COMP 360 - Winter 2016 - Assignment 4

Due: 6pm Mar 16th.

General rules: In solving these questions you may consult books but you may not consult with each other. You should drop your solutions in the assignment drop-off box located in the Trottier Building.

1. (20 points) Write the duals of the following Linear Program: An independent set in $G$ is a set of vertices, no two of which are adjacent. Let $\mathcal{I}$ denote the set of all independent sets in $G$.

$$\begin{align*}
\min & \quad \sum_{S \in \mathcal{I}} x_S \\
\text{s.t.} & \quad \sum_{S : u \in S} x_S \geq 1 \quad \forall u \in V \\
& \quad x_S \geq 0 \quad S \in \mathcal{I}
\end{align*}$$

2. (5 points) Show that the solution to the linear program in Question 1 is a lower-bound for the minimum number of colours required to colour the vertices of $G$ so that no two adjacent vertices receive the same colour.

3. (25 points) Prove that if $P = NP$, then $P = NP = CoNP$.

4. Recall that in 3COL, given an undirected graph $G$, we want to know whether it is possible to colour its vertices with 3 colors so that adjacent vertices receive different colours. Let $X$ be the following problem: Given a graph $G$, we want to know whether there is an edge $e$ in $G$ such that $G - e$ is 3-colourable.

(a) (15 points) Show that “$X \leq_p 3COL$”.
(b) (15 points) Show that “$3COL \leq_p X$”.

5. (20 points) Let 3SAT denote the non-satisfiability problem for 3CNF’s. Show that 3SAT $\leq_p$ UNQ where in UNQ, given a CNF $\phi$ we want to know whether there is a unique satisfying assignment for $\phi$. (Hint: for every 3CNF $\psi$ construct a CNF $\phi$ (in polytime) such that $\psi$ is a NO input for 3SAT if and only if $\phi$ is a YES input for UNQ.)