Assignment 3

COMP 531: Theory of Computation (Winter 06)

Due April 10 (Mon)

Instructions. Follow the instructions from the first assignment. If you think there's an error in some problem, please let me know asap (navin@cs.mcgill.ca). If you need hints for some problem please feel free to ask me. All problems are worth 10 points.

Problem 1. What is the complexity of counting the number of shortest paths between two given vertices in a directed graph? Is this problem #P-complete? Is it in P?

Problem 2. Show that the problem of computing the number of isomorphisms between two graphs is as hard as the problem of deciding whether two graphs are isomorphic in the following sense: the former problem reduces to the latter via a polytime reduction.

This could be interpreted as a hint that perhaps graph isomorphism problem is not NP-complete because for the NP-complete problems the corresponding counting problem is #P-complete.

Problem 3. (This is Problem 10 in Chapter 13 of the text) Consider the following problem about computing a relation. Associate the following communication problem with any function $f : \{0,1\}^n \to \{0,1\}$. Player 1 gets any input x such that $f(x) = 0$ and player 2 gets any input y such that $f(y) = 1$. They have to communicate in order to determine a bit position i such that $x_i \neq y_i$.

Show that the communication complexity of this problem is *exactly* the minimum depth of any circuit that computes f. (The maximum fan-in of each gate is 2.)

Problem 4. Construct a circuit C with three inputs x_1, x_2, x_3, and three outputs $\bar{x}_1, \bar{x}_2, \bar{x}_3$. You may use any number of AND and OR gates, but you may use at most *two* NOT gates.

*Remark.*There is a general theorem asserting that circuits for negating n variables need about $\log_2 n$ NOT gates, and this is known to be sufficient.

Problem 5. Prove that if a language L has a single prover interactive proof with perfect soundness then $L \in \mathsf{NP}$.