1. (a)

(b)

(c)
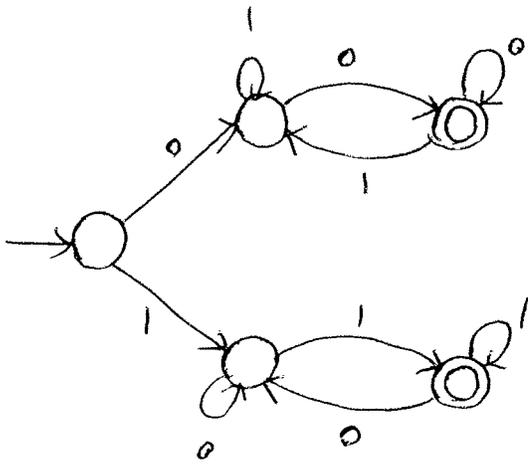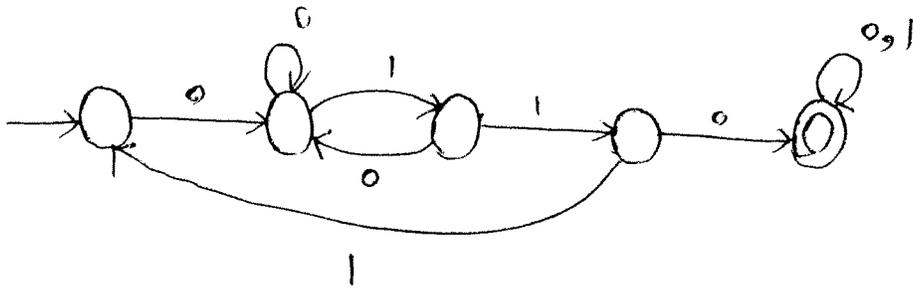
1. (d)

2.

$$Q = \{1, 2, 3\}$$

$$\Sigma = \{a, b, c, d\}$$

$$q_0 = 1$$

$$F = \{3\}$$

δ	a	b	c	d
1	$\{2\}$	$\{1\}$	$\{1\}$	$\{1\}$
2	$\{3\}$	$\{2\}$	$\{1\}$	$\{2\}$
3	$\{3\}$	$\{3\}$	$\{3\}$	$\{3\}$

3. Let m be the length of the longest string in L.

Consider the following DFA which is similar to a binary tree of depth $m+1$.
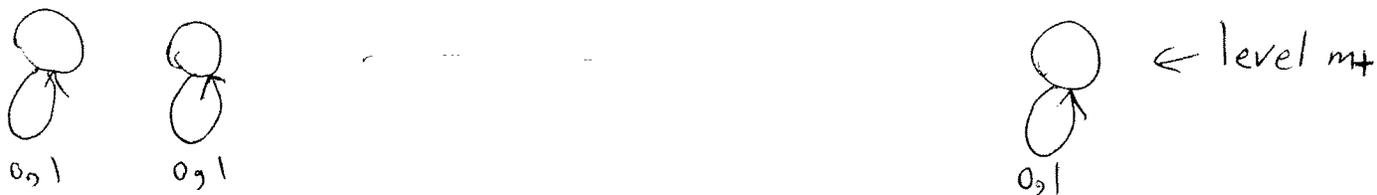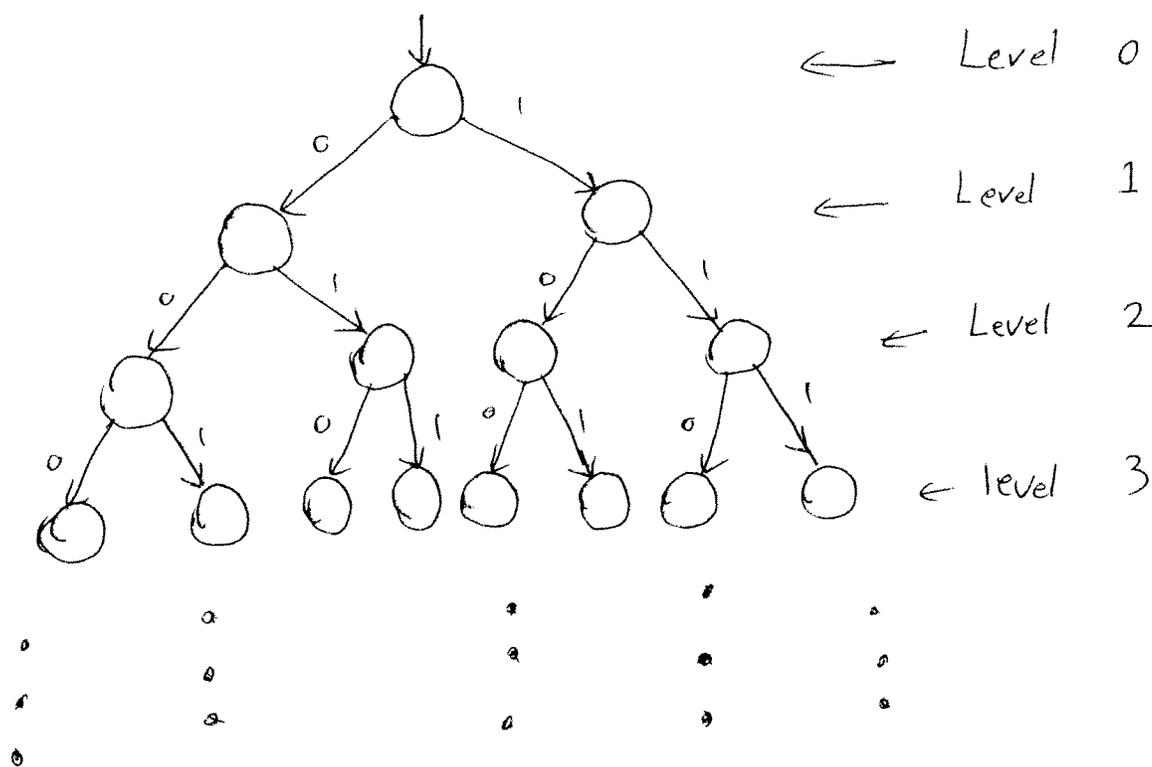
- The leaves have self-loops with labels 0 and 1.
- We turn a state into an accept state iff the path from the root to that state corresponds to a word in L.

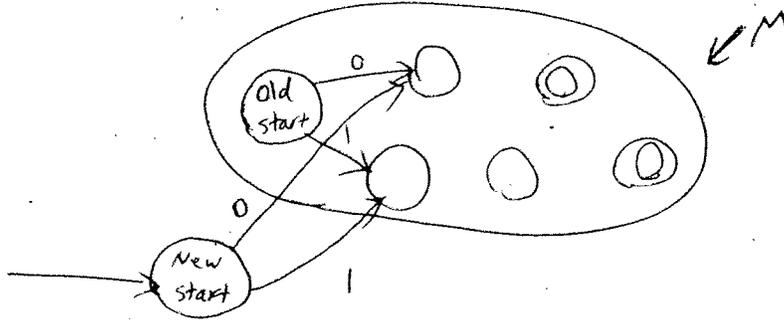# Question 4 Assignment 1

4(a) Add a new start state and connect to it as in the picture:

.4(b) — Convert every accept state to non-accept.

— Make the old start state an accept state.

— Reverse the direction of every arrow.

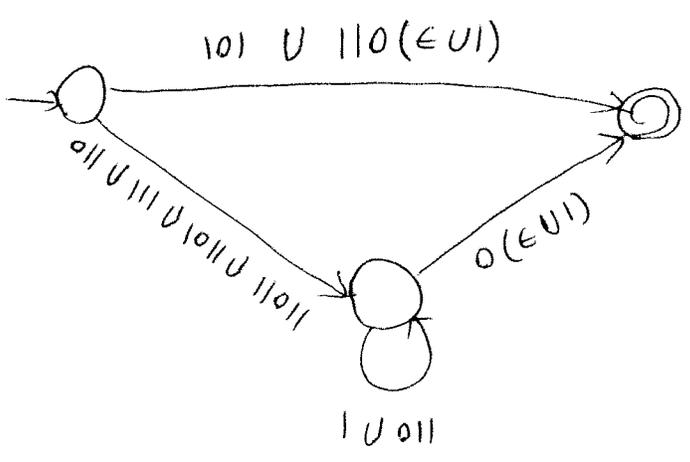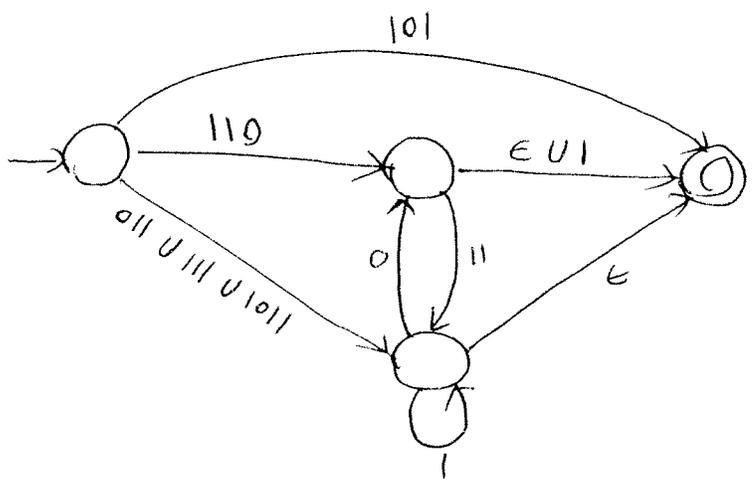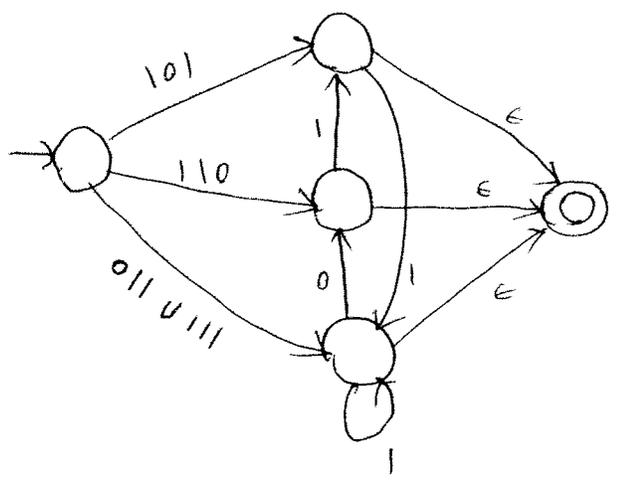— Add a new start stat and put ε-arrows from it to the old accept state.

4(c) Put three copies of M and a new start state and modify them as in the picture.

The new start state is an accept State.

Convert the old accept states to non-accept states.
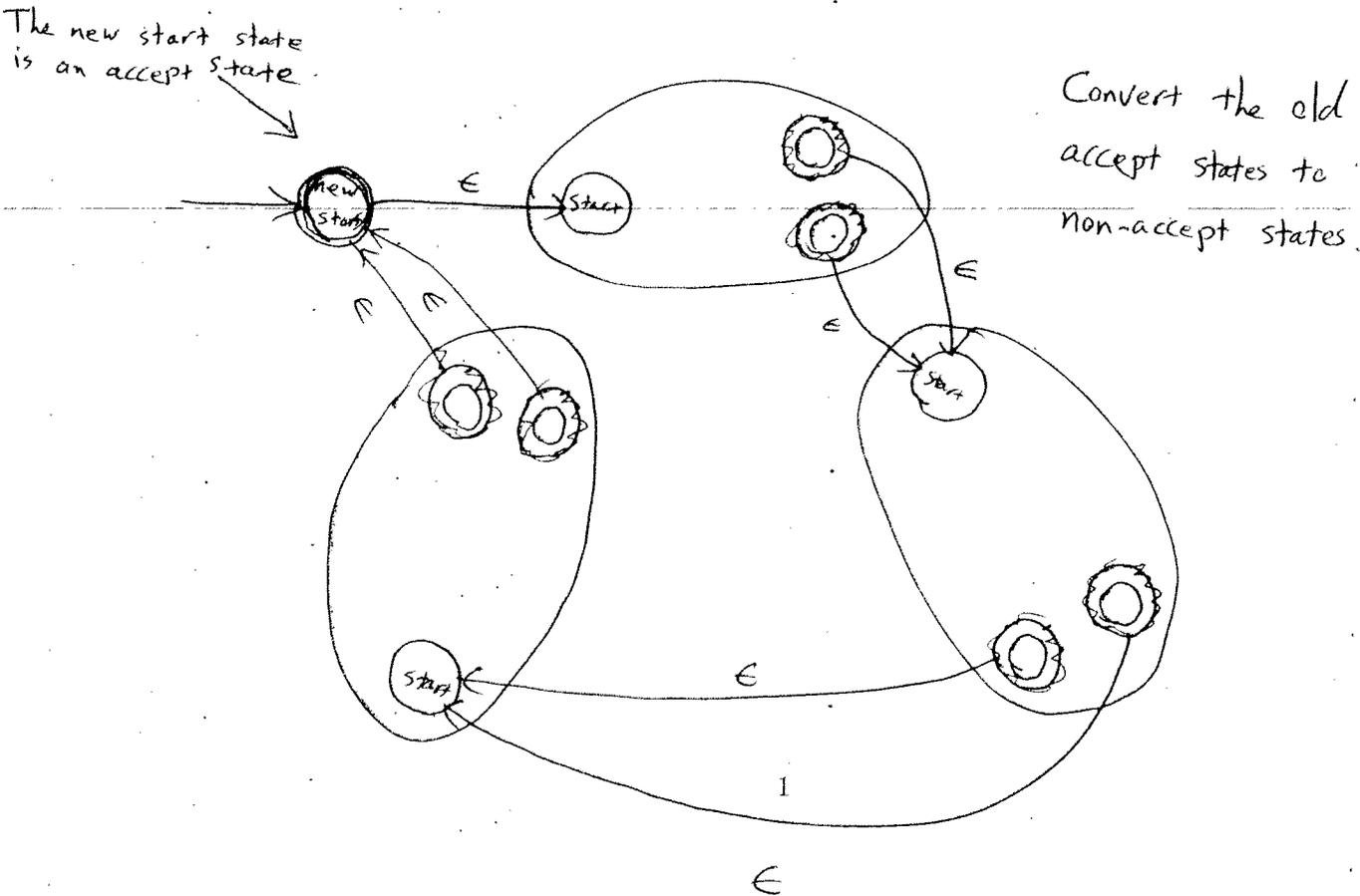
Q5. (a) False. Take $r = 1$ and $s =$. Then ε belongs to the language of the left-hand side, but it does not belong to the language of the right-hand side.

(b) False. Take $s = 0$ and $r = 1$. Then every word in the language of the left-hand side starts with 0 while every word in the language of the right-hand side starts with 1.

(c) False. Take $s = 0$ and $r = 1$. Then 01 belongs to the language of the left-hand side, but not to the language of the right-hand side.

Q6. $X = r^*s$.

7. We start from the following GNFA:

First GNFA with states and transitions labeled: 101, 110, 011 ∪ 111, 1, 0, 1, ε, ε, ε

Second GNFA with labels: 101, 110, ε ∪ 1, 011 ∪ 111 ∪ 1011, 0, 11, ε, 1

Third GNFA with labels: 101 ∪ 110(ε ∪ 1), 011 ∪ 111 ∪ 1011 ∪ 11011, 0(ε ∪ 1), 1 ∪ 011

Final GNFA (two states):

$$101 \cup 110(\epsilon \cup 1) \cup \left(\left(011 \cup 111 \cup 1011 \cup 11011\right)(1 \cup 011)^* 0\,(\epsilon \cup 1)\right)$$

$$= 101 \cup 110 \cup 1101 \cup \left(\left(011 \cup 111 \cup 1011 \cup 11011\right)(1 \cup 011)^* 0\,(\epsilon \cup 01)\right)$$