

- or a recipe

Input $\rightarrow$ algorithm $\rightarrow$ output

Example algorithm: baking a cake



What is the input?

algorithm?

output?

Pseudocode

Pseudocode is a universal and informal language to describe algorithms from humans to humans

It is not a programming language (it can't be executed by a computer), but it can easily be translated by a programmer to any programming language

It uses variables, control-flow operators (while, do, for, if, else, etc.)

Example Python statements

```
students = ["Kris", "David", "JC", "Emmanuel"]
grades = [75, 90, 45, 100]
for student, grade in zip(students, grades):
    if grade >= 60:
        print(student, "has passed")
    else:
        print(student, "has failed")
#output:
#Kris has passed
#David has passed
#JC has failed
#Emmanuel has passed
```

Example pseudocode

Algorithm 1 Student assessment

```
for each student do
    if student's grade ≥ 60 then
        print 'student has passed'
    else
        print 'student has failed'
    end if
end for
```

Search algorithms

Search algorithms locate an item in a data structure
Input: a list of (un)sorted items and value of item to be searched

Algorithms: linear and binary search algorithms will be covered

- images if search algorithms taken from: `http://www.tutorialspoint.com/data_structures_algorithms/`

Output: if value is found in the list, return index of item
Example:

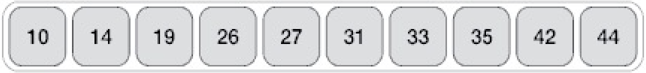
- search (key = 5, list = [3, 7, 6, 2, 5, 2, 8, 9, 2]) should return 4.
- search (key = 1, list = [3, 7, 6, 2, 5, 2, 8, 9, 2]) should return nothing.

Linear search

Look at each item in the list, one by one, from first to last, until the key is found.

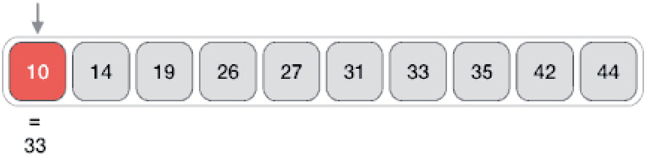
- a sequential search is made over all items one by one
- every item is checked
- if a match is found, then index is returned
- otherwise the search continues until the end of the sequence

Example: search for the item with value 33

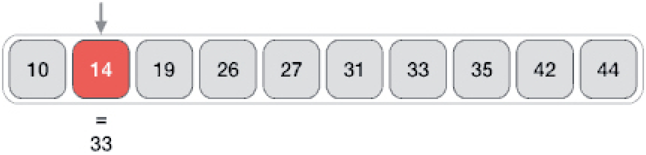


Linear search #2

Starting with the first item in the sequence:

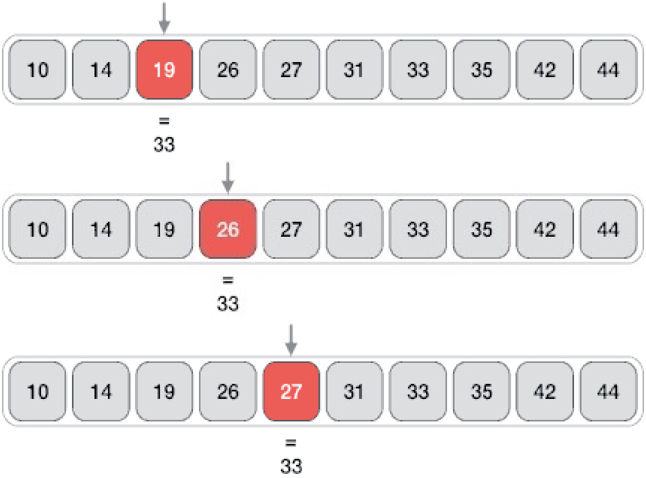


Then the next:



Linear search #3

And so on and so on...



Linear search #4

Until an item with a matching value is found:

If no item has a matching value, the search continues until the end of the sequence

Linear search: pseudocode

Algorithm 2 Linear search

```
procedure LINEAR_SEARCH(sequence, key)
    for index = 0 to length(sequence) do
        if  sequence[index] == key then
            return index
        end if
    end for
    return None
end procedure
```

Linear search: Python implementation

```
def linear_search(sequence, key):
    for index in range(0, len(sequence)):
        if sequence[index] == key:
            return index
    return None

#import random
#L = random.sample(range(1,10**9),10**7)
#import time
#time_start = time.time()
#print(f"start: {time.asctime(time.localtime(time_start))}
#index = linear_search(L, -1)
#print(index)
#time_finish = time.time()
#print(f"end:   {time.asctime(time.localtime(time_finish))
#print("time taken (seconds):", time_finish-time_start)
```

Issues with linear search

Running time: If the sequence to be searched is very long, the function will run for a long time.

Example: The list of all medical records in Quebec contains more than 8 Million elements!

Much of computer science is about designing *efficient* algorithms, that are able to yield a solution quickly even on large data sets.

See experimentation on Spyder (linear_vs_binary_search.py)...

Binary search

A faster search algorithm (compared to linear)

- the sequence of items must be sorted
- works on the principle of 'divide and conquer'

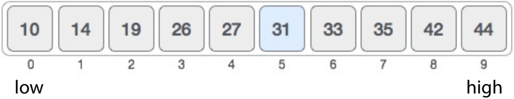
Analogy: Searching for a word (called the key) in an English dictionary.

To look for a particular word:

- Compare the word in the middle of the dictionary to the key
- If they match, you've found the word! Stop.
- If the middle word is greater than the key, then the key is searched for in the left half of the dictionary
- Otherwise, the key is searched for in the right half of the dictionary
- This repeated halves the portion of the dictionary that needs to be considered, until either the word is found, or we've narrowed it down to a portion that contains zero word, and we conclude that the key is not in the dictionary

Binary search #2

Example: let's search for the value 31 in the following sorted sequence



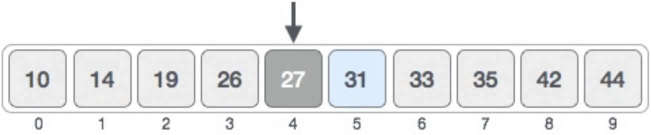
First, we need to determine the middle item:

```python
sequence = [10, 14, 19, 26, 27, 31, 33, 35, 42, 44]
low = 0
high = len(sequence) - 1
mid = low + (high-low)//2     # integer division
print (mid) # prints: 4
```

Binary search #3

Since *index* $= 4$ is the midpoint of the sequence

- we compare the value stored (27)
- against the value being searched (31)



The value at index 4 is 27, which is not a match

- the value being search is greater than 27
- since we have a sorted array, we know that the target value can only be in the upper portion of the list

Binary search #4

low is changed to *mid* + 1

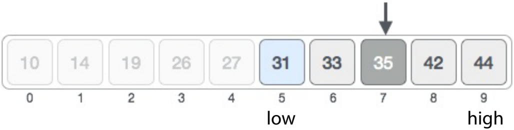
Now, we find the new *mid*

```
low = mid + 1    # 5
mid = low + (high-low)//2     # integer division
print (mid) # prints: 7
```

Binary search #4

mid is 7 now

- compare the value stored at index 7 with our value being searched (31)



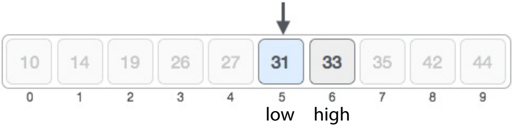
The value stored at location 7 is not a match

- 35 is greater than 31
- since it's a sorted list, the value must be in the lower half
- set *high* to *mid* - 1

Binary search #5

Calculate the mid again

- *mid* is now equal to 5



We compare the value stored at index 5 with our value being searched (31)

- It is a match!

Binary search #6

Remember,

- binary search halves the searchable items
- improves upon linear search, but...
- <u>requires a sorted collection</u>

<u>Useful links</u>

bisect - Python module that implements binary search

- `https://docs.python.org/2/library/bisect.html`

Visualization of binary search

- `http://interactivepython.org/runestone/static/pythonds/SortSearch/TheBinarySearch.html`

Binary search: pseudocode

Algorithm 3 Binary search

```
procedure BINARY_SEARCH(sequence, key)
    low = 0, high = length(sequence) − 1
    while low ≤ high do
        mid = (low + high) / 2
        if sequence[mid] > key then
            high = mid - 1
        else if sequence[mid] < key then
            low = mid + 1
        else
            return mid
        end if
    end while
    return 'Not found'
end procedure
```

Binary search: Python implementation

```python
def binary_search(sequence, key):
    low = 0
    high = len(sequence) - 1
    while low <= high:
        mid = (low + high)//2
        if sequence[mid] > key:
            high = mid - 1
        elif sequence[mid] < key:
            low = mid + 1
        else:
            return mid
    return None
```

Linear vs Binary search efficiency

Try linear_and_binary_search.py to see for yourself the difference in running time for large lists!

For a list of 10 Million elements:

- linear search takes about 3 seconds
- binary search takes about 0.0002 seconds.
- binary search is more than 100,000 times faster than linear search.

In general,

- the running time of linear search is proportional to the length of the list being searched.
- the running time of linear search is proportional to the **logarithm** of the length of the list being searched.

Binary search versus Linear search

```python
import random
import time
from decimal import Decimal
from linear_search import linear_search
from binary_search import binary_search

# generate list of 10 Million elements,
# where each element is a random number between 0 and 1,000,000,000
print("Generating list...")
n = 10**7
L = random.sample(range(10**9), n)

L.append(111111111) # for testing purpose
L.append(555555555)
L.append(999999999)

print("Sorting list...")
L.sort()

while True:
    key = int(input("Enter key for linear search: "))

    # perform linear search
    print("Starting linear search ...")
    time_start = time.time()
    index = linear_search(L, key)
    time_finish = time.time()
    linear_search_time = time_finish-time_start
    print(f"Found at position: {index}; time taken:", \
          "{:.2e}".format(linear_search_time), "seconds")

    print("Starting binary search ...")
    time_start = time.time()
    index = binary_search(L, key)
```