Approximation Algorithms **Homework 4**
Date: March 9, 2010 **Due:** March 18, 2010

1. (15 points) Problem 7.3 (page 209).

2. (10 points) Contrary to what I claimed in class, we can actually remove the edges in any order in the primal dual algorithm for the generalized Steiner tree problem. Problem 7.4.

3. (10 points) Problem 7.5.

4. (10 points) Can you also give a local-ratio algorithm for the minimum cost branching problem based on the following ideas. Consider the graph $Z = (V, A_Z)$ where $A_Z = \{a \in A : c_a = 0\}$ is the set of zero cost arcs.

   - If every node can reach r in Z, then we are done.
   - Else, there is a strongly connected component of Z, say C, not containing r such that all incoming arcs into C have strictly positive weight. Divide the weight function w in two parts w' and w'' in the following manner. Let $\alpha = min_{a \in \delta^-(C)} w(a)$. Let $w'(a) = \alpha$ for each $a \in \delta^-(C)$ and 0 for all other arcs. Let $w''(a) = w(a) - w'(a)$ for each arc a. Inductively find an optimal branching under the weight w'' in the graph obtained by shrinking component C into a singleton vertex. Augment this solution with a set of zero cost edges inside C to return a feasible solution.

   Give a recursive algorithm based on the above outline and show its optimality.

5. (10 points) Given a set of intervals on the real line $[a_i, b_i]$ for each $1 \leq i \leq n$ and a weight function w on intervals, the *maximum weight k-interval packing* problem asks for a subset J of intervals of maximum weight such that there are at most k intervals in J at any point on the line.

   (a) Formulate a linear program for the maximum weight k-interval packing problem.

   (b) Show that only $n - 1$ *point* constraints need to be imposed apart from the bound constraints.

   (c) Show that the linear program is integral.