# ASSIGNMENT 1
# Solving linear equations

COMP-202, Summer 2010

Due: Thursday, May 13th, 2010

If you have taken Math133 (or any other introductory linear algebra course), then you have performed the somewhat tedious task of solving a medium-sized system of linear Equations. While some students like to do it, a computer scientist would immediately question whether it wouldn't be better to write a program to perform this task. So in this first assignment, we will solve a 4x4 system using Cramer's rule. Note that the process you learn to do by hand, Gaussian Elimination, is also an algorithm that you could write a program for. But it requires some programming 'building blocks' we haven't covered yet.

Given a 4x4 matrix A, we know its determinant is computed via

$$
|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix}
$$

$$
= a_{11} \begin{vmatrix} a_{22} & a_{23} & a_{24} \\ a_{32} & a_{33} & a_{34} \\ a_{42} & a_{43} & a_{44} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} & a_{24} \\ a_{31} & a_{33} & a_{34} \\ a_{41} & a_{43} & a_{44} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} & a_{24} \\ a_{31} & a_{32} & a_{34} \\ a_{41} & a_{42} & a_{44} \end{vmatrix} - a_{14} \begin{vmatrix} a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{vmatrix}
$$

$$
= a_{11}(a_{22} \begin{vmatrix} a_{33} & a_{34} \\ a_{43} & a_{44} \end{vmatrix} - a_{23} \begin{vmatrix} a_{32} & a_{34} \\ a_{42} & a_{44} \end{vmatrix} + a_{24} \begin{vmatrix} a_{32} & a_{33} \\ a_{42} & a_{43} \end{vmatrix})
$$

$$
- a_{12}(a_{21} \begin{vmatrix} a_{33} & a_{34} \\ a_{43} & a_{44} \end{vmatrix} - a_{23} \begin{vmatrix} a_{31} & a_{34} \\ a_{41} & a_{44} \end{vmatrix} + a_{24} \begin{vmatrix} a_{31} & a_{33} \\ a_{41} & a_{43} \end{vmatrix})
$$

$$
+ a_{13}(a_{21} \begin{vmatrix} a_{32} & a_{34} \\ a_{42} & a_{44} \end{vmatrix} - a_{22} \begin{vmatrix} a_{31} & a_{34} \\ a_{41} & a_{44} \end{vmatrix} + a_{24} \begin{vmatrix} a_{31} & a_{32} \\ a_{41} & a_{42} \end{vmatrix})
$$

$$
- a_{14}(a_{21} \begin{vmatrix} a_{32} & a_{33} \\ a_{42} & a_{43} \end{vmatrix} - a_{22} \begin{vmatrix} a_{31} & a_{33} \\ a_{41} & a_{43} \end{vmatrix} + a_{23} \begin{vmatrix} a_{31} & a_{32} \\ a_{41} & a_{42} \end{vmatrix}),
$$

where

$$
\begin{vmatrix} a_{31} & a_{33} \\ a_{41} & a_{43} \end{vmatrix} = a_{31} \cdot a_{43} - a_{33} \cdot a_{43}.
$$

If we have a system $Ax = b$, where $A$ and $b$ are given, we can compute $x$ as follows:

$$
x_1 = \frac{d_1}{d} = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} & a_{14} \\ b_2 & a_{22} & a_{23} & a_{24} \\ b_3 & a_{32} & a_{33} & a_{34} \\ b_4 & a_{42} & a_{43} & a_{44} \end{vmatrix}}{|A|}, x_1 = \frac{d_2}{d} = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} & a_{14} \\ a_{21} & b_2 & a_{23} & a_{24} \\ a_{31} & b_3 & a_{33} & a_{34} \\ a_{41} & b_4 & a_{43} & a_{44} \end{vmatrix}}{|A|}, ..., x_4 = \frac{d_4}{d} = \frac{\begin{vmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \\ a_{41} & a_{42} & a_{43} & b_4 \end{vmatrix}}{|A|}
$$

Write a java program in SolveSystem.java with a class called SolveSystem that takes in all the values of A (row by row), and b, and then outputs x. At the beginning the program should output what it does, and query every value that is asked. The program should then output the determinant of A, and the values of $x_1$ though $x_4$.

The program, if run on a terminal, should take in the values and output its text exactly as follows:

```
>> java SolveSystem                          please insert a32: 7
This program solves the system              please insert a33: 6
    [a11 a12 a13 a14] [x1]    [b1]           please insert a34: 12
    [a21 a22 a23 a24]*[x2] =  [b2]           please insert a41: 4
    [a31 a32 a33 a34] [x3]    [b3]           please insert a42: 14
    [a41 a42 a43 a44] [x4]    [b4]           please insert a43: 15
given all a's and b's.                       please insert a44: -1
please insert a11: 16                        please insert b1:  -9
please insert a12: 2                         please insert b2:  1
please insert a13: 3                         please insert b3:  -7
please insert a14: 13                        please insert b4:  19
please insert a21: 5                         determinant of A: 272.0
please insert a22: 11                        x1:              1.0
please insert a23: 10                        x2:              2.0
please insert a24: 8                         x3:              -1.0
please insert a31: 9                         x4:              -2.0
```

## Do's and Don'ts:

- Use a filename exactly as specified (including capitalization), and with a classname exactly as specified
- Put your name and student ID at the top of every file as a comment
- Give all variables useful/descriptive names according to the conventions
- Submit your java file (just the source file) via WebCT.
- You may discuss the assignment with other students, put do not share any code. All submitted work has to be your own.

## Hints:

- It's a good idea to separate declaration of variables, inputting values, computation and outputting values in separate sections. They should have comments at the top saying what the section does.
- You will need to use double variables, because most of the time you will have fractional values involved.
- You should first write the code that just computes the determinant of $A$, because it's easy to get the large formula wrong. Once you are sure that part is right, add the code for the other determinants.
- Use variable names exactly as in the formulas, otherwise it's easier to get confused.
- The formulas can be written on multiple lines, like it was done above. It's advisable to use temporary variables to store $d$, $d_1$, $d_2$, $d_3$ in, to make the program more readable.
- Go to the tutorial to get started.

## Example systems:

$$A \cdot x = b, |A|$$

$$\begin{bmatrix} 16 & 2 & 3 & 13 \\ 5 & 11 & 10 & 8 \\ 9 & 7 & 6 & 12 \\ 4 & 14 & 15 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ -1 \\ -2 \end{bmatrix} = \begin{bmatrix} -9 \\ 1 \\ -7 \\ 19 \end{bmatrix}, 272$$

$$\begin{bmatrix} -8.4 & -2.9 & 3.8 & -9.2 \\ -11.8 & -1.5 & 8.2 & -10.8 \\ -11.7 & -4.0 & 13.0 & 5.9 \\ -13.1 & 7.9 & 14.2 & -12.2 \end{bmatrix} \cdot \begin{bmatrix} 0.8 \\ 0.9 \\ 10.8 \\ -0.5 \end{bmatrix} = \begin{bmatrix} 36.31 \\ 83.17 \\ 124.49 \\ 156.09 \end{bmatrix}, -894.79$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 2 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 9 \\ 20 \end{bmatrix}, 24$$

$$\begin{bmatrix} 0.9 & -11.6 & 13.9 & -2.0 \\ -12.3 & 8.5 & -2.0 & 4.7 \\ -2.8 & -6.3 & 5.8 & -11.7 \\ -11.9 & 3.1 & 7.7 & 13.0 \end{bmatrix} \cdot \begin{bmatrix} -9.4 \\ -7.0 \\ 8.9 \\ -0.4 \end{bmatrix} = \begin{bmatrix} 197.25 \\ 36.44 \\ 126.72 \\ 153.49 \end{bmatrix}, 54.03$$