COMP 250: Practice Midterm

October 22th, 2015, 6:05pm – 7:25pm

- This is a short answer exam. While the real exam will be multiple choice, this exam is intended to prepare you for what may be expected from you on the real exam.
- You have 80 minutes to write the exam.
- This exam contains one page, and is out of 40 marks.

1. **Java (3):**
   Write a java method that sorts an array using any method. List input, output, preconditions, and post conditions.

2. **Divide and Conquer (10):**
   Write an algorithm which sorts a set of n numbers using at *most* log(n!) + n number of comparisons.
   (You may use any operation that does not compare a pair of elements as many times as you wish)

3. **Induction (6):**
   Given a set of n>2 distinct points on a 2 dimensional plane, show that it is always possible to draw a polygon with n sides containing all points as vertices, such that no two sides intersect.
   Assume that not all points lie on a single line.

4. **Landau Symbols (Big O notation) (12):**
   The notation o (read: small o) can be interpreted to mean the following:
   If $f(n) = o(g(n))$,
   then $f(n) = O(g(n))$ but $g(n) \neq O(f(n))$

   a) Find a function f(n) such that:
      $f(n) = o(n)$
      $\log(n) = o(f(n))$

   b) Find a function f(n) such that:
      $f(n) = o(n^c)$, for any c>0
      $(\log(n))^k = o(f(n))$, for any k>0

   c) Find a function f(n) that uses only the binary operations +,-,*,/,^,log, such that f(n)=O(n*n!) and n!/n = O(f(n)).
      (Recall that $\int ln(x)dx = x*ln(x) - x$)

5. **Quicksort (8):**
   There exists an algorithm which can find the k_{th} element in a list in O(n) time, and suppose that it is in place. Using this algorithm, write an in place sorting algorithm that runs in worst case time O(n*log(n)), and prove that it does. Given that this algorithm exists, why is mergesort still used?

6. **ADTs (1):**
   Complete the following table with *optimal* big O running times, given the data structure:

	Array of size n	Linked list of size n
Get i_{th} entry in list, where i is any number between 1 and n		
Concatenate two lists		