The CKY Parsing Algorithm and PCFGs

COMP-550

Oct 12, 2017

Announcements

I'm out of town next week:

- **Tuesday** lecture: Lexical semantics, by TA Jad Kabbara
- **Thursday** lecture: Guest lecture by Prof. Timothy O'Donnell (Linguistics)

Corollary: no office hours on Tuesday

- TA office hours about A2 will be announced

Outline

CYK parsing

PCFGs

Probabilistic CYK parsing

Markovization

Parsing

Input sentence, grammar → output parse tree

Parsing into a CFG: **constituent parsing**

Parsing into a dependency representation: **dependency parsing**

Difficulty: need an efficient way to search through plausible parse trees for the input sentence

Parsing into a CFG

Given:

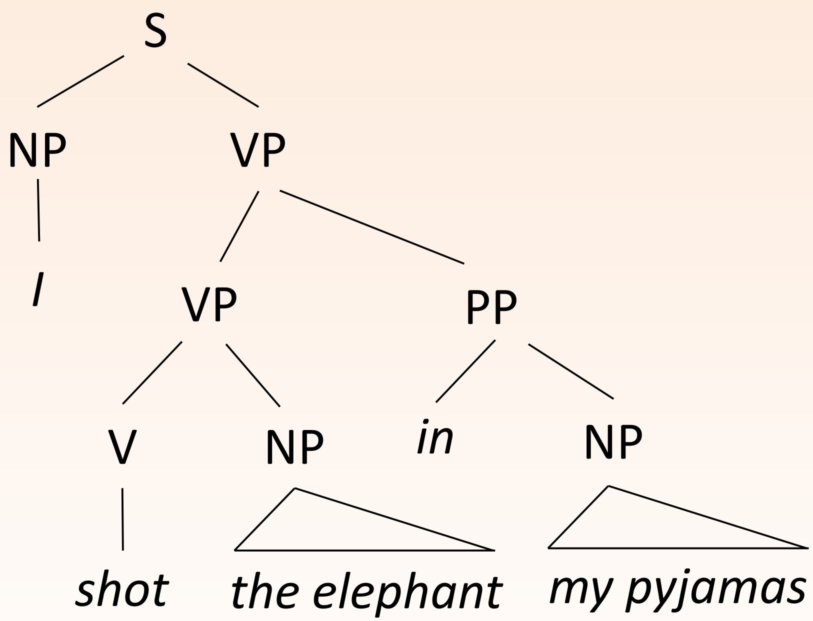
1. CFG
2. A sentence made up of words that are in the terminal vocabulary of the CFG

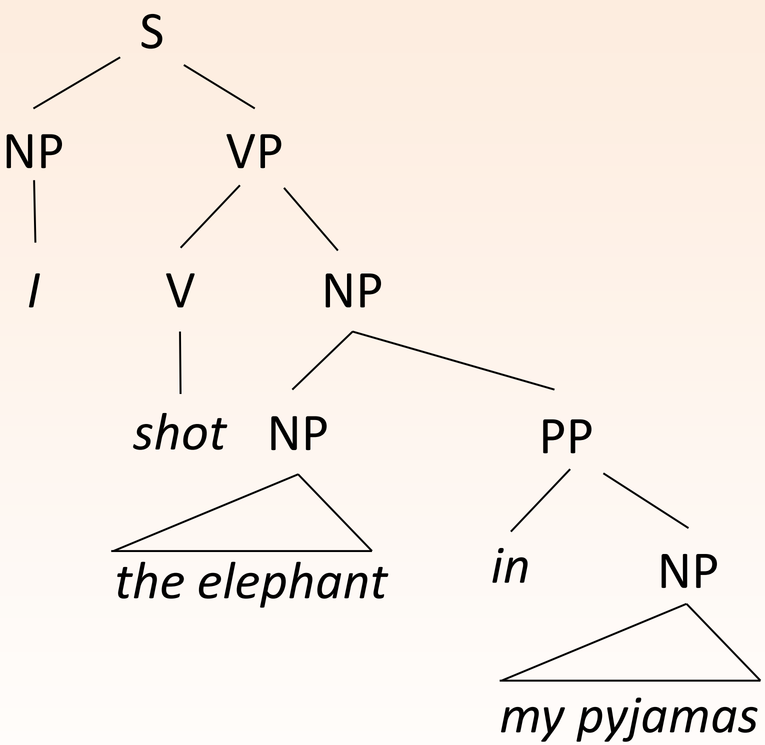
Task: Recover all possible parses of the sentence.

Why *all* possible parses?

Syntactic Ambiguity

I shot the elephant in my pyjamas.





Types of Parsing Algorithms

Top-down

Start at the top of the tree, and expand downwards by using rewrite rules of the CFG to match the tokens in the input string

e.g., Earley parser

Bottom-up

Start from the input words, and build ever-bigger subtrees, until a tree that spans the whole sentence is found

e.g., **CYK algorithm**, shift-reduce parser

Key to efficiency is to have an efficient search strategy that avoids redundant computation

CYK Algorithm

Cocke-Younger-Kasami algorithm

- A **dynamic programming** algorithm – partial solutions are stored and efficiently reused to find all possible parses for the entire sentence.
- Also known as the CKY algorithm

Steps:

1. Convert CFG to an appropriate form
2. Set up a table of possible constituents
3. Fill in table
4. Read table to recover all possible parses

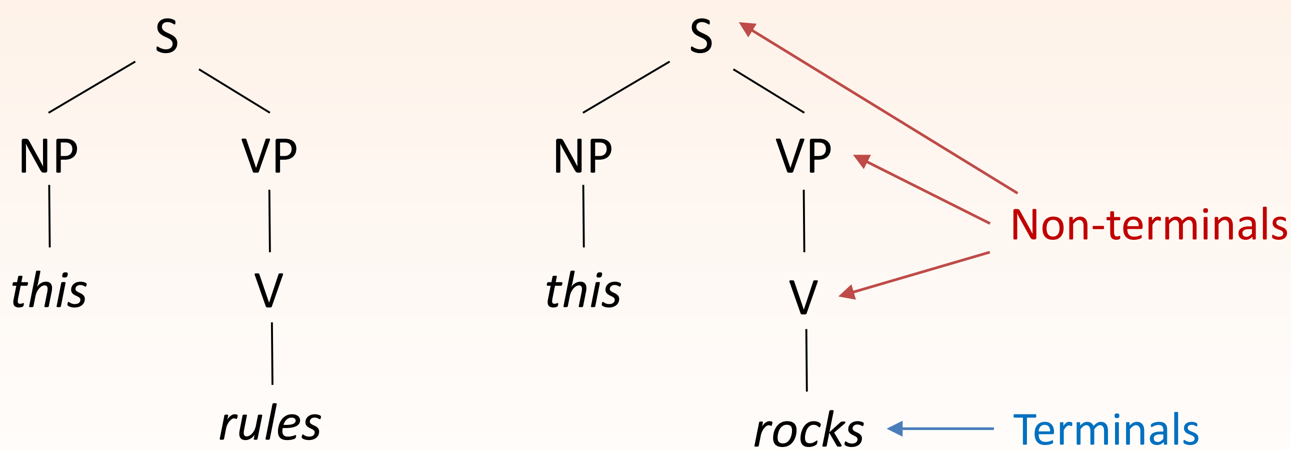
CFGs and Constituent Trees

Rules/productions:

S → NP VP　　　　NP → *this*

VP → V　　　　V → *is* | *rules*| *jumps* | *rocks*

Trees:



CYK Algorithm

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Chomsky Normal Form

To make things easier later, need all productions to be in one of these forms:

1. A → B C, where A, B, C are nonterminals
2. A → *s*, where A is a non-terminal *s* is a terminal

This is actually not a big problem.

Converting to CNF (1)

Rule of type A → B C D …

- Rewrite into: A → X1 D … and X1 → B C

Rule of type A → *s* B

- Rewrite into: A → X2 B and X2 → *s*

Rule of type A → B

- Everywhere in which we see B on the LHS, replace it with A

Examples of Conversion

Let's convert the following grammar fragment into CNF:

S → NP VP
VP → V NP PP
VP → V NP
NP → N
NP → Det N
NP → Det N PP
PP → *in* NP

N → *I* | *elephant* | *pyjamas*
V → *shot*
Det → *my* | *the*

Next: Set Up a Table

This table will store all of the constituents that can be built from contiguous spans within the sentence.

Let sentence have N words. w[0], w[1], … w[N-1]

Create table, such that a cell in row i column j corresponds to the span from w[i:j+1], zero-indexed.

- Since i < j, we really just need half the table.

The entry at each cell is a list of non-terminals that can span those words according to the grammar.

Parse Table

I$_0$	*shot*$_1$	*the*$_2$	*elephant*$_3$	*in*$_4$	*my*$_5$	*pyjamas*$_6$
[0:1]	[0:2]	[0:3]	[0:4]	[0:5]	[0:6]	[0:7]
	[1:2]	[1:3]	[1:4]	[1:5]	[1:6]	[1:7]
		[2:3]	[2:4]	[2:5]	[2:6]	[2:7]
			[3:4]	[3:5]	[3:6]	[3:7]
				[4:5]	[4:6]	[4:7]
					[5:6]	[5:7]
						[6:7]

S → NP VP
VP → X1 PP
X1 → V NP
VP → V NP
NP → Det N
NP → X2 PP
X2 → Det N
PP → P NP
P → *in*
NP → *I* | *elephant* | *pyjamas*
N → *I* | *elephant* | *pyjamas*
V → *shot*
Det → *my* | *the*

Filling in Table: Base Case

One word (e.g., cell [0:1])

- Easy – add all the lexical rules that can generate that word

Base Case Examples (First 3 Words)

I_0	$shot_1$	the_2	$elephant_3$	in_4	my_5	$pyjamas_6$
NP N [0:1]	[0:2]	[0:3]	[0:4]	[0:5]	[0:6]	[0:7]
	V [1:2]	[1:3]	[1:4]	[1:5]	[1:6]	[1:7]
		Det [2:3]	[2:4]	[2:5]	[2:6]	[2:7]
			[3:4]	[3:5]	[3:6]	[3:7]
				[4:5]	[4:6]	[4:7]
					[5:6]	[5:7]
						[6:7]

S → NP VP
VP → X1 PP   X1 → V NP
VP → V NP
NP → Det N
NP → X2 PP   X2 → Det N
PP → P NP
P → *in*
NP → *I* | *elephant* | *pyjamas*
N → *I* | *elephant* | *pyjamas*
V → *shot*
Det → *my* | *the*

Filling in Table: Recursive Step

Cell corresponding to multiple words

- eg., cell for span [0:3]   *I shot the*
- Key idea: all rules that produce phrases are of the form

  A → B C
- So, check all the possible break points *m* in between the start *i* and the end *j*, and see if we can build a constituent with a rule in the form, A [*i*:*j*] → B [*i*:*m*] C [*m*:*j*]

Recurrent Step Example 1

I_0	$shot_1$	the_2	$elephant_3$	in_4	my_5	$pyjamas_6$
NP N [0:1]	? [0:2]	[0:3]	[0:4]	[0:5]	[0:6]	[0:7]
	V [1:2]	[1:3]	[1:4]	[1:5]	[1:6]	[1:7]
		[2:3]	[2:4]	[2:5]	[2:6]	[2:7]
			[3:4]	[3:5]	[3:6]	[3:7]
				[4:5]	[4:6]	[4:7]
					[5:6]	[5:7]
						[6:7]

S → NP VP
VP → X1 PP
VP → V NP
NP → Det N
NP → X2 PP
PP → P NP
P → *in*
NP → *I* | *elephant* | *pyjamas*
N → *I* | *elephant* | *pyjamas*
V → *shot*
Det → *my* | *the*

X1 → V NP
X2 → Det N

Recurrent Step Example 2

I$_0$	*shot*$_1$	*the*$_2$	*elephant*$_3$	*in*$_4$	*my*$_5$	*pyjamas*$_6$
[0:1] NP N	[0:2]	[0:3]	[0:4]	[0:5]	[0:6]	[0:7]
	[1:2] V	[1:3]	[1:4]	[1:5]	[1:6]	[1:7]
		[2:3] Det	[2:4] ?	[2:5]	[2:6]	[2:7]
			[3:4] NP N	[3:5]	[3:6]	[3:7]
				[4:5]	[4:6]	[4:7]
					[5:6]	[5:7]
						[6:7]

S $\rightarrow$ NP VP

VP $\rightarrow$ X1 PP

X1 $\rightarrow$ V NP

VP $\rightarrow$ V NP

NP $\rightarrow$ Det N

NP $\rightarrow$ X2 PP

X2 $\rightarrow$ Det N

PP $\rightarrow$ P NP

P $\rightarrow$ *in*

NP $\rightarrow$ *I* | *elephant* | *pyjamas*

N $\rightarrow$ *I* | *elephant* | *pyjamas*

V $\rightarrow$ *shot*

Det $\rightarrow$ *my* | *the*

Backpointers

I_0	$shot_1$	the_2	$elephant_3$	in_4	my_5	$pyjamas_6$
[0:1] NP N	[0:2]	[0:3]	[0:4]	[0:5]	[0:6]	[0:7]
	[1:2] V	[1:3]	[1:4]	[1:5]	[1:6]	[1:7]
		[2:3] Det	[2:4] NP	[2:5]	[2:6]	[2:7]
			[3:4] NP N	[3:5]	[3:6]	[3:7]
				[4:5]	[4:6]	[4:7]
					[5:6]	[5:7]
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S → NP VP
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N → *I* | *elephant* | *pyjamas*
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Store where you came from!

Putting It Together

I_0	$shot_1$	the_2	$elephant_3$	in_4	my_5	$pyjamas_6$
[0:1] NP N	[0:2]	[0:3]	[0:4]	[0:5]	[0:6]	[0:7]
	[1:2] V	[1:3]	[1:4]	[1:5]	[1:6]	[1:7]
		[2:3] Det	[2:4] NP (Det 2:3 ,N 3:4)	[2:5]	[2:6]	[2:7]
			[3:4] NP N	[3:5]	[3:6]	[3:7]
				[4:5]	[4:6]	[4:7]
					[5:6]	[5:7]
						[6:7]

S → NP VP
VP → X1 PP   X1 → V NP
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Fill the table in the correct order!

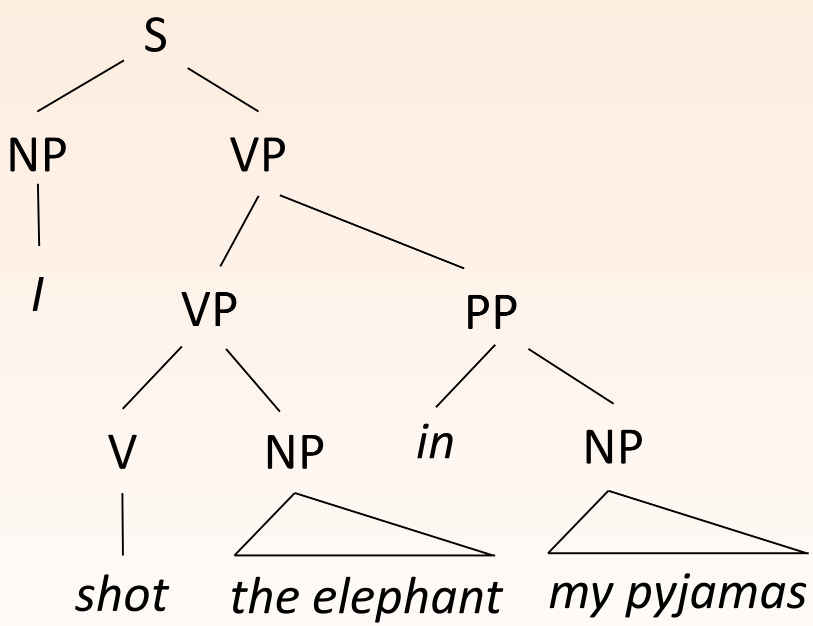
Finish the Example

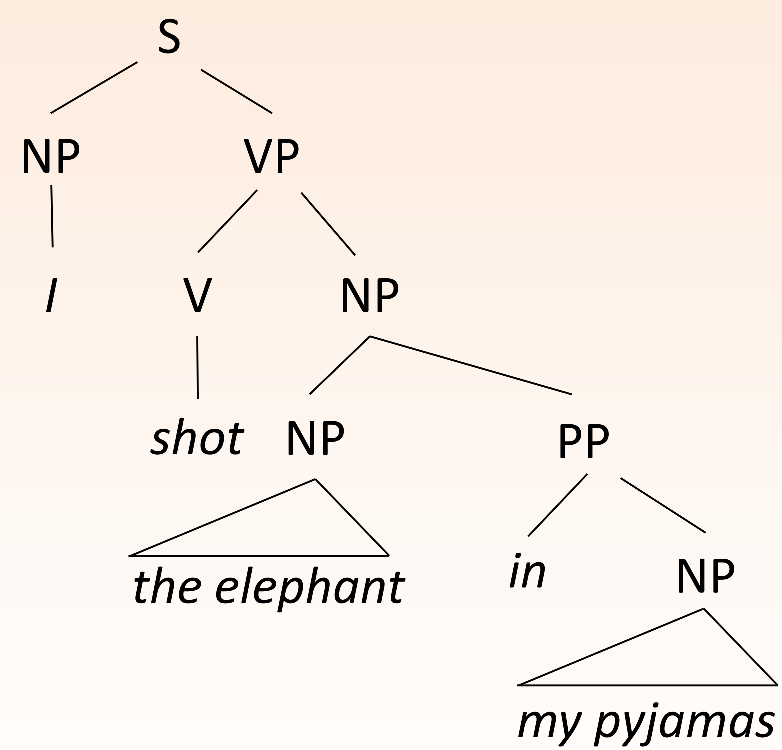
Let’s finish the example together for practice

How do we reconstruct the parse trees from the table?

Dealing with Syntactic Ambiguity

In practice, one of these is more likely than the other:





How to distinguish them?

Probabilistic CFGs

Associate each rule with a probability:

e.g.,

NP → NP PP	0.2
NP → Det N	0.4
NP → *I*	0.1
…	
V → *shot*	0.005

Probability of a parse tree for a sentence is the product of the probabilities of the rules in the tree.

Formally Speaking

For each nonterminal $A \in N$,

$$\sum_{\alpha \to \beta \in R \ s.t. \alpha = A} \Pr(\alpha \to \beta) = 1$$

- i.e., rules for each LHS form a probability distribution

If a tree t contains rules $\alpha_1 \to \beta_1, \alpha_2 \to \beta_2, \ldots,$

$$\Pr(t) = \prod_i \Pr(\alpha_i \to \beta_i)$$

- Tree probability is product of rule probabilities

Probabilistic Parsing

Goal: recover the best parse for a sentence, along with its probability

For a sentence, sent,

let $\tau(\text{sent})$ be the set of possible parses for it,

we want to find

$$\underset{t \in \tau(\text{sent})}{\operatorname{argmax}} \Pr(t)$$

Idea: extend CYK to keep track of probabilities in table

Extending CYK to PCFGs

Previously, cell entries are nonterminals (+ backpointer)

e.g., table[2:4] = {{NP, Det[2:3] N[3:4] }}

table[3:4] = {{NP, } {N, }}

Now, cell entries include the (best) probability of generating the constituent with that non-terminal

e.g., table[2:4] = {{NP, Det[2:3] N[3:4], 0.215}}

table[3:4] = {{NP, , 0.022} {N, , 0.04}}

Equivalently, write as 3-dimensional array

table[2, 4, NP] = 0.215 (Det[2:3], N[3:4])

table[3, 4, NP] = 0.022

table[3, 4, N] = 0.04

New Recursive Step

Filling in dynamic programming table proceeds almost as before.

During recursive step, compute probability of new constituents to be constructed:

Pr(A[i:j]→B[i:m] C[m:j]) = Pr(A→BC) × table[i,m,B] × table[m,j,C]

From PCFG

From previously filled cells

There could be multiple rules that form constituent A for span [i:j]. Take max:

table[i,j,A] =

$$\max_{A \to BC,\text{ break at } m} \Pr(A[i:j] \to B[i:m]C[m:j])$$

Example

I_0	$shot_1$	the_2	$elephant_3$	in_4	my_5	$pyjamas_6$
[0:1] NP, 0.25 N, 0.625	[0:2]	[0:3]	[0:4]	[0:5]	[0:6]	[0:7]
	[1:2] V, 1.0	[1:3]	[1:4]	[1:5]	[1:6]	[1:7]
		[2:3] Det, 0.6	[2:4] NP, ?	[2:5]	[2:6]	[2:7]
			[3:4] NP, 0.1 N, 0.25	[3:5]	[3:6]	[3:7]
				[4:5]	[4:6]	[4:7]
					[5:6]	[5:7]
						[6:7]

New value:

0.6 * 0.25 * Pr(NP → Det N)

Bottom-Up vs. Top-Down

CYK algorithm is **bottom-up**

- Starting from words, build little pieces, then big pieces

Alternative: **top-down** parsing

- Starting from the start symbol, expand non-terminal symbols according to rules in the grammar.
- Doing this efficiently can also get us all the parses of a sentence (**Earley algorithm**)

How to Train a PCFG?

Derive from a treebank, such as WSJ.

Simplest version:

- each LHS corresponds to a categorical distribution
- outcomes of the distributions are the RHS
- MLE estimates:

$$\Pr(\alpha \to \beta) = \frac{\#(\alpha \to \beta)}{\#\alpha}$$

- Can smooth these estimates in various ways, some of which we've discussed

Vanilla PCFGs

Estimate of rule probabilities:

- MLE estimates:

$$\Pr(\alpha \to \beta) = \frac{\#(\alpha \to \beta)}{\#\alpha}$$

- e.g., Pr(S -> NP VP) = #(S -> NP VP) / #(S)
    - Recall: these distributions are normalized by LHS symbol

Even with smoothing, doesn't work very well:

- Not enough context
- Rules are too sparse

Subject vs Object NPs

NPs in subject and object positions are not identically distributed:

- Obvious cases – pronouns (*I* vs *me*)
  - But both appear as NP -> PRP -> *I/me*
- Less obvious: certain classes of nouns are more likely to appear in subject than object position, and vice versa.
  - For example, subjects tend to be **animate** (usually, humans, animals, other moving objects)

Many other cases of obvious dependencies between distant parts of the syntactic tree.

Sparsity

Consider subcategorization of verbs, with modifiers

• *ate*	VP -> VBD
• *ate quickly*	VP -> VBD AdvP
• *ate with a fork*	VP -> VBD PP
• *ate a sandwich*	VP -> VBD NP
• *ate a sandwich quickly*	VP -> VBD NP AdvP
• *ate a sandwich with a fork*	VP -> VBD NP PP
• *quickly ate a sandwich with a fork*	VP -> AdvP VBD NP PP

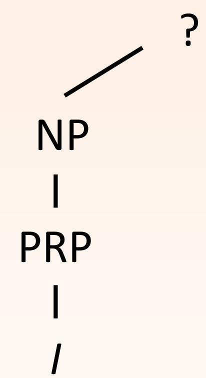
We should be able to factorize the probabilities:

- of having an adverbial modifier, of having a PP modifier, etc.

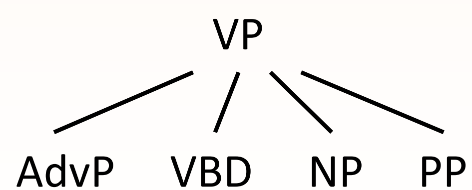
Wrong Independence Assumptions

Vanilla PCFGs make independence assumptions that are too strong AND too weak.

Too strong: *vertically*, up and down the syntax tree



Too weak: *horizontally*, across the RHS of a production



Adding Context

Add more context vertically to the PCFG

- Annotate with the parent category

Before: NP -> PRP, NP -> Det NN, etc.

Now:

Subjects:

NP^S -> PRP, NP^S -> Det NN, etc.

Objects:

NP^VP -> PRP, NP^VP -> Det NN, etc.

Learn the probabilities of the rules separately (though they may influence each other through interpolation/smoothing)

Example

Let’s help Pierre Vinken find his ancestors.

```
( (S
    (NP
      (NP (NNP Pierre) (NNP Vinken) )
      (, ,)
      (ADJP
        (NP (CD 61) (NNS years) )
        (JJ old) )
      (, ,) )
    (VP (MD will)
      (VP (VB join)
        (NP (DT the) (NN board) )
        (PP (IN as)
          (NP (DT a) (JJ nonexecutive) (NN director) ))
        (NP (NNP Nov.) (CD 29) )))
    (. .) ))
```

Note that the tree here is given in bracket parse format, rather than drawn out as a graph.

Removing Context

Conversely, we break down the RHS of the rule when estimating its probability.

Before: Pr(VP -> START AdvP VBD NP PP END) as a unit

Now: Pr(VP -> START AdvP) *

Pr(VP -> AdvP VBD) *

Pr(VP -> VBD NP) *

Pr(VP -> NP PP) *

Pr(VP -> PP END)

- In other words, we're making the same N-gram assumption as in language modelling, only over non-terminal categories rather than words.
- Learn probability of factors separately

Example

Let's help Pierre Vinken find his children.

```
( (S
    (NP
      (NP (NNP Pierre) (NNP Vinken) )
      (, ,)
      (ADJP
        (NP (CD 61) (NNS years) )
        (JJ old) )
      (, ,) )
    (VP (MD will)
      (VP (VB join)
        (NP (DT the) (NN board) )
        (PP (IN as)
          (NP (DT a) (JJ nonexecutive) (NN director) ))
        (NP (NNP Nov.) (CD 29) )))
    (. .) ))
```

Markovization

Vertical markovization: adding ancestors as context

- Zeroth order – vanilla PCFGs
- First order – the scheme we just described
- Can go further:
  - e.g., Second order: NP^VP^S -> …

Horizontal markovization: breaking RHS into parts

- Infinite order – vanilla PCFGs
- First order – the scheme we just described
- Can choose any other order, do interpolation, etc.

Effect of Category Splitting

Vertical Order		Horizontal Markov Order $h = 0$	$h = 1$	$h \leq 2$	$h = 2$	$h = \infty$
$v = 1$	No annotation	71.27 (854)	72.5 (3119)	73.46 (3863)	72.96 (6207)	72.62 (9657)
$v \leq 2$	Sel. Parents	74.75 (2285)	77.42 (6564)	77.77 (7619)	77.50 (11398)	76.91 (14247)
$v = 2$	All Parents	74.68 (2984)	77.42 (7312)	77.81 (8367)	77.50 (12132)	76.81 (14666)
$v \leq 3$	Sel. GParents	76.50 (4943)	78.59 (12374)	79.07 (13627)	78.97 (19545)	78.54 (20123)
$v = 3$	All GParents	76.74 (7797)	79.18 (15740)	79.74 (16994)	79.07 (22886)	78.72 (22002)

Figure 2: Markovizations: F_1 and grammar size.

WSJ results by Klein and Manning (2003)

- With additional linguistic insights, they got up to 87.04 F1
- Current best is around 94-95 F1