A 4-Approximation Algorithm for Guarding 1.5D Terrains

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Outline

What is a 1.5D Terrain?

- Also known as an x-monotone chain.
- The terrain intersects any vertical line at most once.
- No caves or overhangs.

- Points on the terrain 'see' each other if the line segment connecting them is never below the terrain.

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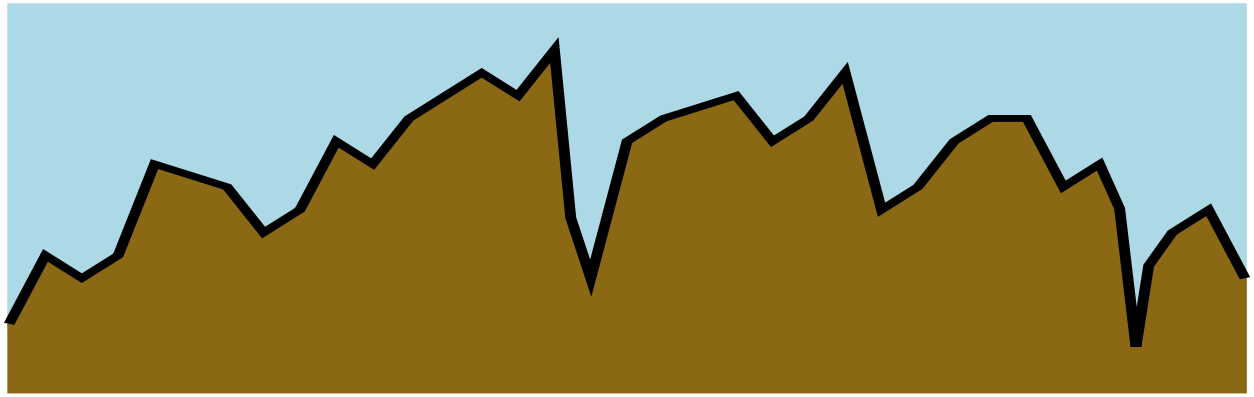

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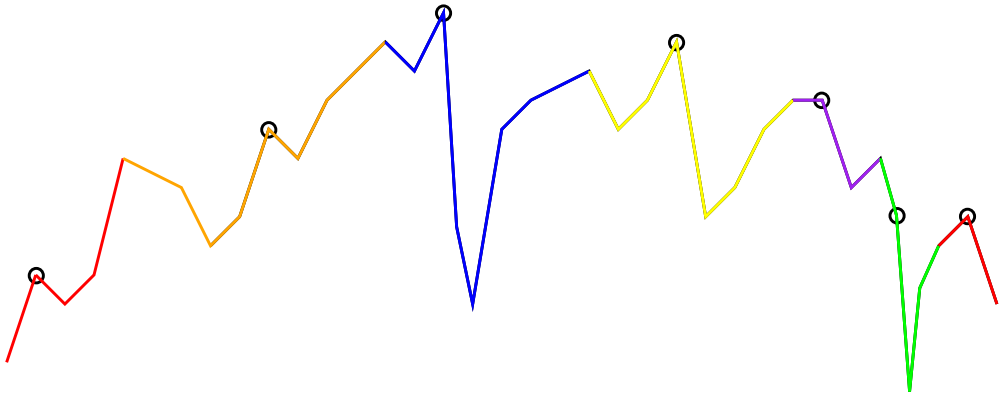
The Terrain Guarding Problem

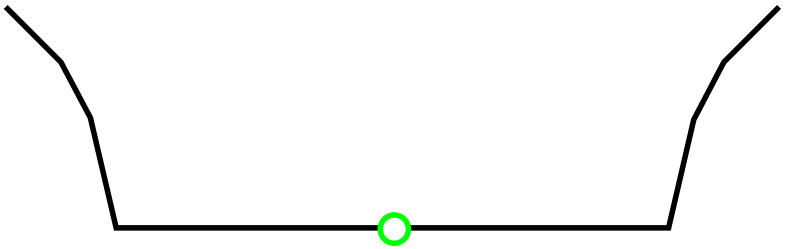
- We want a minimum set of guards that see the entire terrain.
- This is very similar to the *Art Gallery Problem*.



The Terrain Guarding Problem

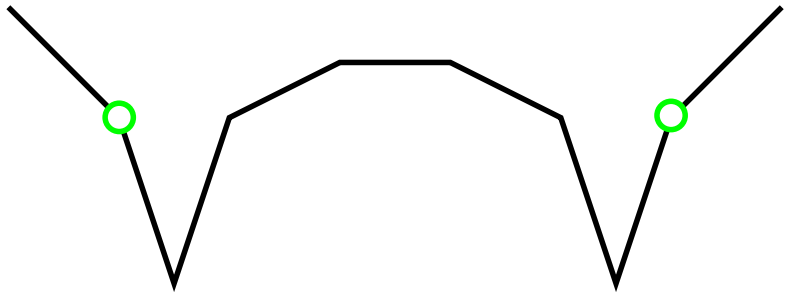
The **continuous** problem:

- The entire terrain must be guarded.
- Guards can be placed anywhere.
- Closer to real-life applications.



The **discrete** problem:

- Only vertices need to be guarded.
- Guards can only be placed on vertices.
- Closely related to non-geometric problems (*e.g.* Vertex Cover).



Outline

Is 1.5D Terrain Guarding NP-Complete?

- We don't know.
- Many related problems are NP-complete:
  - Art Gallery Problem.
  - Vertex Domination.
  - Set Cover.
  - 2.5D Terrain Guarding.
- However, 1.5D Terrains seem to forbid complex constructions.

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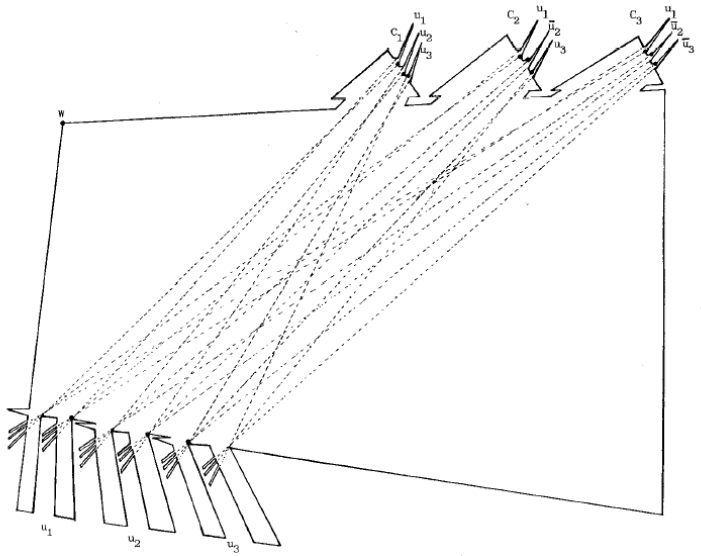
Not 1.5D



Terrain Guarding Algorithms

- Constant factor approximation algorithms given by:
  - Ben-Moshe, Katz, and Mitchell.
  - Clarkson and Varadarajan (randomized).
- They did not attempt to minimize the approximation factor.

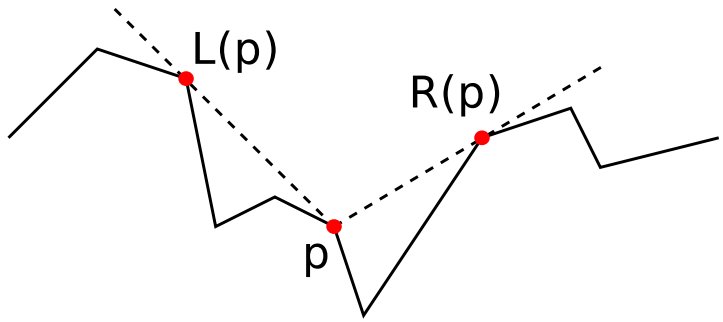
The Terrain Guarding Problem

- Our contribution is a 4-approximation algorithm.
- Simpler than previous algorithms.
- Best approximation factor so far.
- We will present the algorithm for the **discrete** problem.
- Works for the continuous problem with a slight modification.
- Runs in $\Theta(n^2)$ time.

Outline

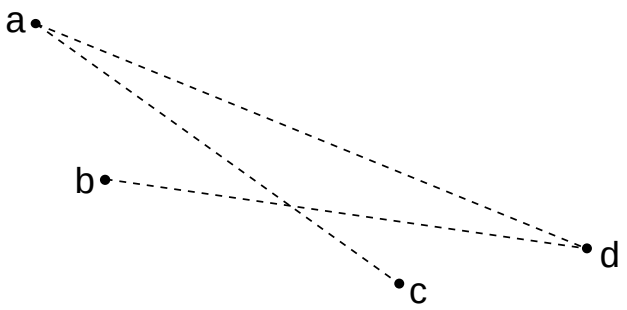
Terminology and Notation

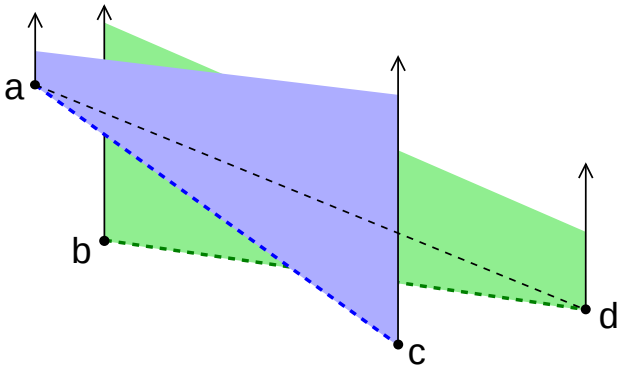
- $p < q$ means p is to the left of q.
- p dominates q means p sees every unguarded point that q sees.
- $L(p)$ is the leftmost point that sees p.
- $R(p)$ is the rightmost point that sees p.



Order Claim

- The fundamental property of 1.5D terrains that we exploit.

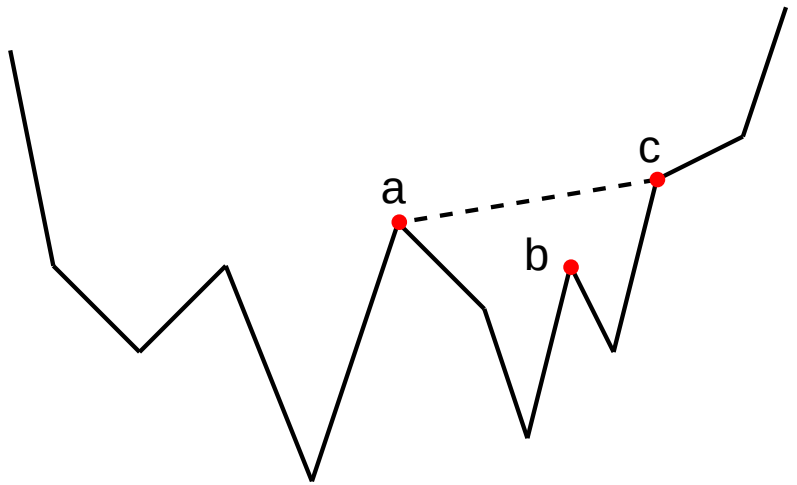




- Consider $a < b < c < d$.
- If a sees c and b sees d then a sees d.

External Domination

- Consider $a < b < c$ such that a sees c.
- $\{a, c\}$ dominates b outside the interval (a, c).



Outline

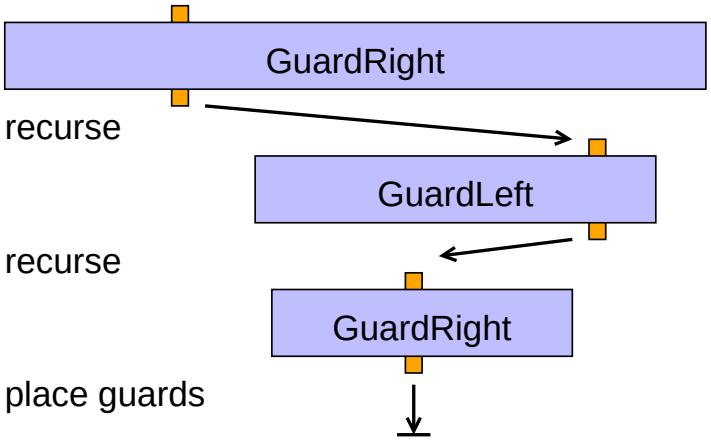
The Algorithm

while unguarded points remain **do**
  find u, $S(u)$ such that:
    - u is unguarded
    - $|S(u)| \leq 4$
    - $S(u)$ dominates any guard that sees u
  place guards at the points in $S(u)$
end while

- This guarantees an approximation factor of 4.
- The real work is finding u and $S(u)$.

Finding u and $S(u)$

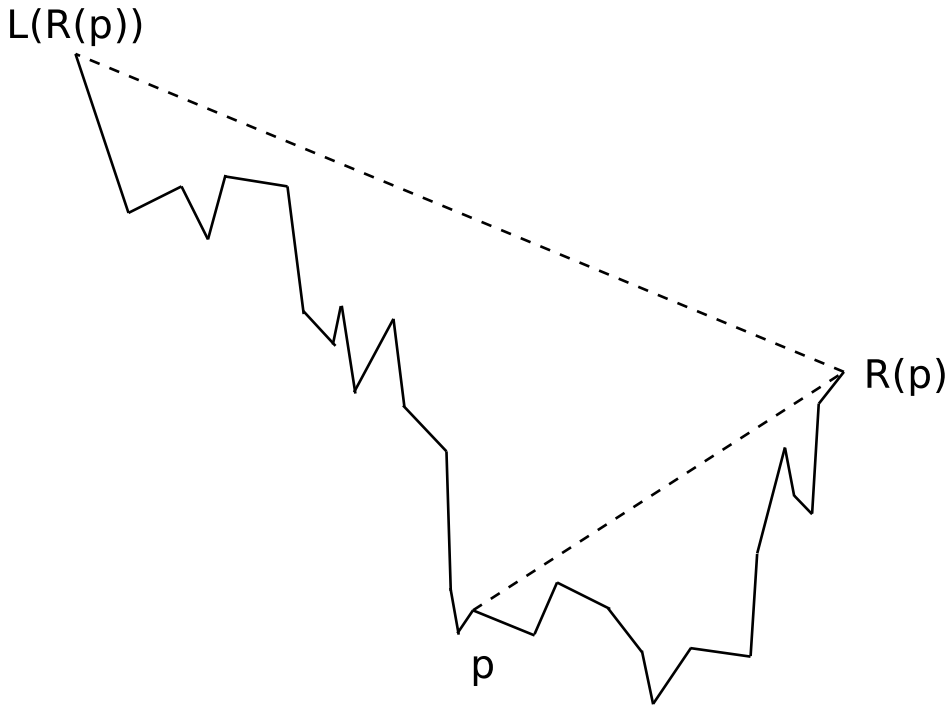
- The work is done by GuardRight, a recursive method.
- GuardLeft is the mirror image of GuardRight.



- GuardRight recurses by calling GuardLeft and vice versa.
- Guards are placed only in the terminal case.

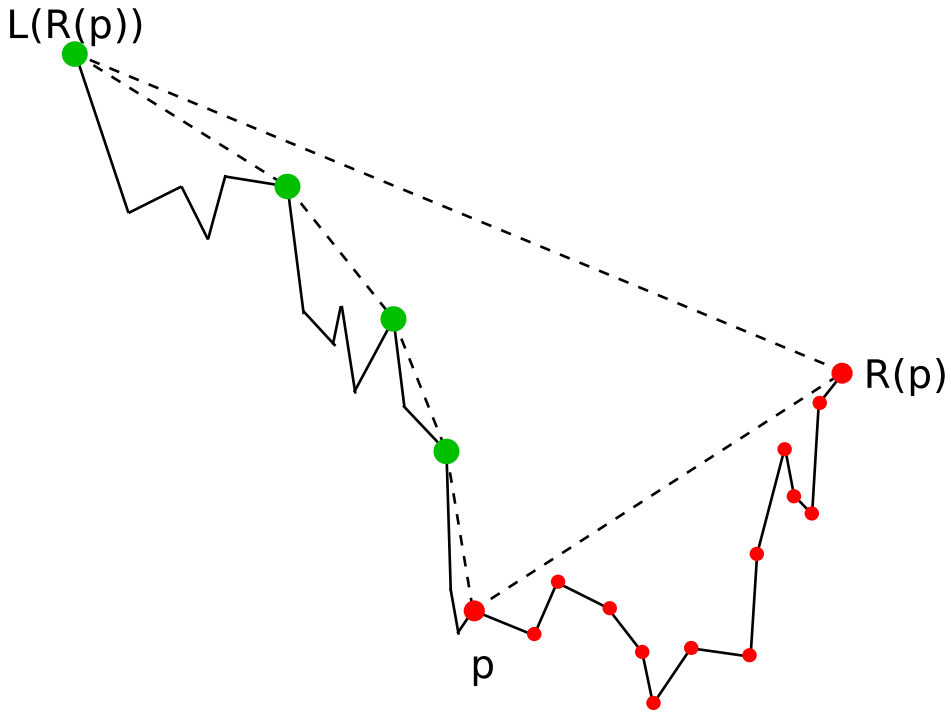
GuardRight(p, c)

- p is an unguarded vertex (not on the convex hull).
- c sees every unguarded vertex in $[L(R(p)), p)$
- We look for u and $S(u)$ in $[L(R(p)), R(p)]$.
- We either find them or isolate a subregion to recurse on.



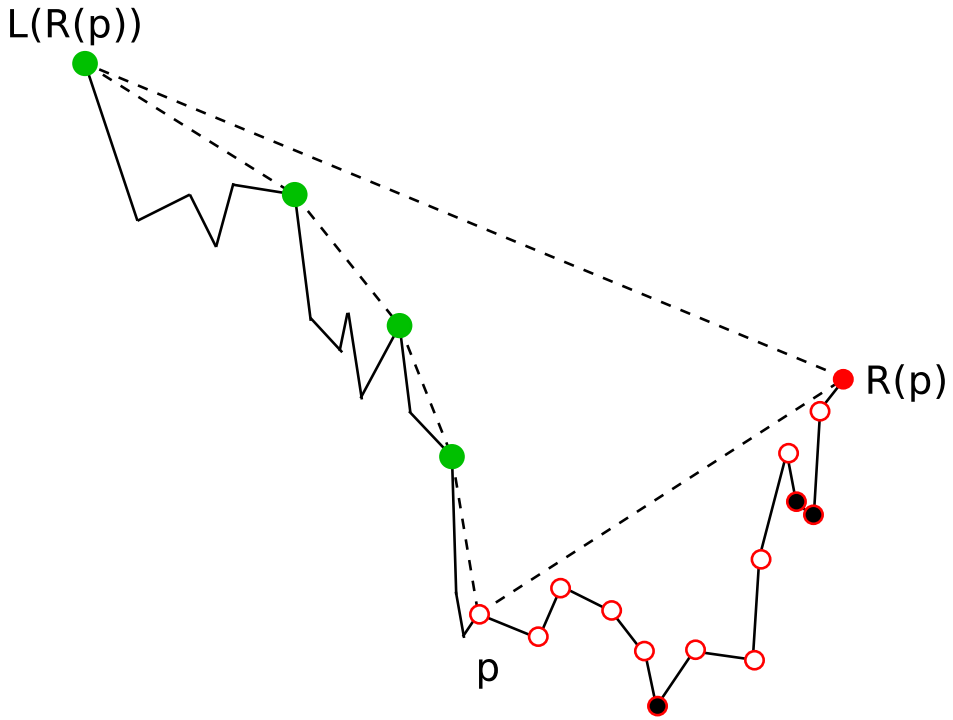
Guards on the Left

- Left vertices (•) are on the convex hull between p and $L(R(p))$.
- Right vertices (•) are between p and $R(p)$.
- If we place a guard at c we don't need to consider placing guards in $[L(R(p)), p)$ except on left vertices.



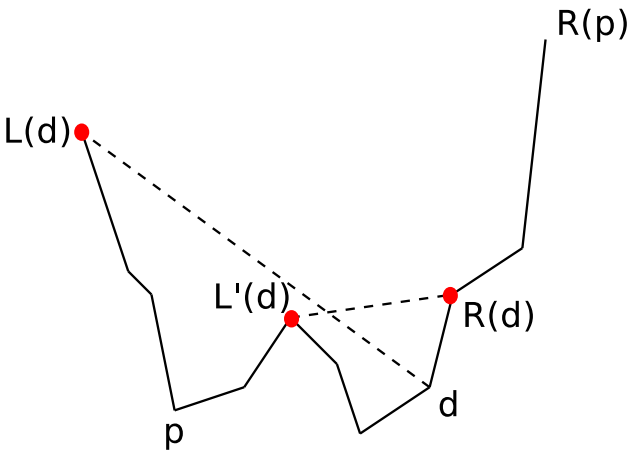
Yet More Terminology

- Unguarded vertices in $[p, R(p))$ are either *exposed* or *sheltered*.
- Exposed vertices (○) can see a left vertex.
- Sheltered vertices (●) cannot see any left vertex.
- p is exposed.
- d is a special exposed vertex that our algorithm finds.



The Terminal Case

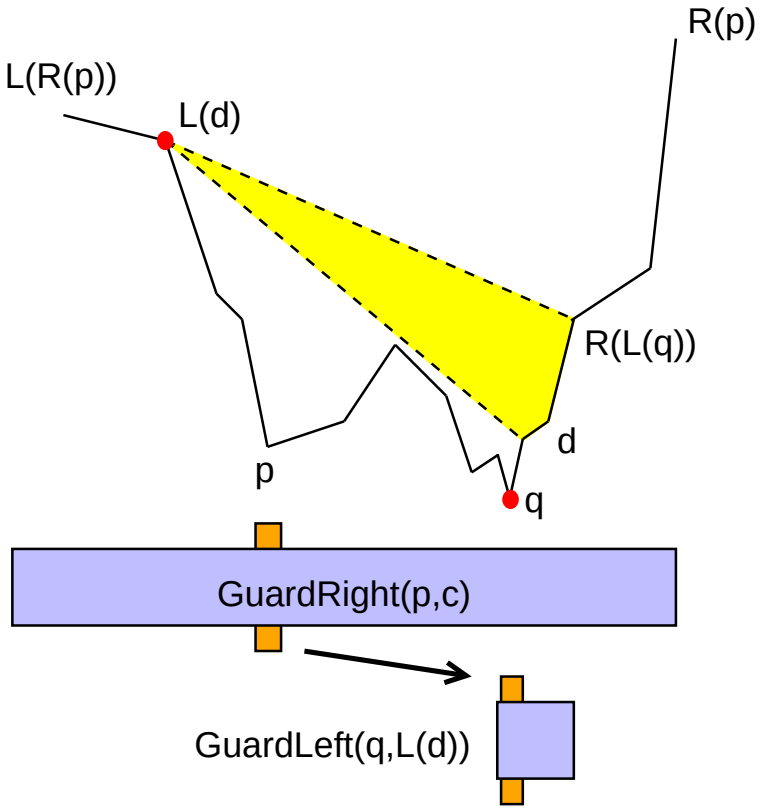
- $L(d)$ sees every exposed vertex to the right of $L'(d)$.
  - $L'(d)$ is the leftmost right vertex that sees d.
- If $\{c, L(d), L'(d), R(d)\}$ sees every unguarded vertex in $[L'(d), R(d)]$ then the set dominates any vertex that sees d.
- This is the **terminal case** so we place guards.



- $u \leftarrow d$.
- $S(u) \leftarrow \{c, L(d), L'(d), R(d)\}$.

The Recursive Case

- If not terminal, there must be a sheltered vertex in $[L'(d), R(p))$
- Define q as the rightmost sheltered vertex.
- $L(d)$ sees every unguarded vertex in $(q, R(L(q))]$.
- We call $\textsc{GuardLeft}(q, L(d))$.



The Recursive Case

- We recurse on $[L(q), R(L(q))]$, which is a proper subterrain of $[L(R(p)), R(p)]$.
- Problem size shrinks, so we reach a terminal case eventually.
- We just repeat this whole process until the entire terrain is guarded.

Outline

Future Work

- Primary question: is 1.5D terrain guarding in P or is it NP-complete?
- Characterize visibility graphs of terrains.
  - The order claim isn't the only tool we can use!
  - What other tools are available?

Thank you!