A Few More Practice Questions

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1. A matrix can be represented by a list of lists of numbers. For example, the matrix:

\[
\begin{pmatrix}
1 & 2 & 3 \\
2 & 3 & 4 \\
3 & 2 & 1
\end{pmatrix}
\]

can be represented by the list \([1;2;3];[2;3;4];[3;2;1]\).

(a) Write a program that takes such a list of lists and checks that it is a properly formatted square matrix: the inner lists are all the same length and the number of inner lists is equal to the size of each inner list.

(b) Write a program that computes the sum of the numbers along the diagonal: this is called the trace of the matrix.

2. Let \(f, g\) be functions of type \(\text{float} \rightarrow \text{float}\). Assume that you have a function \(\text{integrate}\) of type \((\text{float} \rightarrow \text{float}) \rightarrow \text{float} \rightarrow \text{float} \rightarrow \text{float}\) such that \(\text{integrate } f \text{ lo hi}\) computes

\[
\int_{\text{lo}}^{\text{hi}} f(x)\,dx.
\]

There is a step-size used for the numerical integration but it is built in to the integrate function (for simplicity, in a real example it would also be a parameter).

(a) Write a higher-order function that takes two functions like \(f\) and \(g\) above and returns the function \(x \mapsto f(x) \ast g(x)\).
(b) Write a higher-order function that takes a function $f$ and produces the following function
\[ x \mapsto \int_0^x f(y) \, dy. \]

(c) Write a higher-order function that takes $f, g$ and returns the following function
\[ (f * g)(x) = \int_{-\infty}^{\infty} f(x - y) g(y) \, dy. \]
Assume that there is a predefined constant called $\text{infinity}$ that represents the largest representable floating point number.

3. Assume that $A \ll B$.
   (a) Is $(A \to B) \to A$ a subtype of $(B \to B) \to A$?
   (b) Is $A \to (B \to A)$ a subtype of $B \to (B \to A)$?