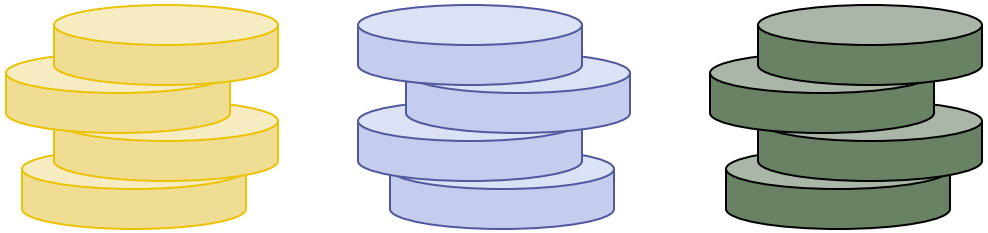
COMP250: Stacks



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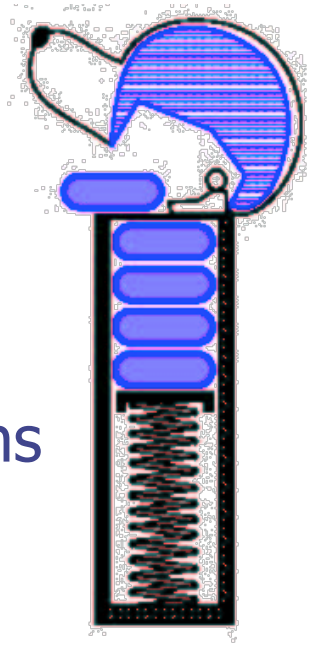
Based on slides from (Goodrich & Tamassia, 2004)

The Stack ADT



- A Stack ADT is a list that allows only operations at one end of the list (called the top)
- Main stack operations:
  - push(object): inserts an element at the top of the stack
  - object pop(): removes the object at the top of the stack
  - object top(): returns the last inserted element without removing it (N.B. In Java, this is called peek())
  - integer size(): returns the number of elements stored
  - boolean isEmpty(): indicates whether no elements are stored
- Last in – First out (LIFO)

Applications of Stacks

- Direct applications
  - Page-visited history in a Web browser
  - Undo sequence in a text editor
  - Chain of method calls in the Java Virtual Machine
- Indirect applications
  - Auxiliary data structure for algorithms
  - Component of other data structures

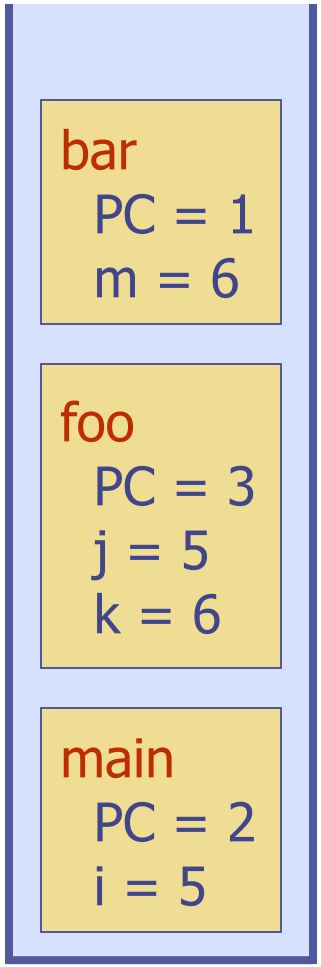
Method Stack in the JVM

- The Java Virtual Machine (JVM) keeps track of the chain of active methods with a stack
- When a method is called, the JVM pushes on the stack a frame containing
  - Local variables and return value
  - Program counter, keeping track of the statement being executed
- When a method ends, its frame is popped from the stack and control is passed to the method on top of the stack
- Allows for **recursion**

```
main() {
   int i = 5;
   foo(i);
   }

foo(int j) {
   int k;
   k = j+1;
   bar(k);
   }

bar(int m) {
   …
   }
```



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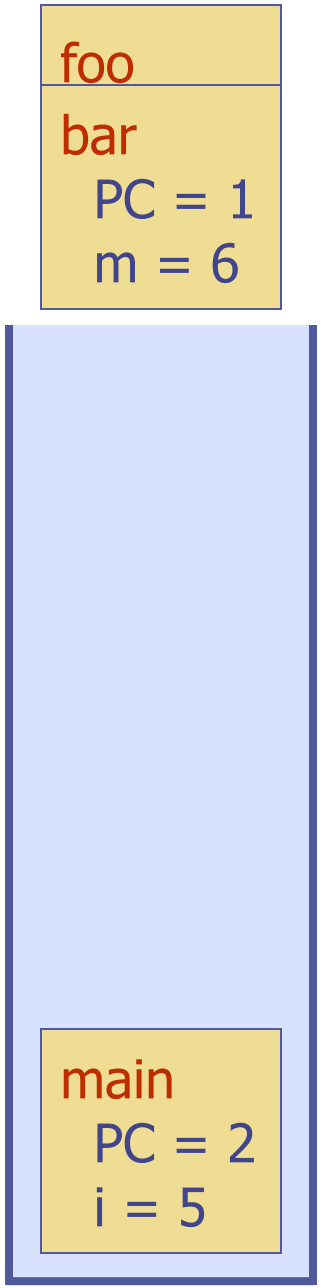
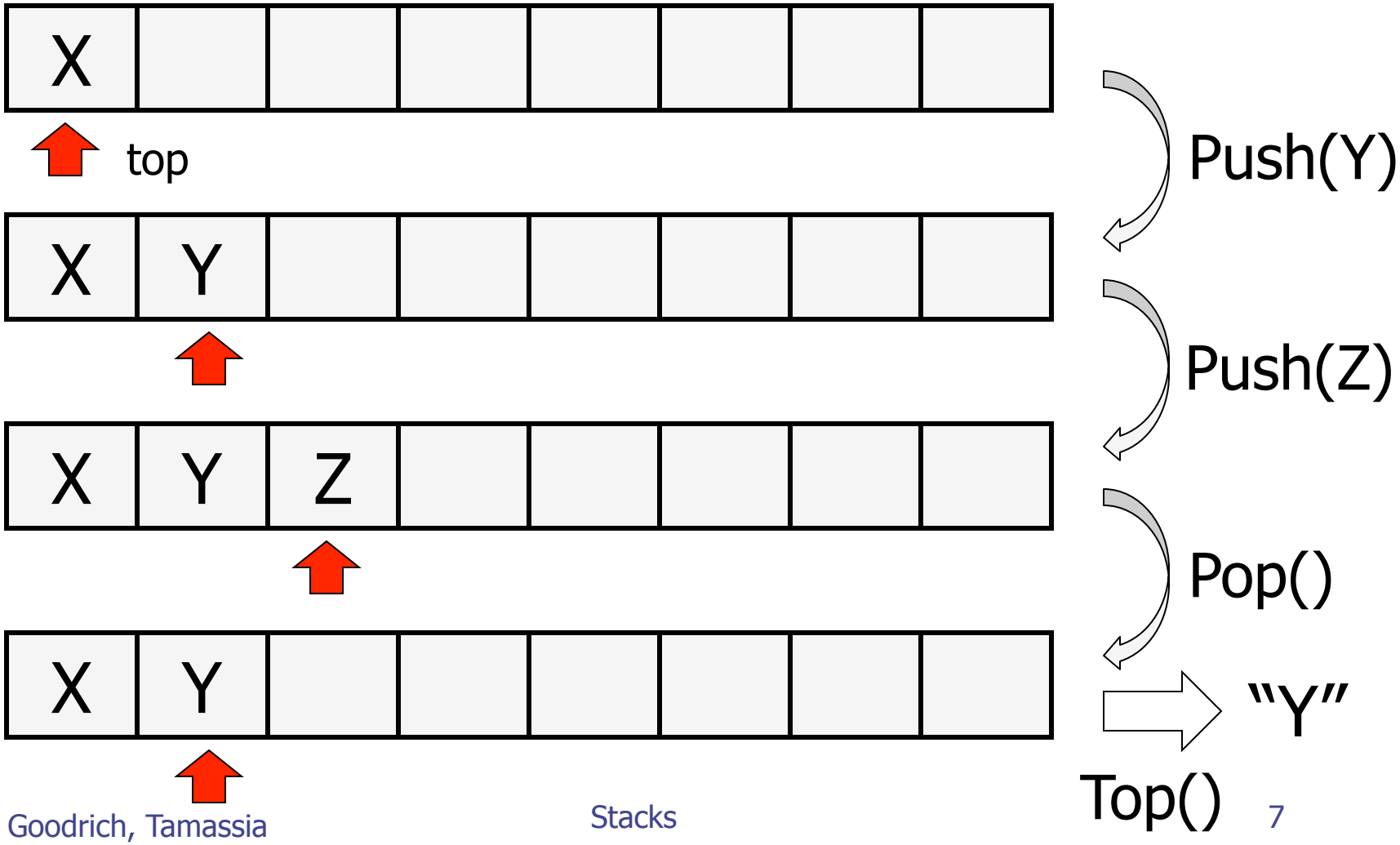


Array-based Stack in Java

```
public class ArrayStack {

   // holds the stack elements
   private Object S[ ];

   // index to top element
   private int top = -1;

   // constructor
   public ArrayStack(int capacity) {
      S = new Object[capacity]);
   }
```

- An integer top keeps track of the top of the stack.
- Capacity fixes the size of the array, hence the stack.

Example of array-based Stack



Array-based Stack (push)

- The array storing the stack elements may become full
- A push operation will then throw a FullStackException
  - Limitation of the array-based implementation
  - Not intrinsic to the Stack ADT

Algorithm *push*(o)
  if $t = S.length - 1$ **then**
    throw *FullStackException*
  else
    $t \leftarrow t + 1$
    $S[t] \leftarrow o$

Array-based Stack (pop)

- A simple way of implementing the Stack ADT uses an array
- We add elements from left to right
- A variable keeps track of the index of the top element

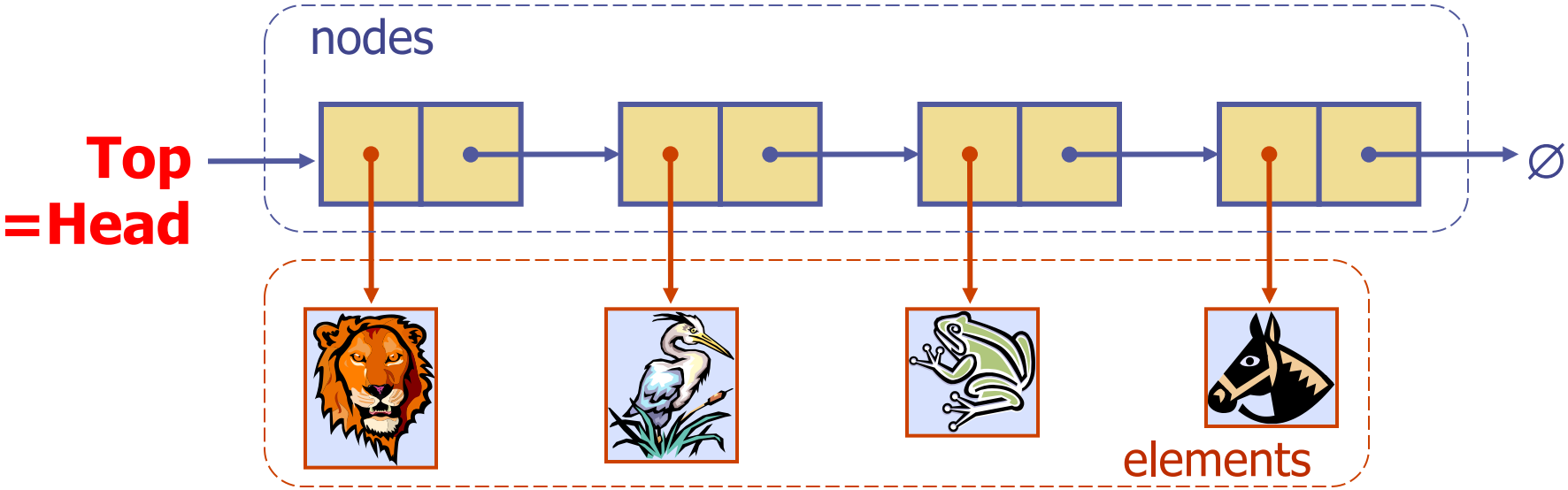
```
Algorithm size()
  return t + 1

Algorithm pop()
  if isEmpty() then
    throw EmptyStackException
  else
    t ← t - 1
```

Performance and Limitations

- Performance
  - Let n be the number of elements in the stack
  - The space used is $O(n)$
  - Each operation runs in time $O(1)$
- Limitations
  - The maximum size of the stack must be defined a priori and cannot be changed
  - Trying to push a new element into a full stack causes an implementation-specific exception

Stack using a Singly Linked List

- We can implement a stack with a singly linked list
- The top element is stored at the first node of the list
- The space used is $O(n)$ and each operation of the Stack ADT takes $O(1)$ time **How?**



Example: Parentheses Matching

- Each "(", "{", or "[" must be paired with a matching ")", "}", or "["
  - Correct: `( ) ( ( ) ) { ( [ ( ) ] ) }`
  - Incorrect: `) ( (  ) ) { ( [ (  ) ] ) }`
  - Incorrect: `( { [  ] ) }`
  - Incorrect: `(`

Parentheses Matching Algorithm

```
Algorithm ParenMatch(X,n):
Input: An array X of n tokens, each of which is either
       a grouping symbol.
Output: true if and only if X is well-balanced
S ← new Stack()
for i = 0 to n-1 do
...
```

Model

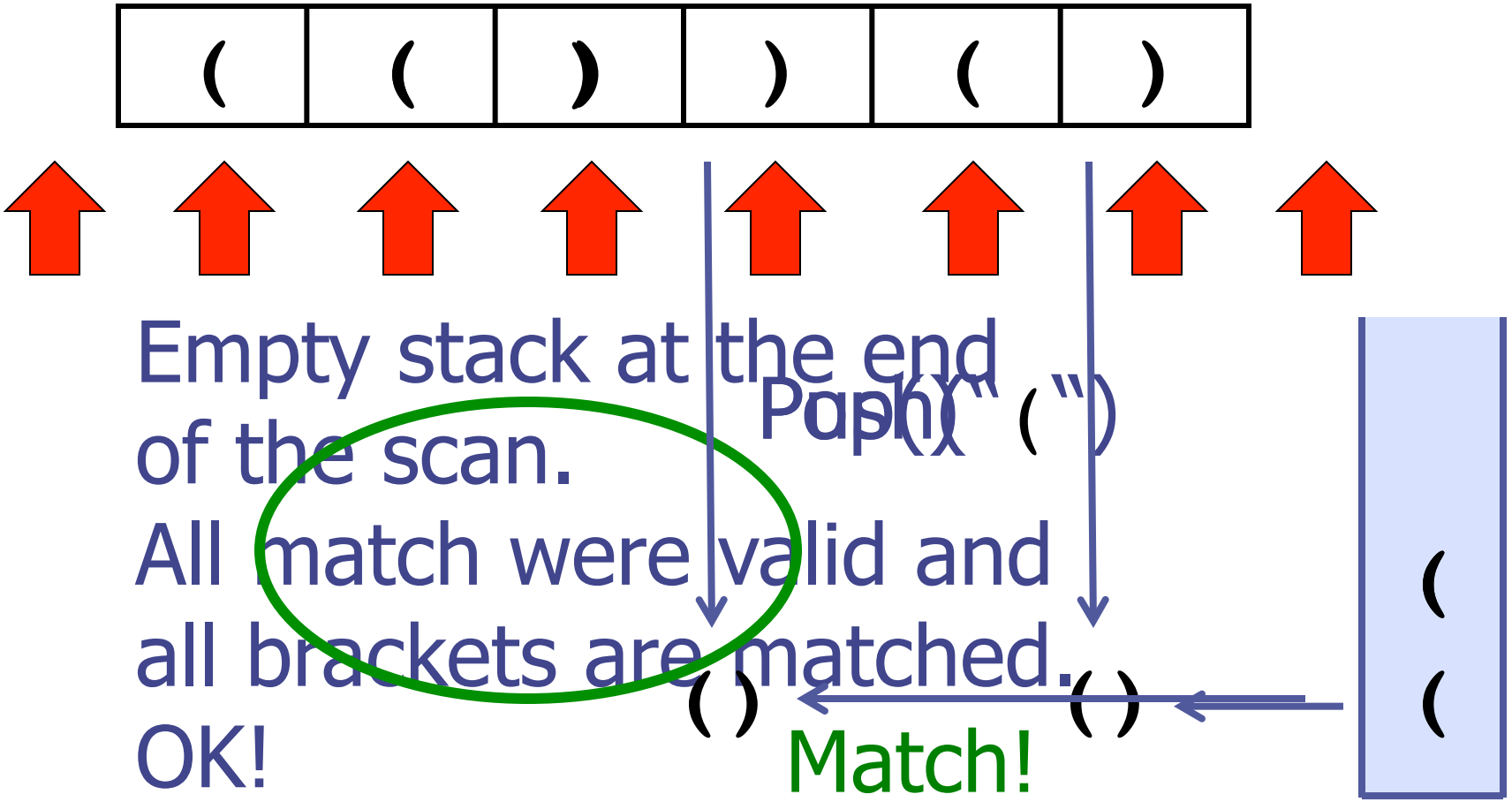
What are well-bracketed words?

- Each opening bracket (e.g. "[") is associated to one and only one closing bracket (e.g. "]") .
- Open a bracket before to close it.
- Reading from left to right, a closing bracket is associated to the last opening bracket screened.

Why do we need a stack?

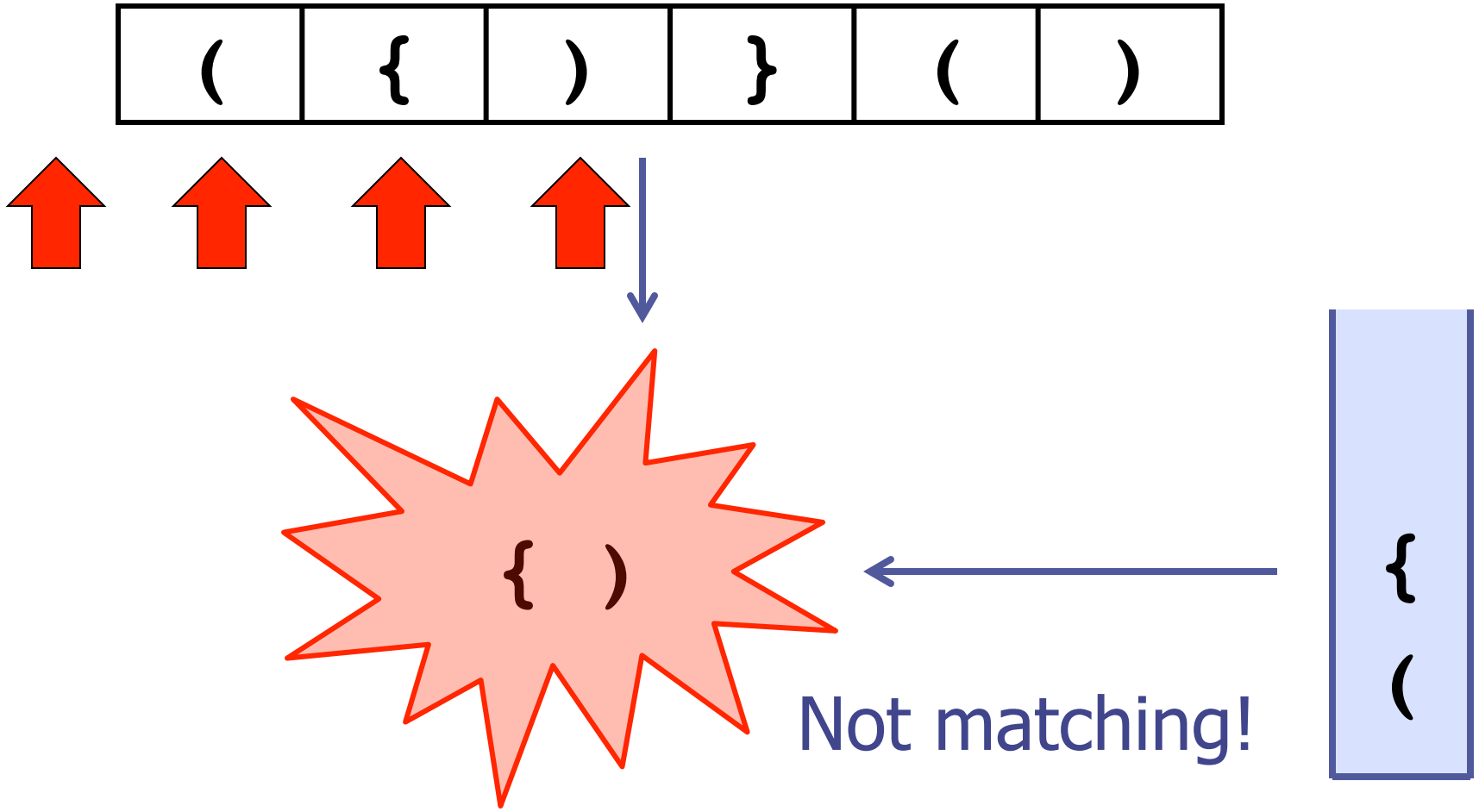
- Store the opening brackets not already matched.
- Last in - first out: The last opening bracket seen is the first one to be matched to a closing bracket.

Example



Empty stack at the end of the scan.
All match were valid and all brackets are matched.
OK!

Example



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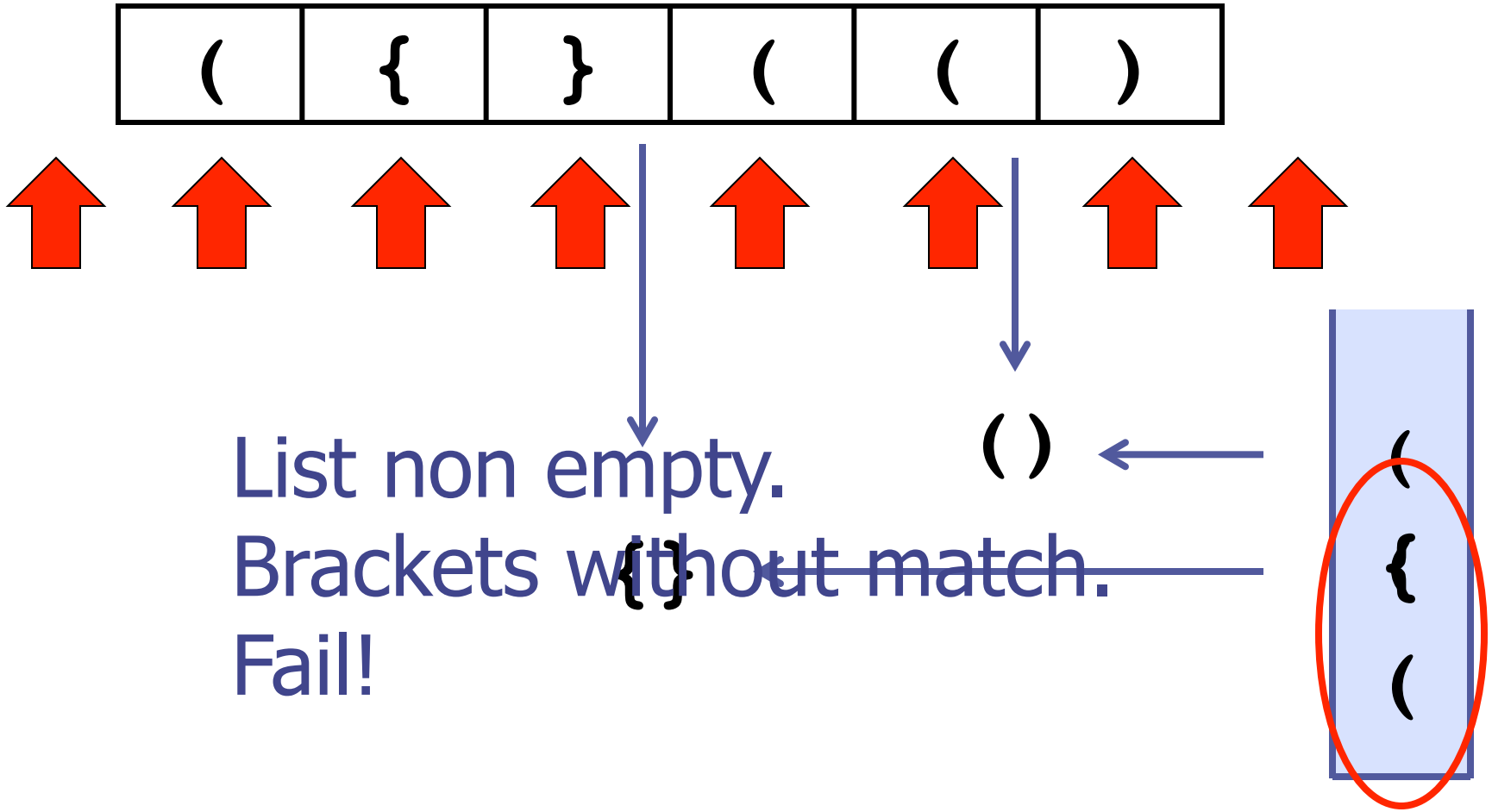
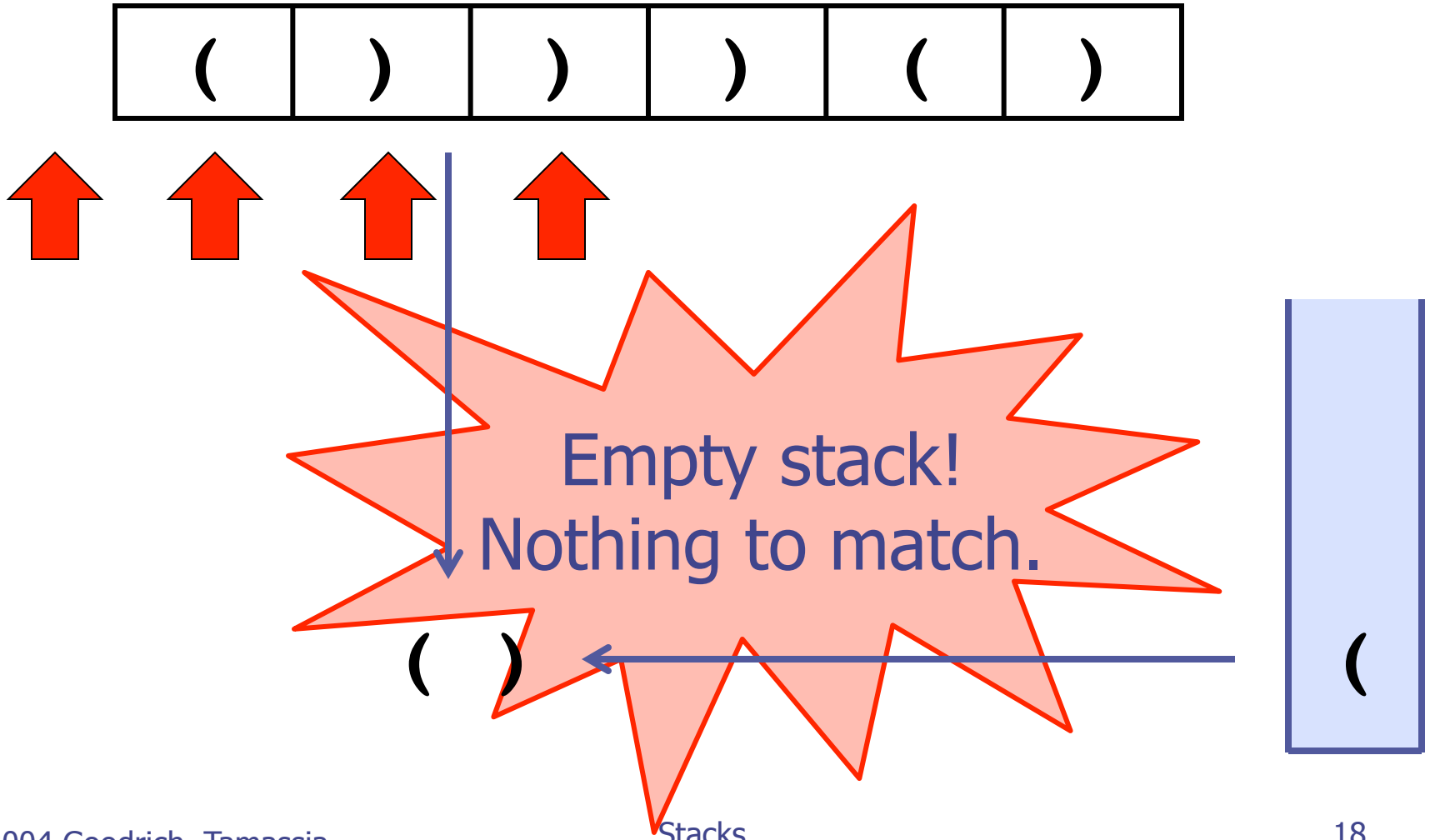


List non empty.
Brackets without match.
Fail!

Example



Parentheses Matching Algorithm

```
Algorithm ParenMatch(X,n):
Input: An array X of n tokens.
Output: true iff all the grouping symbols in X match
Let S be an empty stack
for i=0 to n-1 do
   if X[i] is an opening grouping symbol then
       S.push(X[i])
   else if X[i] is a closing grouping symbol then
       if S.isEmpty() then
             return false {nothing to match with}
       if S.pop() does not match the type of X[i] then
             return false {wrong type}
if S.isEmpty() then
   return true {every symbol matched}
else
   return false {some symbols were never matched}
```

Midterm

- Mid-term covers all topics (including queues)
- 2 rooms (partition based on your last name)
- Review on Wednesday Oct 21
- Vote for topics covered in review session