Evolutionary game theory and cognition

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Zero-sum games

- A game between two players (Alice and Bob) is represented by a matrix $G$ of pairs.

Example

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\begin{bmatrix}
(3, 1) & (2, 3) \\
(-1, 2) & (3, -1)
\end{bmatrix}
$$
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- If Alice plays strategy $i$ and Bob plays strategy $j$ then $(a, b) := G_{ij}$ is the outcome, where $a$ corresponds to the change in Alice's utility and $b$ to Bob's.
### Definition

A game $G$ is a zero-sum game if for each $(a, b) := G_{ij}$ we have $a + b = 0$. 

**Example:**

```plaintext
[(1, -1), (-1, 1), (-1, 1), (1, -1)]
```

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# Coordination games

## Definition

A two-strategy game $G$ is a coordination game if we have

\[
G = \begin{bmatrix}
(a_1, b_1) & (c_2, d_1) \\
(c_1, d_2) & (a_2, b_2)
\end{bmatrix}
\]

And $a_1 > c_1$, $a_2 > c_2$, $b_1 > d_1$, $b_2 > d_2$. **Examples**

- \[
\begin{bmatrix}
(1, 1) & (-1, -1) \\
(-1, -1) & (1, 1)
\end{bmatrix}
\]
- \[
\begin{bmatrix}
(2, 1) & (0, 0) \\
(0, 0) & (1, 2)
\end{bmatrix}
\]
- \[
\begin{bmatrix}
(4, 4) & (0, 2) \\
(2, 0) & (3, 3)
\end{bmatrix}
\]

The diagonals are always better for both players, they just have to figure out how to pick the same strategy. Captures the idea of win-win, lose-lose situations.
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- The diagonals are always better for both players, they just have to figure out how to pick the same strategy.
- Captures the idea of win-win, lose-lose situations.
What do these two types of games tell us?

- Zero-sum and coordination games are mutually exclusive: there is no game that is both zero-sum and a coordination game.

- Upside: zero-sum and coordination provide a good duality between impossibility of cooperation and obvious cooperation.

- Downside: both types of games are really boring. The most interesting games (from a mathematical and modeling point of view) are neither zero-sum nor coordination.

- Being non-zero-sum does not ensure cooperation.
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Prisoner’s dilemma

\[
\begin{pmatrix}
(b - c, b - c) & (-c, b) \\
(b, -c) & (0, 0)
\end{pmatrix}
\]

- \( b \) is the benefit of receiving and \( c \) is the cost of giving.
- Strategy 1 is called cooperate or \( C \) and strategy 2 is called defect or \( D \).
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- The best for the players taken together (or Pareto optimum) is mutual cooperation.
## Nash equilibrium

### Definition

A strategy pair \((p, q)\) is a Nash equilibrium of a game \(G\) if for all other strategies \(r\) we have:

\[
\text{fst}(G(p, q)) \geq \text{fst}(G(r, q))
\]

and

\[
\text{snd}(G(p, q)) \geq \text{snd}(G(p, r))
\]

Informally: neither Alice nor Bob can improve their payoff by unilateral change of strategy.

If we only allow pure strategies then replace \(G(i, j)\) by \(G_{ij}\)

If we allow mixed strategies, then every game has at least one Nash equilibrium.
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Example
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\end{bmatrix}, \quad \begin{bmatrix}
(2, -3) & (-1, 1) \\
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Cognitive demands of rationality

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- To find a Nash equilibrium Alice needs to be able to simulate the game and she must be able to place yourself in Bob’s shoes.
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- Let’s bound rationality and see what happens!
Evolutionary game theory

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- All cognition is stripped away.
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- What happens to rationality?
### Evolutionary stable strategy

**Definition**

A strategy $s$ is an evolutionary stable strategy for a game $G$ if for all other strategies $r$ we have (a) $\text{fst}(G(s, s)) > \text{fst}(G(r, s))$, or (b) $\text{fst}(G(s, s)) = \text{fst}(G(r, s))$ and $\text{fst}(G(s, r)) > \text{fst}(G(r, r))$. 

Consider a population all with strategy $s$, a mutant with strategy $r$ cannot invade the population only if one of the following conditions holds:

1. $r$ has a higher fitness than $s$ in a population of all $s$.
2. $r$ has the same fitness when interacting with $s$ and the same or greater fitness when interacting with other $r$.

Compare this to the Nash equilibrium conditions. The conditions are almost identical: we can think of the evolutionary process as a rational process (entity?).
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- Various augmentations of the model create fascinating results, among them: cooperation.
Cognitively relevant augmentations

- Kin selection: the ability to recognize your children, siblings and parents
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- Direct reciprocity: the ability to remember previous interactions
- Indirect reciprocity: the ability to track social constructs like reputation