COMP250: Dictionary ADT & Binary Search Trees

Lecture 21

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Dictionary ADT

- A dictionary (a.k.a. map) stores a set of pairs (key, value)
  - (word, definition)
  - (studentID, studentRecord)
  - (flightNumber, flightInformation)
- Data is accessed only through key:
  - Object find(key k)
  - void insert(key k, Object v)
  - Object remove(key k)
- If the keys can be ordered
  - Object previous(key k)
  - Object next(key k)

Dictionary ADT

```
Dictionary vehicle = {
     ‘car’:’a road vehicle, typically with
four wheels, powered by an internal
combustion engine and able to carry a small
number of people.’;
     ‘bicyle’:’a vehicle composed of two
wheels held in a frame one behind the other,
propelled by pedals and steered with
handlebars attached to the front wheel.’
}
```

Array implementation

Key	Value
Key 1	**Content 1**
Key 2	**Content 2**
Key 3	**Content 3**
Key 4	**Content 4**
ø	**ø**

Size = 4

Array implementation

Array of pairs (key, value)

- find(key k) : scan array to find key O(n)
- insert(key k, Object v): O(1)
  - Add the pair (k, v) at the end of the array
  - Increase size by one
- remove(key k) O(n)
  - Scan array to find k
  - Shift left remaining elements

Array implementation

Remove('Key 2')

Key	Value
Key 1	**Content 1**
~~**Key 2**~~	~~**Content 2**~~
Key 3	**Content 3**
Key 4	**Content 4**
Ø	**Ø**

Sorted Array implementation

Key	Value
4	**x**
7	**x**
8	**x**
12	**x**
15	**x**
16	**x**
21	**x**
33	**x**
42	**x**
53	**x**
55	**x**
62	**x**

Sorted array implementation

Array of pairs (key, value), *sorted by key*

- find(key k) : binary search to find key — O(log n)
- insert(key k, Object v): — O(n)
  - Binary search to find where to insert, O(log n)
  - Shift element right to insert new element, O(n)
- remove(key k) — O(n)
  - Binary search to find key, O(log n)
  - Shift left remaining elements, O(n)

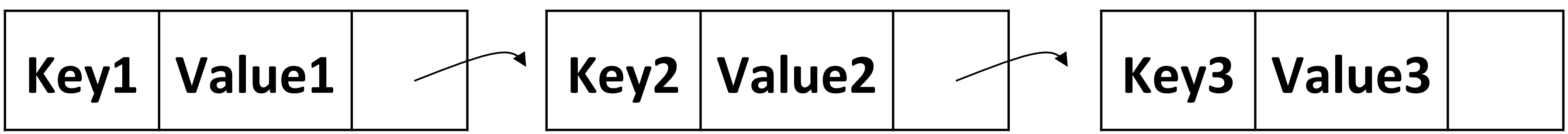
Remove in a sorted Array

Remove('12')

Key	Value
4	x
7	x
8	x
~~12~~	~~x~~
15	x
16	x
21	x
33	x
42	x
53	x
55	x
62	x

Find: O(log n)
+
Remove: O(n)

Linked-list implementation



Linked-list implementation

Linked-list where each node contain a pair (key, value)

- find(key k) : scan list to find key O(n)
- insert(key k, Object v): O(1)
  - Add the pair (k, v) at the end of the list
- remove(key k) O(n)
  - Scan list to find k, O(n)
  - Remove node, O(1)

Note: Keeping the linked-list sorted does not help, as binary search can't be done in time O(log n) in linked lists. Why?

Implementations of dictionary

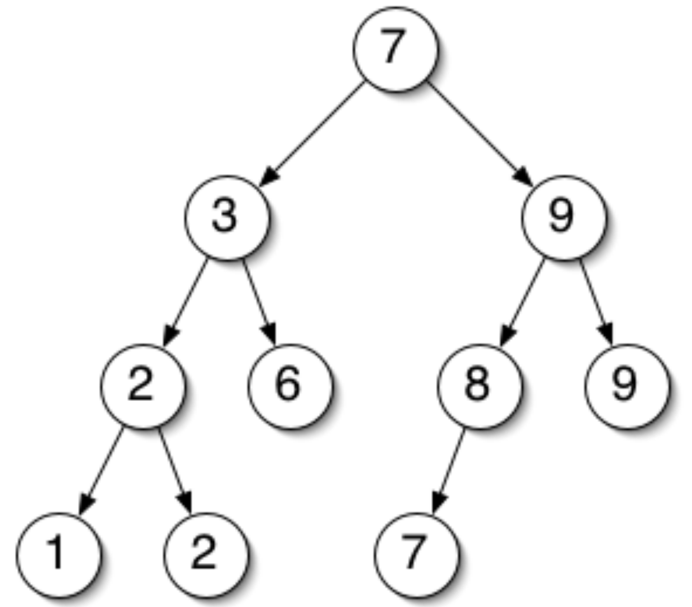
Method	find	insert	remove
Array	O(n)	O(1)	O(n)
Linked-list	O(n)	O(1)	O(n)
Sorted array	O(log n)	O(n)	O(n)

BST - Definition

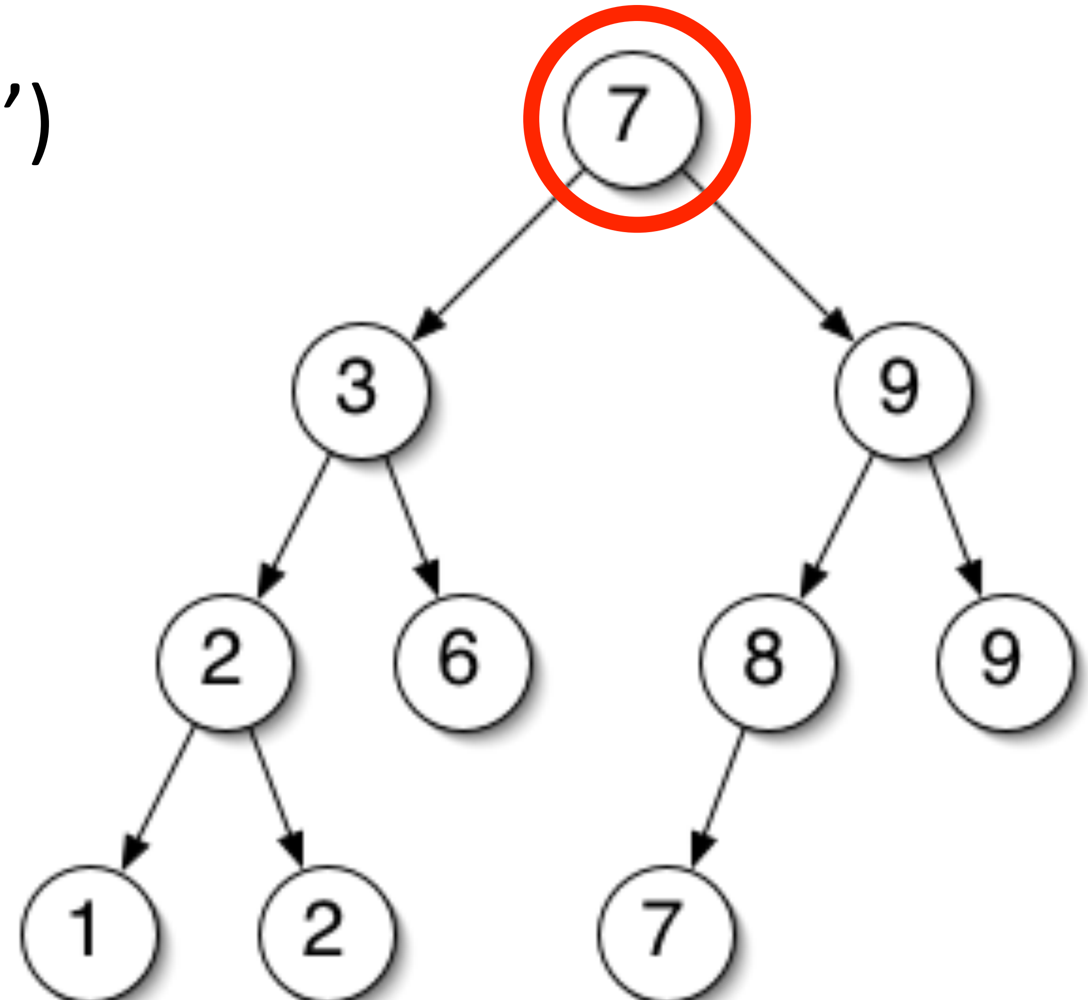
A **binary search tree** (BST) is a binary tree such that for any node n,

- The elements of the left subtree of n have values smaller or equal to n
- The elements of the right subtree of n have values larger of equal to n

(In the figure, we show only the keys)



find('8')



BST - Find

Idea: 1) Start from the root of the tree

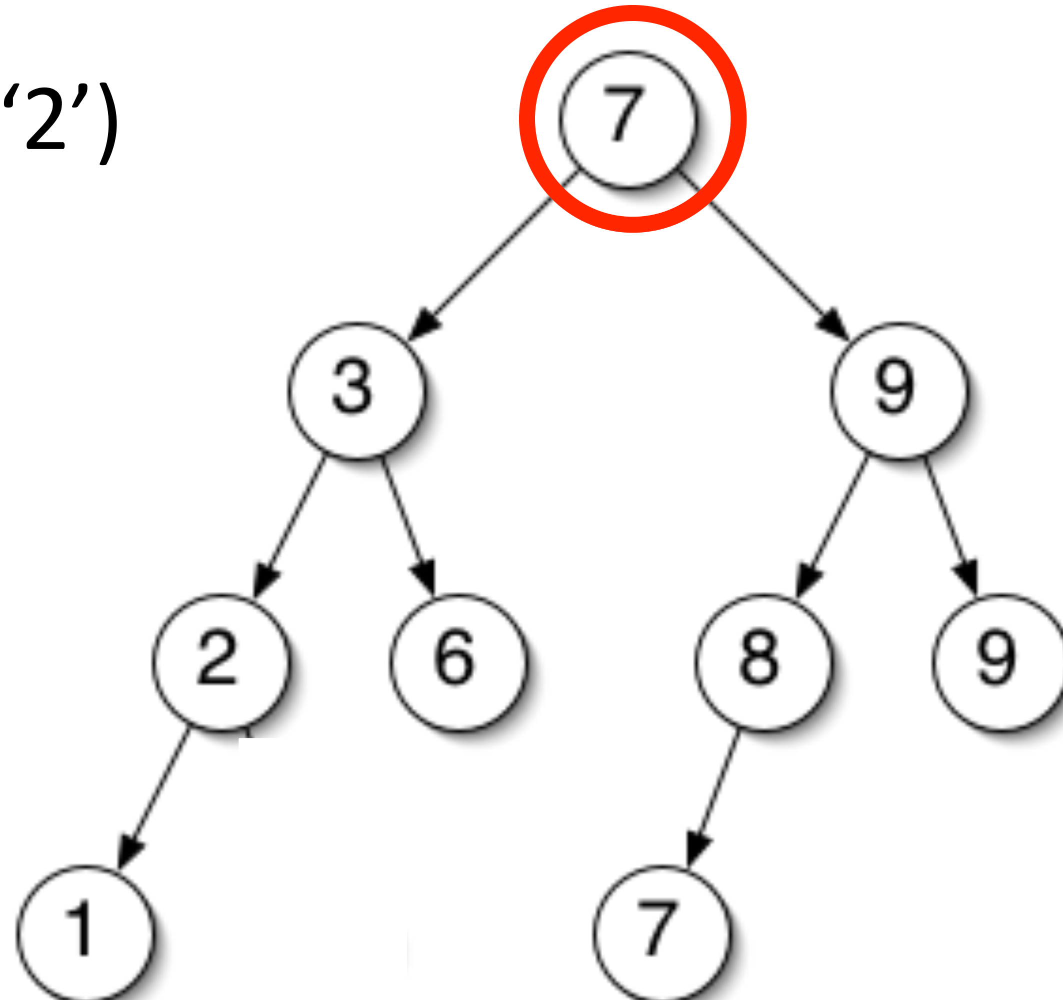
2) Choose if you should go to the left or right child.

3) Repeat until you find the key sought or get to a leaf.

```
Algorithm find(node n, key k)
Input: The node n at the root of the tree to explore.
                 The key k to find
Output: Returns one node with key equal to k
if (n = null) then return null
if (n.key = k) then return n
if (n.key > k) then return find(n.leftChild, k)
if (n.key < k) then return find(n.rightChild, k)
```

Can you write a non-recursive version of this algorithm?

insert('2')



BST - insert

Idea: 1) Find the leaf where the insertion will take place, by going down the tree as for the "find" algo.

2) Add a new left or right child to that leaf

```
Algorithm insert(node n, key k, object v)
Input: The key k and information i to be added to
  the subtree rooted at n. Assumes n!= null
Output: Inserts a new node (k,i) in the subtree
  rooted at n
if (k ≤ n.key) then
   if (n.leftChild != null) then
  insert(n.leftChild, k, v)
   else n.setLeftChild( new node(k,v) );
else
   if (n.rightChild != null) then
  insert(n.rightChild, k, v)
   else n.setRightChild( new node(k,v) );
```

BST - remove

Idea: 1) Find the node N to be removed using the "find" algo

2) - If N is a leaf, simply remove it

- If N is an internal node with only one child, replace N by its child

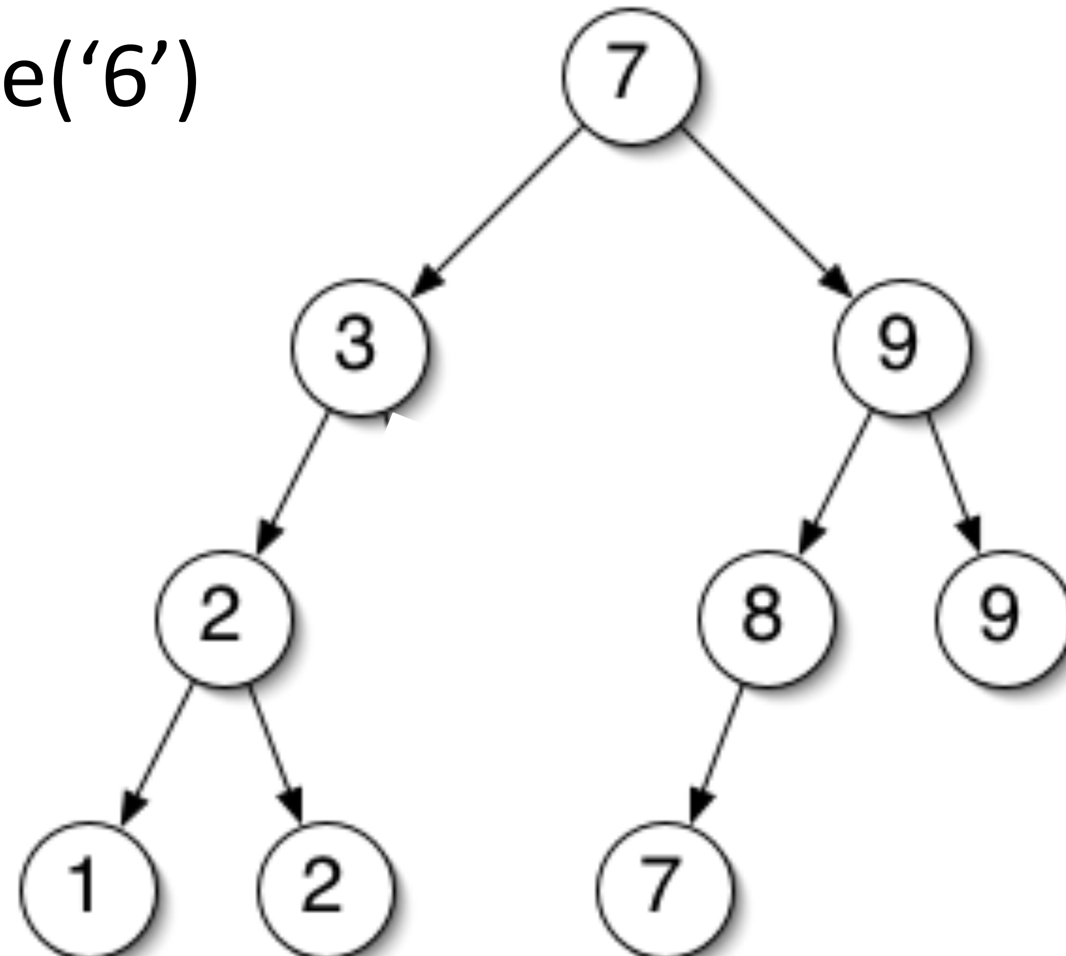
- If N is an internal node with two children, N will be replaced by the node N' that has the next key largest key after N.

To find N':

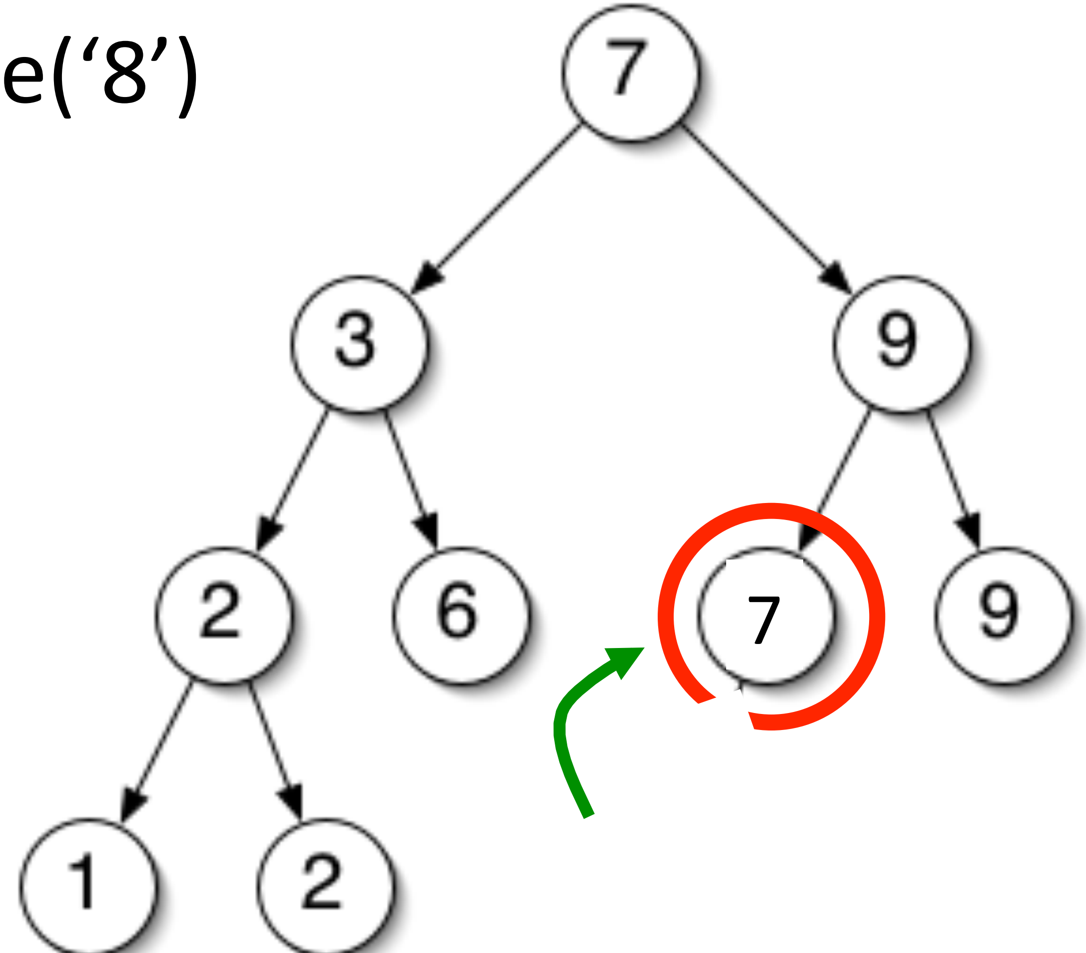
i) Follow the right child of N and then go down left children until no left child is found. The node found is N'

Overwrite N by N'.

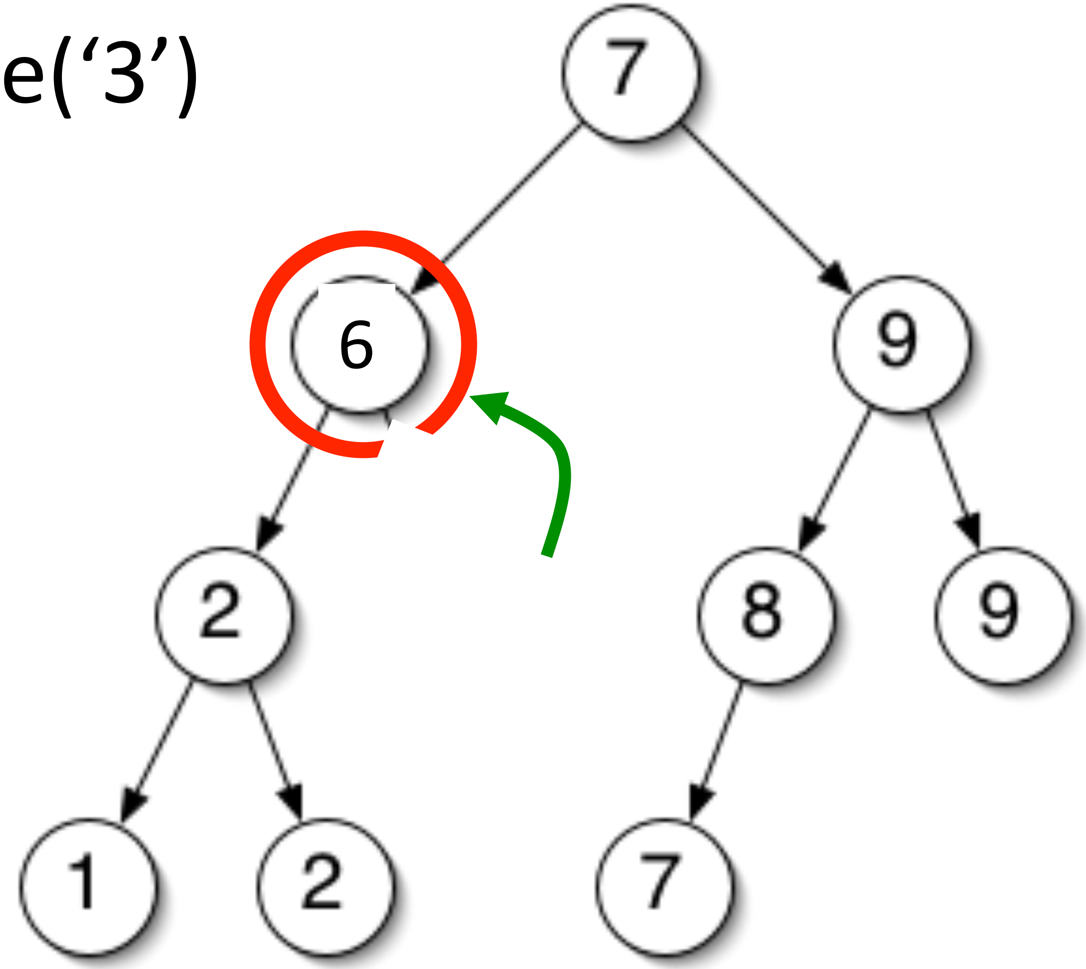
remove('6')



remove('8')



remove('3')



Implementation

```
// A small utility function
Algorithm replace(node x, node y)
Input: Two nodes x and y
Output: Copies node y onto node x, overwriting x.
if (x.parent != null)
   if (x.parent.leftChild = x) then
   x.parent.setLeftChild(y)
      else x.parent.setRightChild(y)
if (y != null) then y.parent ← x.parent
```

```
Algorithm remove(node root, key k)
Input: The key k of the node to be removed from the
  subtree rooted at n
Output: Removes node with key k and returns it.
node x ← find(root, k)
if (x=null) then return null  // key k was not found
if ( x.isALeaf() ) then replace(x, null); return
if (x.leftChild = null or x.rightChild = null)
  then        // x has only one child
  if ( x.leftChild = null ) then
      replace(x, x.rightChild)  // x was right child
  else if ( x.rightChild = null )
      replace(x,  x.leftChild)  // x was left child
  else // x has two children, find successor of x
      suc ← x.rightChild
      while (suc.leftChild != null) do
            suc ← suc.leftChild
            x.value = suc.value
            x.key = suc.key
            replace(suc, suc.rightChild)
```