Rendering falling snow using an inverse Fourier transform sketches_0199

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Keywords: motion parallax, falling snow

Methods for rendering falling snow typically use particle systems [MaAllister 2000] which require tens of thousands of particles (snowflakes), and thus can be expensive. Here we present an alternative method for rendering falling snow that does not use particles but rather uses a global Fourier transform. Our idea is based on a well-known fact that pure image translation with velocity (v_x, v_y) pixels/frame produces a plane of energy,

$$\omega_t \quad = \quad - v_x \, \omega_x - v_y \, \omega_y \qquad (1)$$

in the 3D frequency domain [Watson and Ahumada 1985]. Falling snow differs from pure image translation of Eq. (1) in that falling snow produces motion parallax: the 2D speed and size of each moving object (snow flake) is determined by its 3D depth. That is, snowflakes that are further away from the viewer appear smaller in the image plane but also move more slowly in the image plane.

This correlation between size and speed of falling snowflakes can be captured in the frequency domain as follows. The distance d to a snowflake is proportional to spatial frequency $\sqrt{\omega_x^2 + \omega_y^2}$. Distance d is also inversely proportional to the speed ω_t / ω_y, where we assume (temporarily) that the motion is in the y direction. This leads immediately to the relation between spatial and temporal frequency:

$$\omega_t \quad = \quad c \frac{\omega_y}{\sqrt{\omega_x^2 + \omega_y^2}} \qquad (2)$$

A plot of this tent-like surface is shown in Fig. 1.

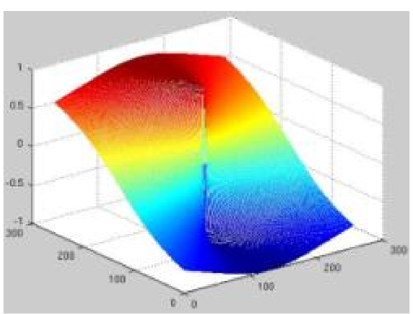

Figure 1: A plot of Equation (2) with constant (c = 1).

To render falling snow, we generate in the frequency domain a set of surfaces of the form of Eq. (2) for a range of constants c. We then take the inverse Fourier transform to get the XYT image.

One remaining question is how to choose the power spectrum as a function of spatial frequency for these surfaces. We consider images of size 512×512 and we limit our tent surfaces to three octaves, ranging from 16 to 128 cycles per frame. Spatial frequencies lower than 16 cycles per frame were not used in order to enforce an upper bound on size of moving image structure – that is, snowflakes are small. Spatial frequencies above 128 cycles per frame were not used in order to stay far from the Nyquist limit.

For those frequencies between 16 and 128 cycles, we assigned power proportional to $\frac{1}{\sqrt{\omega_x^2+\omega_y^2}}$. This puts a constant amount of power in each constant octave band [Field 1987]. For each of these spatial frequencies, we randomized the phase subject to the conjugacy constraint [Bracewell 1965] for image $I(x,y,t)$ and its 3D Fourier transform, $\hat{I}(\omega_x, \omega_y, \omega_t)$, namely:

$$\hat{I}(\omega_x, \omega_y, \omega_t) \quad = \quad \overline{\hat{I}(-\omega_x, -\omega_y, -\omega_t)} \qquad (3)$$

We take the inverse 3D Fourier transform (IFFT) and rescale the intensities to grey levels from 0 to 255.

Fig. 2 shows one frame of the falling snow image sequence, "composited" with a background image of a house, as in Eq. (4).

$$I(x,y,t) \quad = \quad \alpha \, I_{snow}(x,y,t) + (1-\alpha) \, I_{bg}(x,y) \qquad (4)$$

In this example, the foreground "opacity" α of the falling snow is 0.5. The entire image sequence along with other sequences is shown on the enclosed CD ROM.

Figure 2: One frame of the falling snow rendering image sequence from video example *house*.

References

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