COMP250: Thinking Recursively. Examples.

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Based on slides from (Hatami), (Bailey), (Stepp & Martin)

Course credits

c(x) = total number of credits required to complete course x

c(COMP462) = ?

= 3 credits **+ #credits for prerequisites**

COMP462 has 2 prerequisites: COMP251 & MATH323

= 3 credits **+ c(COMP251) + c(MATH323)**

The function c calls itself twice

c(COMP251) = ? c(MATH323) = ?

c(COMP251) = 3 credits + c(COMP250) COMP250 is a prerequisite

Substitute c(COMP251) into the formula:

c(COMP462) = 3 credits + 3 credits + c(COMP250) + c(MATH323)

c(COMP462) = 6 credits + c(COMP250) + c(MATH323)

Course credits

c(COMP462) = 6 credits + c(COMP250) + c(MATH323)

c(COMP250) = ? c(MATH323) = ?

c(COMP250) = 3 credits no prerequisite

c(COMP462) = 6 credits + 3 credits + c(MATH323)

c(MATH323) = ?

c(MATH323) = 3 credits + c(MATH141)

c(COMP462) = 9 credits + 3 credits + c(MATH141)

c(MATH141) = ?

c(MATH141) = 4 credits no prerequisite

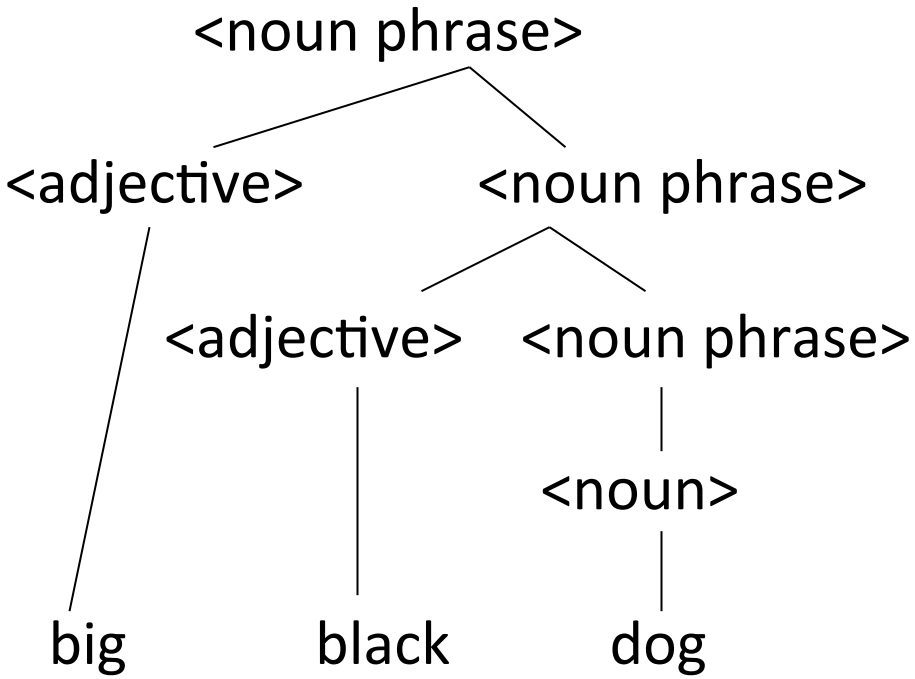
c(COMP462) = 12 credits + 4 credits = 16 credits

Recursive definition

A noun phrase is either

- a noun, or
- an adjective followed by a noun phrase

<noun phrase> → <noun> OR <adjective> <noun phrase>



Definitions

Recursive definition:
A definition that is defined in terms of itself.

Recursive method:
A method that calls itself (directly or indirectly).

Recursive programming:
Writing methods that call themselves to solve problems recursively.

Why using recursions?

- "cultural experience" - A different way of thinking of problems
- Can solve some kinds of problems better than iteration
- Leads to elegant, simplistic, short code (when used well)
- Many programming languages ("functional" languages such as Scheme, ML, and Haskell) use recursion exclusively (no loops)
- Recursion is often a good alternative to iteration (loops).

Iterative algorithms

Definition: Algorithm where a problem is solved by iterating (going step-by-step) through a set of commands, often using loops

```
Algorithm: power(a,n)
Input: non-negative integers a, n
Output: a^n
Product ← 1
for i = 1 to n do
          product ← product * a
Return product
```

i	0	1	2	3	4
product	1	a	$a * a = a^2$	$a^2 * a = a^3$	$a^3 * a = a^4$

Recursive algorithms

Definition: algorithm is recursive if in the process of solving the problem, it calls itself one or more times

```
Algorithm: power(a,n)
Input: non-negative integers a, n
Output: a^n
if (n=0) then
   return 1
else
   return a * power(a,n-1)
```

Example

power(7,4) calls

↳ power(7,3) calls

↳ power(7,2) calls

↳ power(7,1) calls

↳ power(7,0) returns 1

returns 7 * 1 = 7

returns 7 * 7 = 49

returns 7 * 49 = 343

returns 7 * 343 = 2041

Algorithm structure

Every recursive algorithm involves at least 2 cases:

base case: A simple occurrence that can be answered directly.

recursive case: A more complex occurrence of the problem that cannot be directly answered, but can instead be described in terms of smaller occurrences of the same problem.

Some recursive algorithms have more than one base or recursive case, but all have at least one of each.

A crucial part of recursive programming is identifying these cases.

Binary Search

Algorithm binarySearch(array, start, stop, key)
Input: - A sorted array
- the region start...stop (inclusively) to be searched
- the key to be found
Output: returns the index at which the key k has been found, or -1 if it is not in array[start...stop].

Example: Does the following array contains the number 6?

1	1	3	5	6	7	9	9

Binary search example

1	1	3	5	6	7	9	9

Search [0:7]

$5 < 6 \Rightarrow$ look into right half of the array

1	1	3	5	6	7	9	9

Search [4:7]

$7 > 6 \Rightarrow$ look into left half of the array

1	1	3	5	6	7	9	9

Search [4:4]

6 is found. Return 4 (index)

Binary Search Algorithm

```
int bsearch(int[] A, int i, int j, int x) {
   int e = (i+j) % 2;   // Find middle point
   if (A[e] > x) {    // key is bigger, look to left half
      return bsearch(A,i,e-1,x);
   } else if (A[e] < x) { // key is lower, look to right half
      return bsearch(A,e+1,j,x);
   } else {   // value x is found
      return e;
   }
}
```

Is it correct?

Binary Search Algorithm

```
int bsearch(int[] A, int i, int j, int x) {
   if (i<j) {
      int e = ⌊(i+j)/2⌋;
      if (A[e] > x) {
         return bsearch(A,i,e-1);
      } else if (A[e] < x) {
         return bsearch(A,e+1,j);
      } else {
         return e;
      }
   } else { return -1; } // value not found
}
```

Fibonacci numbers

$Fib_0 = 0$ base case

$Fib_1 = 1$ base case

$Fib_n = Fib_{n-1} + Fib_{n-2}$ for $n > 1$ recursive case

i	0	1	2	3	4	5	6	7
Fib_i	0	1	1	2	3	5	8	13

Recursive algorithm

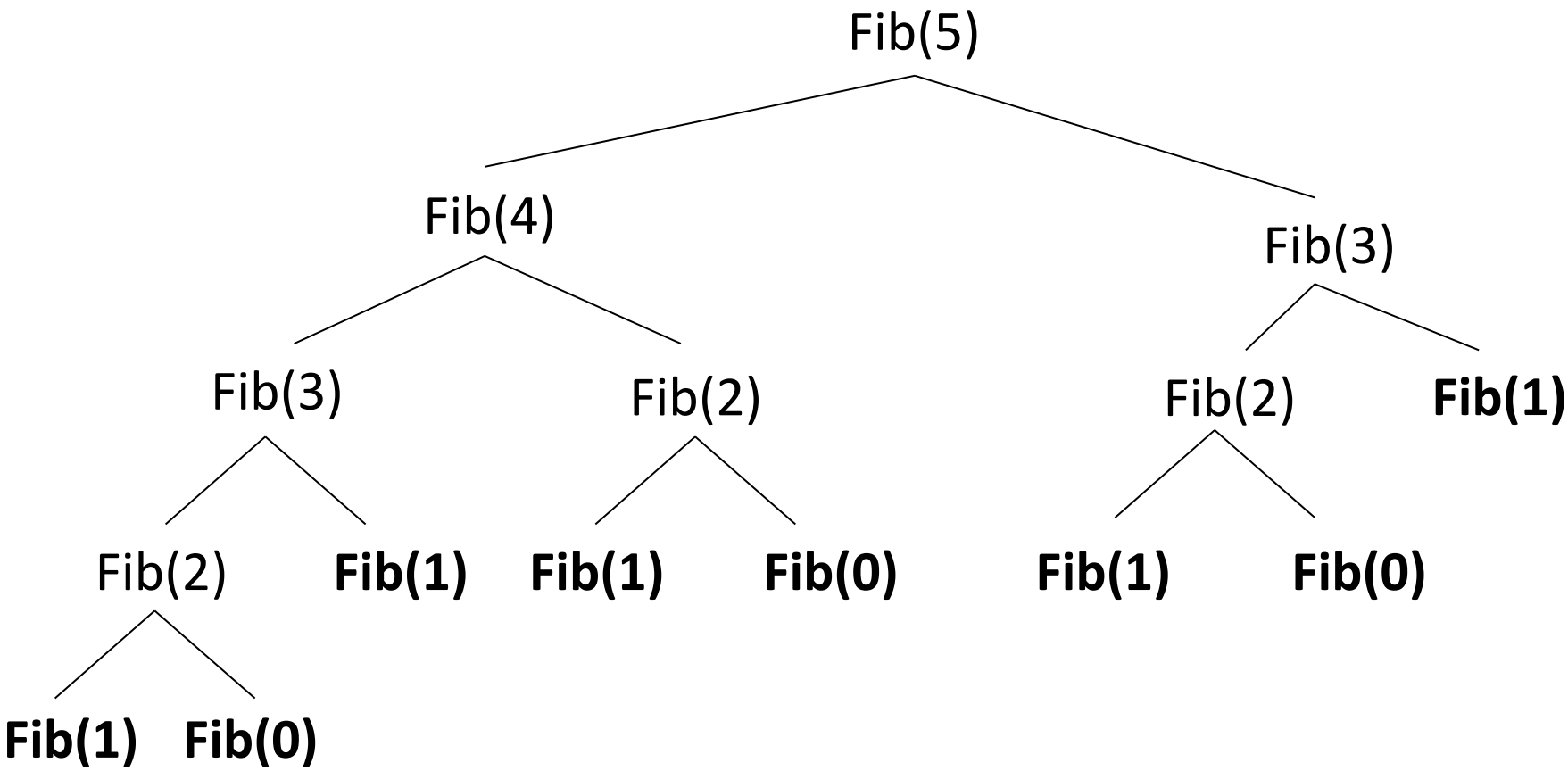
```
// = Fibonacci number n (for n >= 0)
public static int Fib(int n) {
   if (n <= 1) {
      return n;
   }
   // {n > 0}
   return Fib(n-1) + Fib(n-2);
}
```

can handle both base cases together

recursive case (2 recursive calls)

Note: The algorithm follows almost exactly the definition of Fibonacci numbers.

Recursion is not always efficient!



Question: When computing Fib(n), how many times are F(0) or F(1) called?