Towards an Implementation of the 3D Visibility Skeleton

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ABSTRACT

In this note we describe the contents of a video illustrating an algorithm for computing the 3D visibility skeleton of a set of disjoint convex polytopes. The video can be found at `http://www.cs.mcgill.ca/~lzhang15/video/` with file name `socg07visidemo.mov`.

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1. INTRODUCTION

The 3D visibility skeleton is a graph whose vertices correspond to the maximal free line segments that are transversal to four edges of at least three distinct polytopes and tangent to those polytopes; its arcs correspond to sets of maximal free line segments that are tangent to three polytopes [9]. The visibility skeleton has been used for visibility computations such as computing shadow boundaries [10, 8].

This video demonstrates a sweep plane algorithm for capturing the vertices of the 3D visibility skeleton of a set of polytopes in 3D [13, 5].

2. THE ALGORITHM

The input of the algorithm is a set of k disjoint convex polytopes in general position with n edges in total. The output of the algorithm is the set of $O(n^2k^2)$ vertices of the 3D visibility skeleton of the input polyhedra. The algorithm, which runs in $O(n^2k^2 \log n)$ time, can also be used to compute the arcs of the skeleton.

The algorithm performs a rotational plane sweep around each edge e of each polytope, sweeping from one incident face of that edge to the other incident face. The sweep plane intersects the polytopes in at most k disjoint convex polygons, which change their shape as the sweep plane rotates. Figure 1 shows one position of the sweep plane, drawn as a faint grid, as it rotates around edge e of polytope **C**. Polytopes **A** and **B** are intersected by the sweep plane. Polytope **C** lies above the plane, with edge e in the plane. Polytope **D** lies below the plane. Figure 2 shows the view inside the sweep plane. The polytopes **A** and **B** intersect the plane in convex polygons A and B, which support 4 bitangents. Figure 3 shows the 2D visibility skeleton corresponding to Figure 2. The circular cycle of directed arcs gives the ordering of the 4 bitangents around polygon A; the cycle of the remaining directed arcs gives the ordering of the 4 bitangents around polygon B.

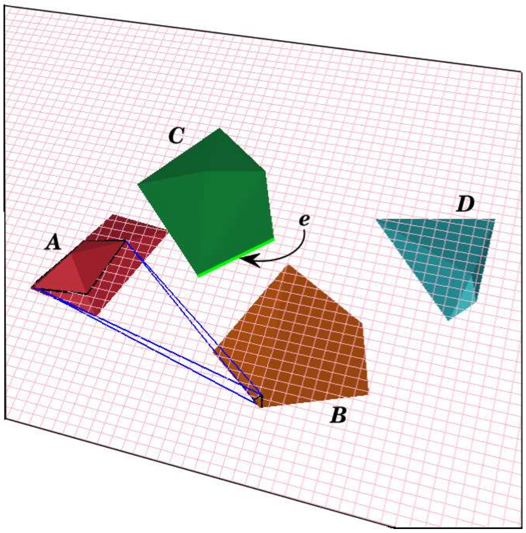

Figure 1: One position of the sweep plane.

During the sweep, the algorithm maintains the 2D visibility skeleton of the intersected polytopes [14]. The 2D visibility skeleton for the convex polygons in the initial sweep plane is computed and then used to determine the initial queue of critical events that will occur during the sweep. The sweep planes at which these critical events occur are called *event planes*. At a critical event, the 2D visibility skeleton may change its topology and the algorithm updates it, as well as the queue of critical events.

There are three types of critical events. A *V-event* occurs when the sweep plane encounters a polytope vertex that supports one or more bitangents in the sweep plane. A *T-event* occurs when two or three bitangents become colinear. An *F-event* occurs when a bitangent becomes colinear with a face of a polytope. There are $O(nk^2)$ events per sweep, and they can be computed and processed in $O(nk^2 \log n)$

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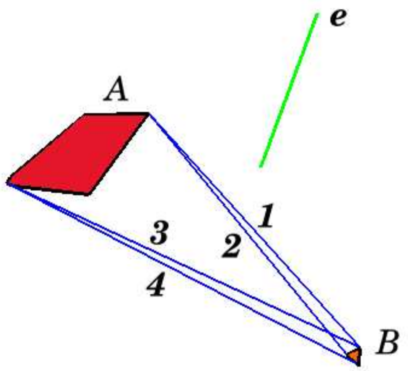

Figure 2: The view inside the sweep plane.

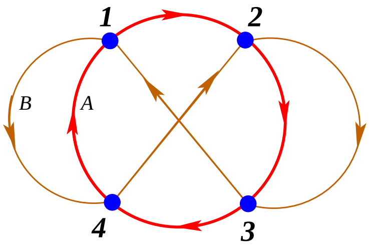

Figure 3: The 2D visibility skeleton for Figure 2.

time in the usual sweep algorithm paradigm.

The vertices of the 3D visibility skeleton are captured during the sweep, as they correspond to the V, T, F-events whose associated bitangents intersect the edge e that the sweep plane is rotating about. After the n sweeps, a description of the arcs of the 3D visibility skeleton can be computed, although the details are not illustrated in the video.

3. IMPLEMENTATION ISSUES AND TECHNICAL DETAILS

Although the algorithm as described in [5] works for any set of possibly intersecting convex polytopes in any configuration, the current implementation requires that the polytopes satisfy certain general position assumptions[1].

The key predicate of the sweep algorithm compares two event planes, to order their occurrences in the sweep. A detailed study of this predicate and its degree is given in [11].

The algorithm was implemented in C++ using the *CGAL* [6] library. We used the *CORE* library [7] to perform exact comparisons of algebraic numbers. To compute the 2D visibility skeleton we used the CGAL-based package due to Angelier and Pocchiola [3], based on the Greedy Flip Algorithm [4, 14].

The graphical output was produced using the *Geomview* software [12] through the interface provided by the *CGAL* library. We took snapshots of the Geomview window display, while rotating the viewpoint to provide a 3D view of the objects in the display window. Finally, we used *iMovie* [2] to assemble all the snapshots together into the final video. We used the *Audacity* [1] software for the audio.

4. ACKNOWLEDGMENTS

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[1] The precise definition of our general position assumptions is straightforward but lengthy. It guarantees, for example, that each critical event corresponds to a unique position of the sweep plane.