1. For each one of the following problems either prove that they are NP-complete or prove that they belong to \( P \).

(a) (15 Points)
- Input: A CNF \( \phi \) with 10 clauses (and \( n \) variables).
- Question: Is there a truth assignment that satisfies \( \phi \)?

(b) (15 Points)
- Input: A CNF \( \phi \) on \( 2^n \) variables.
- Question: Is there a truth assignment that satisfies \( \phi \) and assigns True to exactly \( n \) variables?

(c) (15 Points)
- Input: A 3CNF \( \phi \).
- Question: Is there a truth assignment that satisfies exactly 10 clauses in \( \phi \)?

(d) (15 Points)
- Input: Positive integers \( a_1, \ldots, a_n \) and a positive integer \( M \).
- Question: Is there a subset \( S \subseteq \{1, \ldots, n\} \) such that \( \prod_{i \in S} a_i = M \)?

(e) (15 Points)
- Input: A graph \( G \) and a positive integer \( k \).
- Question: Is there a set \( S \subseteq V(G) \) of size \( k \) such that every vertex of \( G \) either belongs to \( S \) or has at least one neighbour in \( S \)?

2. (25 Points) Consider the following variation of the load balancing problem. Suppose you have a system that consists of \( m \) slow machines and \( k \) fast machines. The fast machine can perform twice as much work per unit of time as the slow machines. Now you are given a set of \( n \) jobs. Job \( i \) takes time \( t_i \) to process on a slow machine and time \( \frac{1}{2} t_i \) on a fast machine. You want to assign each job to a machine, and as before, the goal is to minimize the makespan - that is the maximum, over all machines, of the total processing time of jobs assigned to that machine.

Give a polynomial time 3-factor approximation algorithm for this problem.