Diameter and Rumour Spreading in Real-World Network Models

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joint work with a few people

Nick Wormald

A fundamental question

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Interested in graphs that resemble real-world networks, focus on random graphs with power-law degree distribution.

Part I: DIAMETER

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Mathematical question: which random graphs are small-world?

Generator	Diameter or Avg path len.	Community: Bip. core vs size	Community: $C(k)$ vs k	Clustering coefficient	Remarks
Erdös–Rényi [1960]	$O(\log N)$		Indep.	Low, $CC \propto N^{-1}$	
PLRG [Aiello et al. 2000], PLOD [Palmer and Steffan 2000]	$O(\log N)$	Indep.		$CC \to 0$ for large N	
Exponential cutoff [Newman et al. 2001]	$O(\log N)$			$CC \to 0$ for large N	
BA [Barabási and Albert 1999]	$O(\log N)$ or $O(\frac{\log N}{\log \log N})$			$CC \propto N^{-0.75}$	
Initial attractiveness [Dorogovtsev and Mendes 2003]					
AB [Albert and Barabási 2000]					
Edge copying [Kleinberg et al. 1999], [Kumar et al. 1999]		Power-law			
GLP [Bu and Towsley 2002]				Higher than AB, BA, PLRG	Internet only
Accelerated growth [Dorogovtsev et al. 2001], [Barabási et al. 2002]				Non-monotonic with N	
Fitness model [Bianconi and Barabási 2001]					
Aiello et al. [2001]					
Pandurangan et al. [2002]					
Inet [Winick and Jamin 2002]					Specific to the AS graph
Forest Fire [Leskovec et al. 2005]	"shrinks" as N grows				
Pennock et al. [2002]					
Small-world [Watts and Strogatz 1998]	$O(N)$ for small N, $O(\ln N)$ for large N, depends on p			$CC(p) \propto (1-p)^3$, Indep of N	N = num nodes p = rewiring prob
Waxman [1988]					
BRITE [Medina et al. 2000]	Low (like in BA)			like in BA	BA + Waxman with additions
Yook et al. [2002]					
Fabrikant et al. [2002]					Tree, density 1
R-MAT [Chakrabarti et al. 2004]	Low (empirically)				

Chakrabarti & Faloutsos, ACM Computing Surveys 2006.

Our contribution

We developed a versatile technique for proving that certain random graphs are small-world.

Theorem (M'14)

The following random graph models are small-world.

- ✓ *The (edge) copying model* *[Kumar et al.'00]*
- ✓ *Aiello-Chung-Lu models* *[Aiello,Chung,Lu'01]*
- ✓ *The Cooper-Frieze model* *[Cooper,Frieze'01]*
- ✓ *The generalized linear preference model* *[Bu,Towsley'02]*
- ✓ *The PageRank-based selection model* *[Pandurangan et al.'02]*
- ✓ *Directed scale-free graphs* *[Bollobás et al.'03]*
- ✓ *The forest fire model* *[Leskovec,Kleinberg,Faloutsos'05]*

The Cooper-Frieze model

- ✓ In each step, either a new vertex is born and edges are added from it to the existing graph, or edges are added between the existing vertices.
- ✓ The number of added edges is a bounded random variable.
- ✓ One endpoint of each added edge is either the new vertex, or a uniformly random vertex, or a vertex sampled according to the degrees.
- ✓ The other endpoint is either a uniformly random vertex or a vertex sampled according to the degrees.

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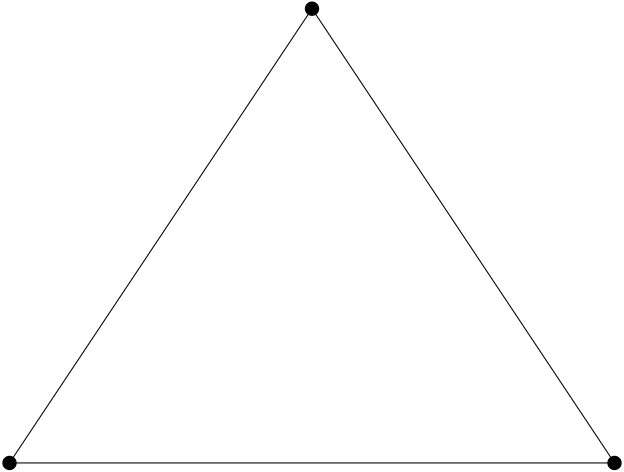
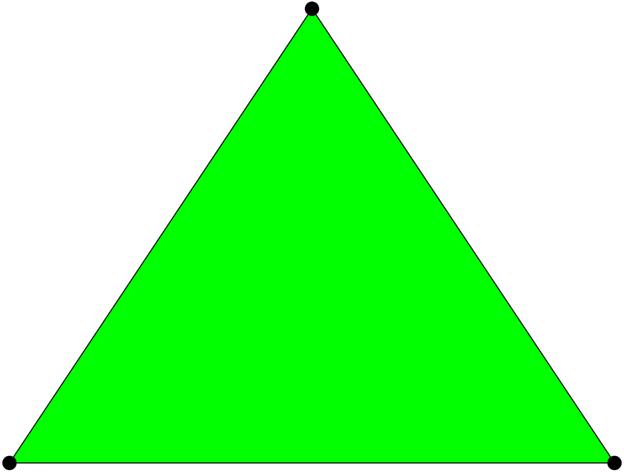
Random Apollonian Networks



$t = 0$



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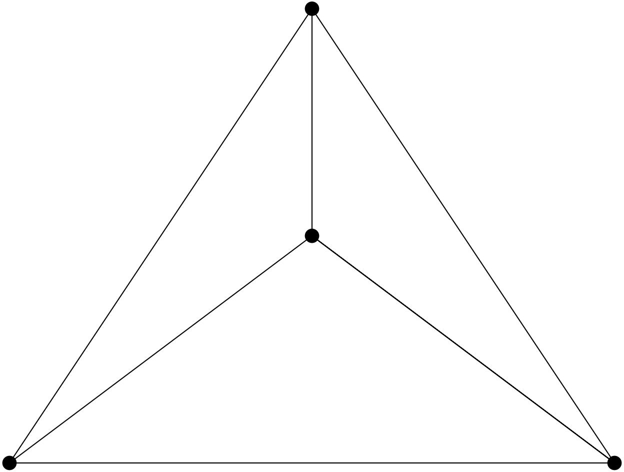
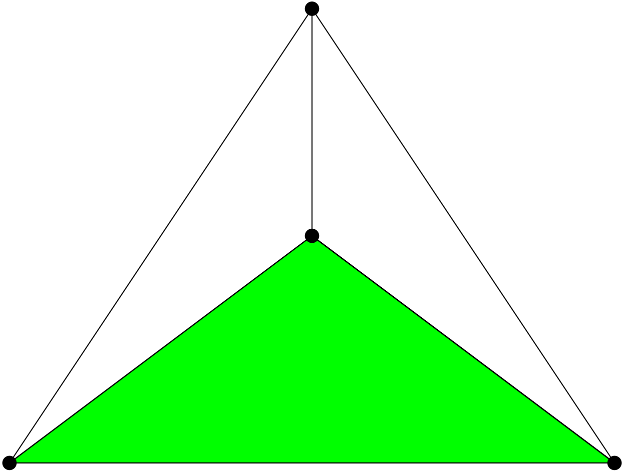


$t = 1$



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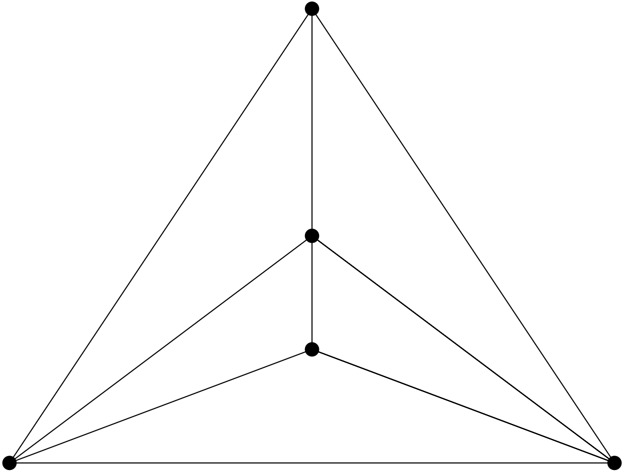
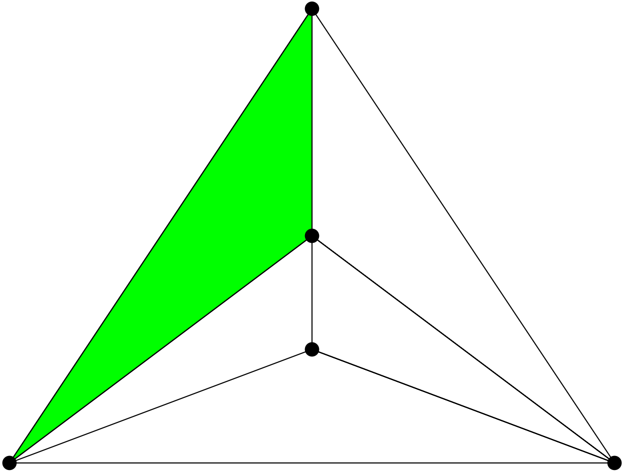


$t = 2$



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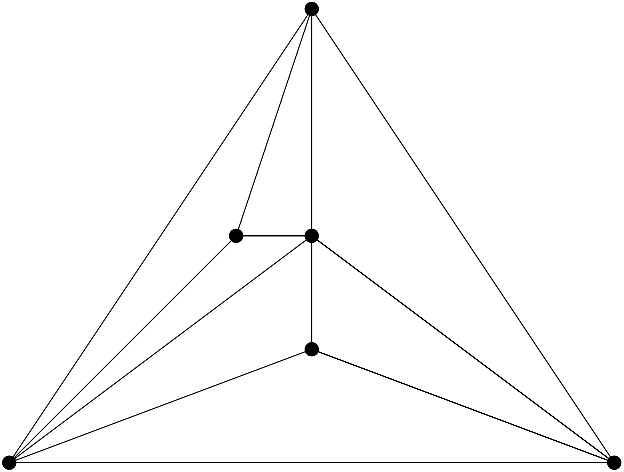

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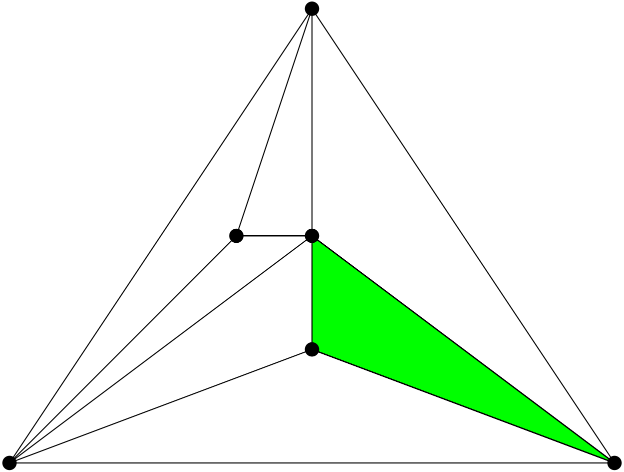

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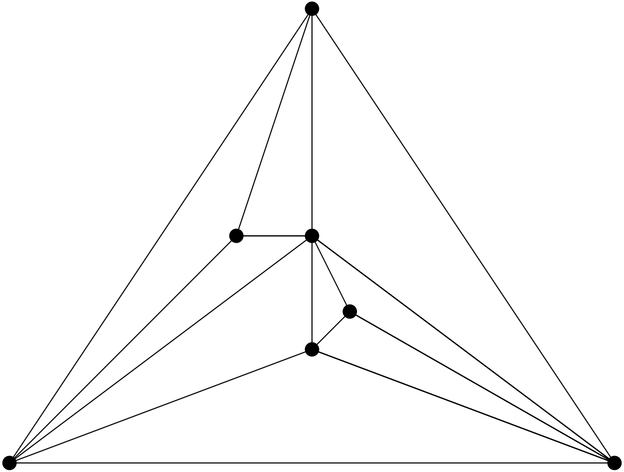

$t = 4$

Old and new results

After t steps, a random triangulated plane graph with $n = t + 3$ vertices, called a Random Apollonian Network (RAN).

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A similar result was proved independently by
Cooper, Frieze, Uehara'13 and Kolossváry, Komjáty, Vágó'13.

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$\mathcal{L}_n :=$ length of a longest path

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Theorem (EFGMSWZ'13)

We have

$$\mathcal{L}_n > n^{0.63}$$

and

$$\mathbb{E}\left[\mathcal{L}_n\right] = \Omega\left(n^{0.88}\right)$$

Random-Surfer Webgraphs

Model definition

- ✓ Parameters: p and d
- ✓ Consider a pool of independent $\text{Geo}(p)$ random variables.
- ✓ Build a random graph with out-degree d: start with one vertex with d loops, add a new vertex in each step.

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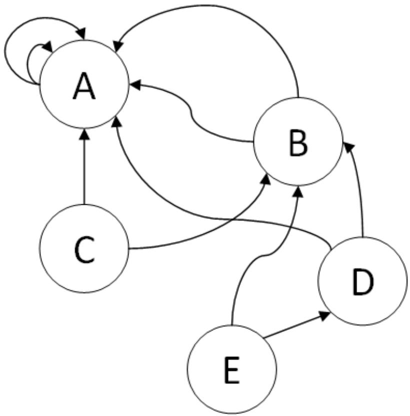
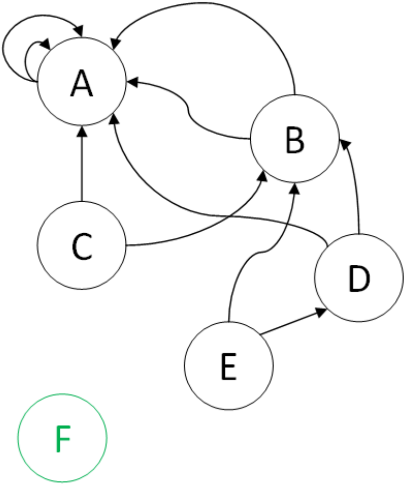
Say $p = 1/2$ and $d = 2$


A
B
C
D
E


A
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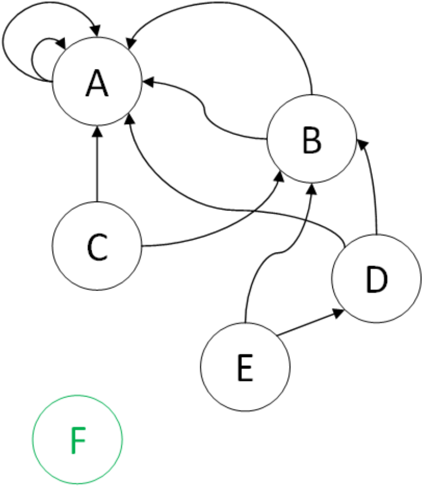
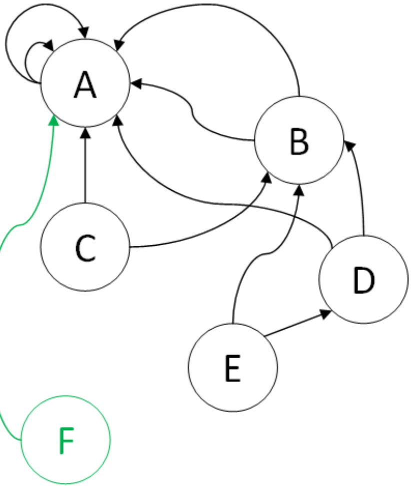
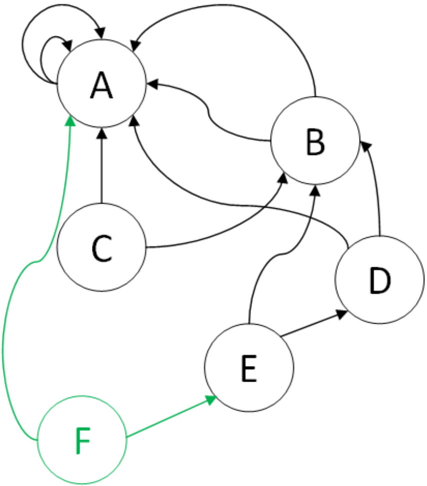
C
D
E
F



C, 2



C, 2



C, 2

E, 0

Blum, Chan and Rwebangira'06.

Our result

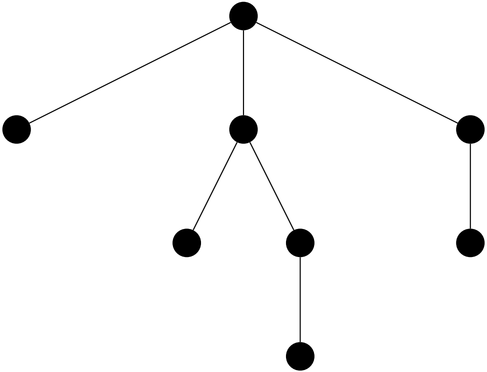
Previous work focused on the degree distribution.

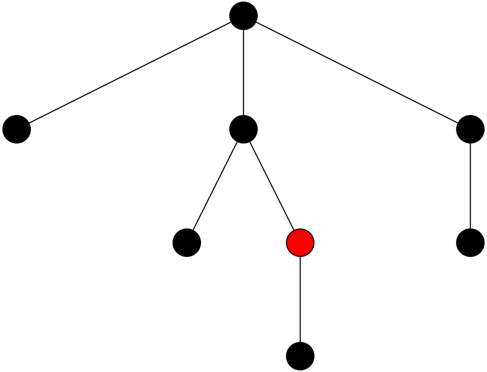
Theorem (M, Wormald'14)

A.a.s. the diameter of the underlying graph $\leq (8e^p/p)\log n$

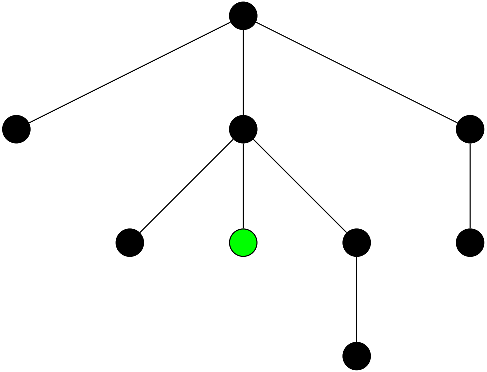
✓ The small-world phenomenon holds for this model.

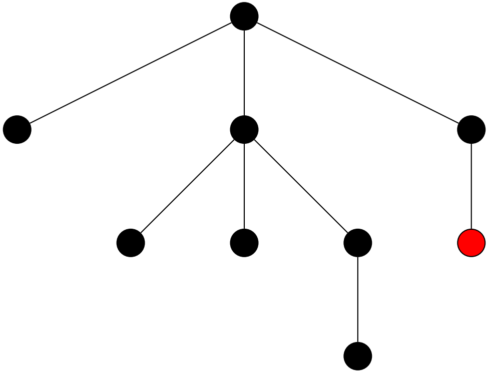
Random-surfer trees ($d = 1$)



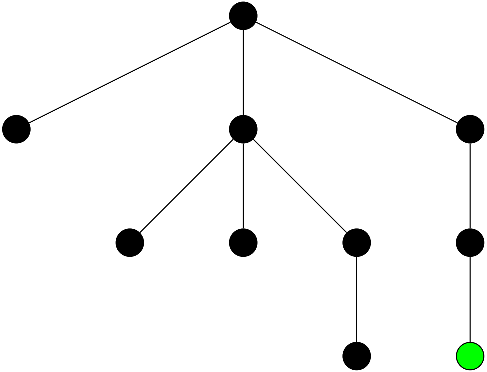


next geom.r.v. $= 1$





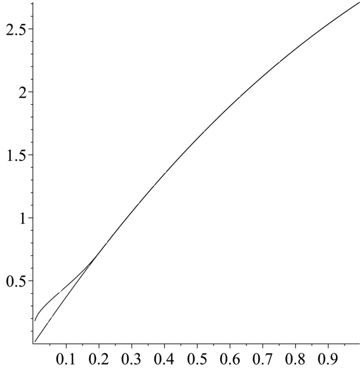
next geom.r.v. $= 0$



Our result

Theorem (M, Wormald'14)

A.a.s. the height is between $(L(p) - o(1)) \log n$ *and* $(U(p) + o(1)) \log n$*, and the diameter is between twice these values.*



Part II: RUMOUR SPREADING

Protocols definition

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- ✓ $s(G)$ and $a(G)$: average time it takes to broadcast the rumour.

Applications and known results

Replicated databases and distributed computing; news propagation in social networks and spread of viruses on the Internet.

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Graph G	$s(G)$	$a(G)$
Path	$(4/3)n + O(1)$	$n + O(1)$
Star	2	$\log n + O(1)$
Complete	$(1 + o(1)) \log_3 n$ [Karp et al.'01]	$\log n + o(1)$
$\mathcal{G}(n, p)$ $1 < np$ fixed	$\Theta(\log n)$ [Feige et al.'90]	$(1 + o(1)) \log n$ [Panagiotou,Speidel'13]

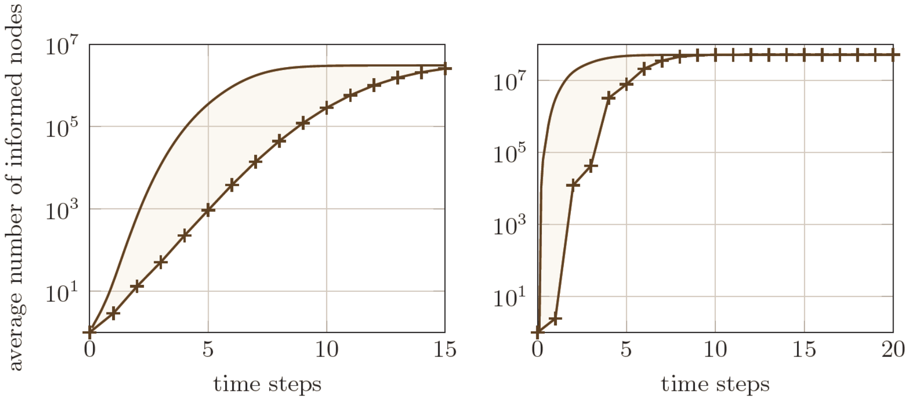
Our results

Theorem (Acan, Collevecchio, M, Wormald'14)

For any connected G we have

$$1 \leq s(G) \leq 4.6n$$
$$\log(n)/3 \leq a(G) \leq 4n$$

Comparison of the two protocols on the same graph: experiments



(a) Orkut network **(b)** Twitter network

From Doerr, Fouz, and Friedrich. MedAlg 2012.

Comparison of the two protocols on the same graph: our results

Theorem (Acan, Collevecchio, M, Wormald'14)

We have

$$\frac{C_1}{\log^2 n} \leq \frac{s(G)}{a(G)} \leq C_2 n^{2/3} \log n$$

Moreover, there exist infinitely many graphs for which this ratio is $\Omega\left((n/\log n)^{1/3}\right)$.

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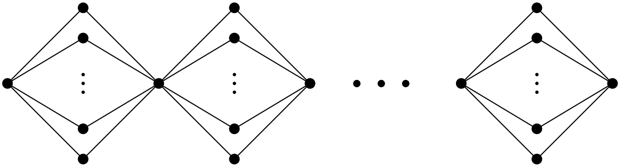


This graph has $\approx n^{1/3}$ diamonds, each consisting of $\approx n^{2/3}$ paths of length 2. It satisfies $a(G) = O(\log n)$ and $s(G) = \Omega(n^{1/3}(\log n)^{2/3})$.


GRANTS PICNIC GRD
VIA NEUMANN TK 2 km
GRANTS PICNIC GRD
VIA PADDY TK 4.6 km
MANCHESTER
UNITED

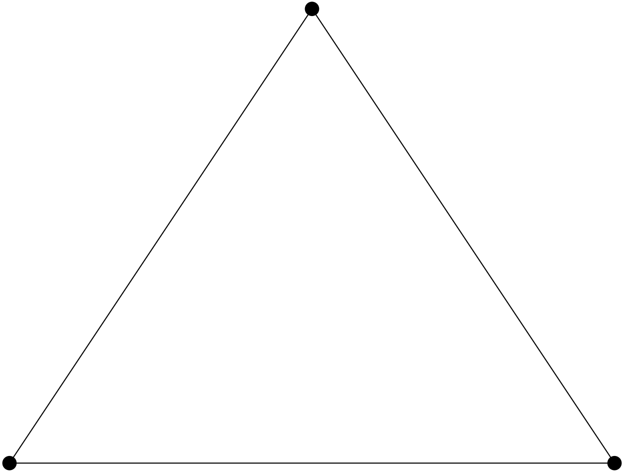
Synchronous Push&Pull on RANs and random k-trees

Synchronous Push&Pull on RANs

Theorem (M, Pourmiri'14)

If initially a random vertex of a RAN knows a rumour, a.a.s. after $O\left(\log^2 n\right)$ rounds, 99 percent of the vertices will get informed.

Random 3-tree



$t = 0$

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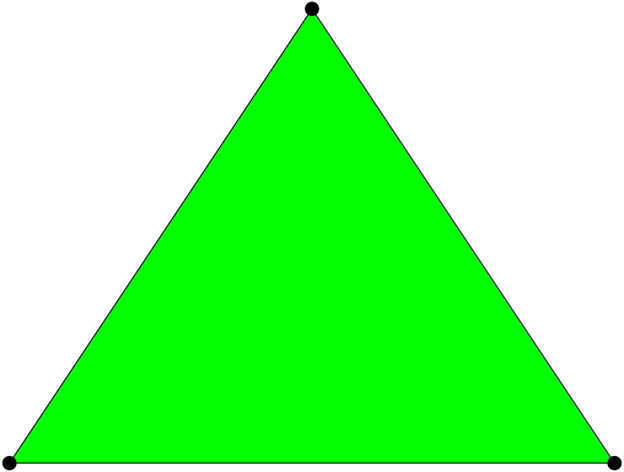

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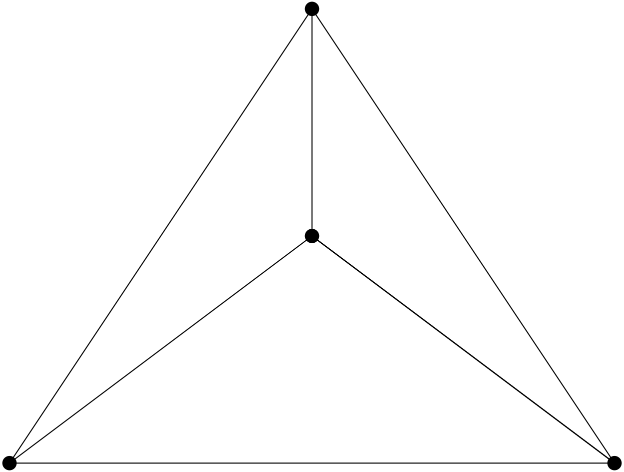
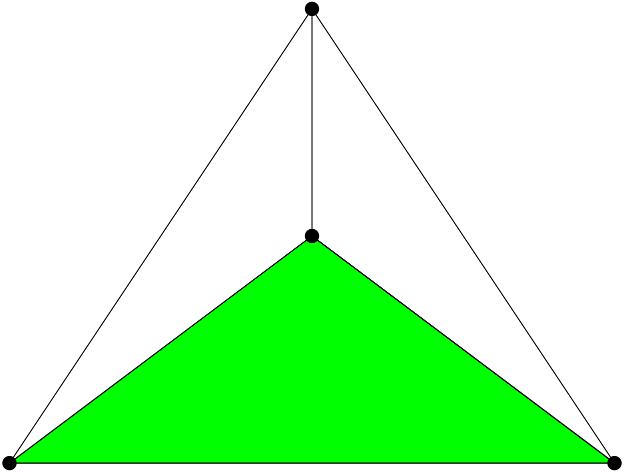


$t = 1$

Random 3-tree



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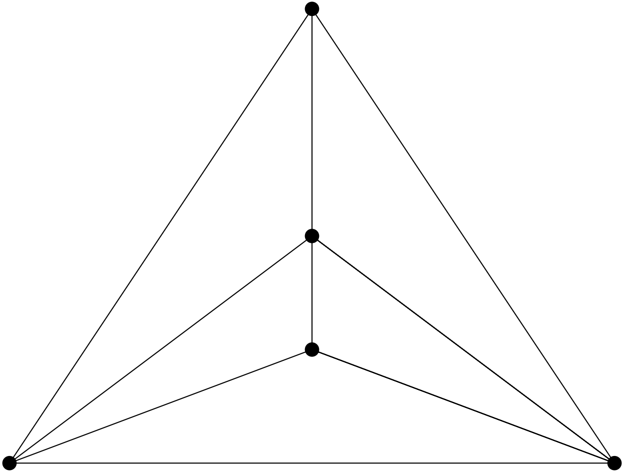
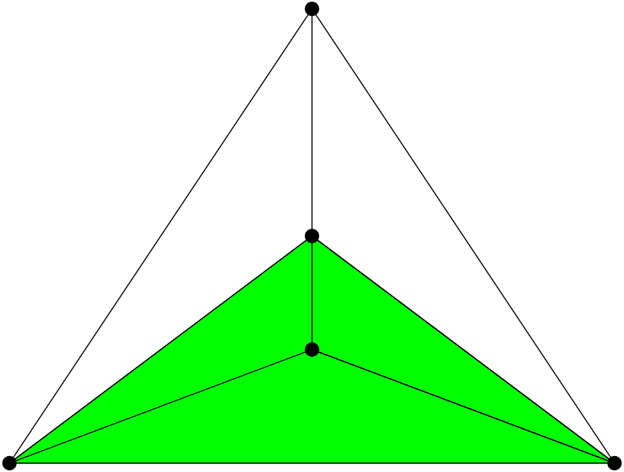


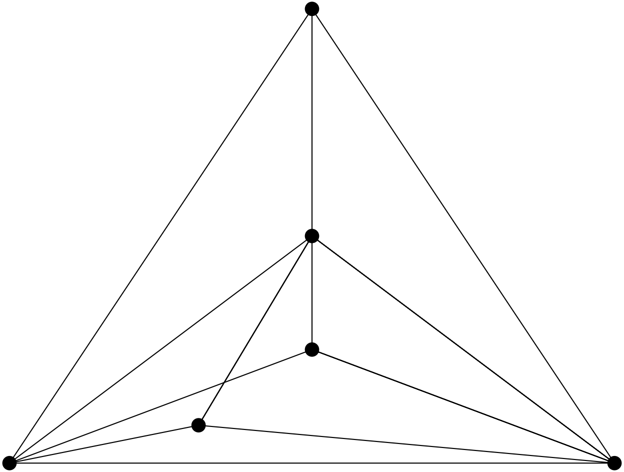
$t = 2$

Random 3-tree



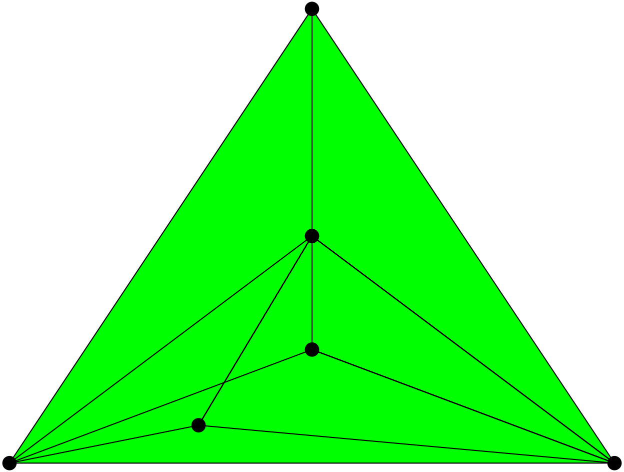
$t = 2$

Random 3-tree



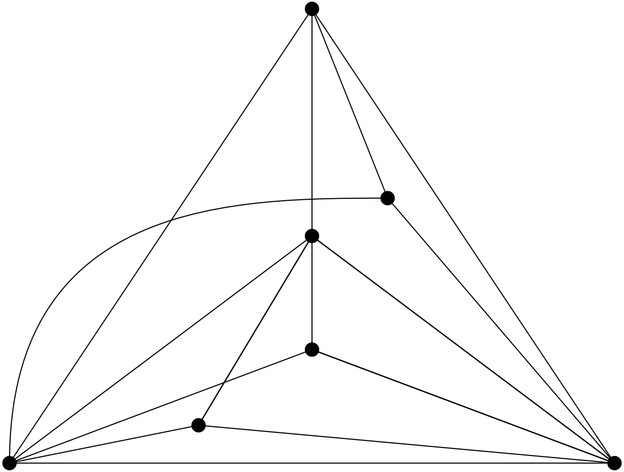
$t = 3$

Random 3-tree



$t = 3$

Random 3-tree



$t = 4$

Synchronous push&pull on random k-trees

Random k-trees were defined in 2009 by Gao, who proved their degree distribution is power-law.

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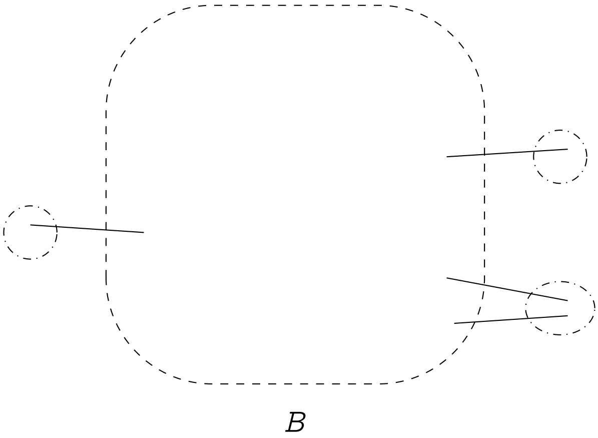
Theorem (M, Pourmiri'14, lower bound)

The time required to inform all vertices is $> n^{1/3k}$ a.a.s.

Exponential blow up if informing each and every vertex is required.

The picture



...

Thanks for listening!

APPENDIX

Versatile technique

Theorem (M'14)

The following random graph models have diameter $O(\log n)$ a.a.s.

- ✓ *The (edge) copying model* *[Kumar et al.'00]*
- ✓ *Aiello-Chung-Lu models* *[Aiello,Chung,Lu'01]*
- ✓ *The Cooper-Frieze model* *[Cooper,Frieze'01]*
- ✓ *The generalized linear preference model* *[Bu,Towsley'02]*
- ✓ *The PageRank-based selection model* *[Pandurangan et al.'02]*
- ✓ *Directed scale-free graphs* *[Bollobás et al.'03]*
- ✓ *The forest fire model* *[Leskovec,Kleinberg,Faloutsos'05]*

Theorem (M'14, 3.24)

The PARID model of Deijfen et al.'09 has diameter $O(\log^3 n)$ a.a.s. if the initial degrees' distribution has an exponential decay.

Diameter of RANs

Theorem (EFGMSWZ'13, 4.1)

$$f(x) := \frac{12x^3}{1-2x} - \frac{6x^3}{1-x}\,,$$

$y :=$ unique solution to

$$x(x-1)f'(x) = f(x)\log f(x), \quad x \in (0, 1/2)\,,$$

$$c := (1-y^{-1})/\log f(y) \approx 1.668$$

Then for every fixed $\varepsilon > 0$,

$$\mathbb{P}\left[(1-\varepsilon)c\log n \le \text{diameter of a RAN} \le (1+\varepsilon)c\log n\right] \to 1$$

Longest paths in RANs

$\mathcal{L}_n :=$ length of a longest path in a RAN

Theorem (EFGMSWZ'13, 4.2)

We have

$$\mathcal{L}_n > n^{0.63}$$

and

$$\mathbb{E}\left[\mathcal{L}_n\right] = \Omega\left(n^{0.88}\right)$$

Theorem (Collevecchio, M, Wormald'14, 4.4)

A.a.s. we have $\mathcal{L}_n < n^{0.99999996}$

The random-surfer Webgraph model

Theorem (M, Wormald'14, 5.2)

A.a.s. the diameter of the underlying graph $\leq (8e^p/p)\log n$

The random-surfer tree model

Theorem (M, Wormald'14, 5.3 and 5.4)

Given p and $\varepsilon > 0$, a.a.s. the height is between $(L(p) - \varepsilon)\log n$ and $(U(p) + \varepsilon)\log n$, and the diameter is between twice these values. Let $p_0 \approx 0.206$ be the unique solution in $(0, 1/2)$ to

$$\log\left(\frac{1-p}{p}\right) = \frac{1-p}{1-2p}\,.$$

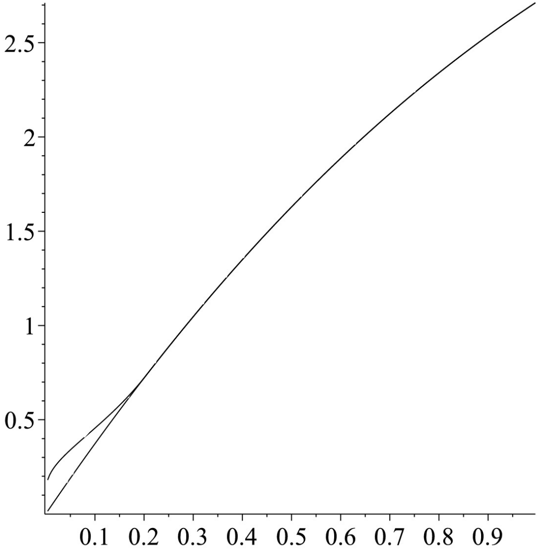
Let s be the solution in $(0,1)$ to

$$s\log\left(\frac{(1-p)(2-s)}{1-s}\right) = 1\,.$$

Then $L(p) = e^{1/s}s(2-s)p$ and

$$U(p) = \begin{cases} L(p) & \text{if } p_0 \le p < 1 \\ \left(\log\left(\frac{1-p}{p}\right)\right)^{-1} & \text{if } 0 < p < p_0\,. \end{cases}$$

The random-surfer tree model (cont'd)



Extremal spread times

Theorem (Acan, Collevecchio, M, Wormald'14, 6.3)

For any connected G,

$$(1-1/n)\,\mathrm{wast}_{\mathsf{a}}(G) \le \mathrm{gst}_{\mathsf{a}}(G) \le e\,\mathrm{wast}_{\mathsf{a}}(G)\log n\,, \tag{1}$$

$$\mathrm{wast}_{\mathsf{a}}(G) = \Omega(\log n) \quad \text{and} \quad \mathrm{wast}_{\mathsf{a}}(G) = O(n)\,, \tag{2}$$

$$\mathrm{gst}_{\mathsf{a}}(G) = \Omega(\log n) \quad \text{and} \quad \mathrm{gst}_{\mathsf{a}}(G) = O(n\log n)\,. \tag{3}$$

Theorem (Acan, Collevecchio, M, Wormald'14, 6.4)

For any connected G,

$$(1-1/n)\,\mathrm{wast}_{\mathsf{s}}(G) \le \mathrm{gst}_{\mathsf{s}}(G) \le e\,\mathrm{wast}_{\mathsf{s}}(G)\log n\,, \tag{4}$$

$$\mathrm{wast}_{\mathsf{s}}(G) = O(n)\,, \tag{5}$$

$$\mathrm{gst}_{\mathsf{s}}(G) = O(n\log n)\,. \tag{6}$$

Comparison of the two protocols on the same graph

Theorem (Acan, Collevecchio, M, Wormald'14, 6.9)

We have

$$\frac{C_1}{\log n} \leq \frac{\mathrm{gst}_{\mathsf{s}}(G)}{\mathrm{gst}_{\mathsf{a}}(G)} \leq C_2 n^{2/3},$$

and the left-hand bound is asymptotically best possible, up to the constant factor. Moreover, there exist infinitely many graphs for which this ratio is $\Omega\left(n^{1/3}(\log n)^{-4/3}\right)$.

Rumour spreading on random k-trees

Theorem (M,Pourmiri'14, 7.3)

Let $k \geq 2$ be fixed and let $f(n) = o(\log \log n)$ be any function going to infinity with n. If initially a random vertex of a random k-tree knows a rumour, then a.a.s. after $O\left((\log n)^{1+\frac{2}{k}} \cdot \log \log n \cdot f(n)\right)$ rounds of the synchronous push&pull protocol, $n - o(n)$ vertices will know the rumour.

Theorem (M,Pourmiri'14, 7.5)

Let $k \geq 2$ be fixed and let $f(n) = o(\log \log n)$ be any function going to infinity with n. Suppose that initially one vertex in the random k-tree knows the rumour. Then, a.a.s. the synchronous push&pull protocol needs at least $n^{(k-1)/(k^2+k-1)}/f(n)$ rounds to inform all vertices.

Rumour spreading on random k-Apollonian networks

Theorem (M,Pourmiri'14, 7.6)

Let $k \geq 3$ be fixed and let $f(n) = o(\log\log n)$ be any function going to infinity with n. Assume that initially a random vertex of a random k-Apollonian network knows a rumour. Then, a.a.s. after

$$O\left((\log n)^{(k^2-3)/(k-1)^2} \cdot \log\log n \cdot f(n)\right)$$

rounds of the synchronous push&pull protocol, at least $n - o(n)$ vertices will know the rumour.

Power-law degree distribution

Definition

A graph has power-law degree distribution with exponent β if the fraction of vertices of degree k is proportional to $k^{-\beta}$.

Examples:

- ✓ The Webgraph (in 2000) had $\beta = 2.1$
- ✓ Collaboration graph of mathematicians (MathSciNet 2000) had $\beta = 2.46$.