1. (c)
2. 

\[ Q = \{1, 2, 3\} \]

\[ \Sigma = \{a, b, c, d\} \]

\[ q_0 = 1 \]

\[ F = \{3\} \]

\[
\begin{array}{cccccc}
\text{ } & a & b & c & d \\
1 & \{2\} & \{1\} & \{1\} & \{1\} \\
2 & \{3\} & \{2\} & \{1\} & \{2\} \\
3 & \{3\} & \{3\} & \{3\} & \{3\} \\
\end{array}
\]
3. Let $m$ be the length of the longest string in $L$.

Consider the following DFA which is similar to a binary tree of depth $m+1$.

- The leaves have self-loops with labels 0 and 1.
- We turn a state into an accept state iff the path from the root to that state corresponds to a word in $L$. 
Question 4 Assignment 1

4(a) Add a new start state and connect to it as in the picture:

4(b) Convert every accept state to non-accept.
- Make the old start state an accept state.
- Reverse the direction of every arrow.
- Add a new start stat and put $\varepsilon$-arrows from it to the old accept state.

4(c) Put three copies of $M$ and a new start state and modify them as in the picture.
Q5. (a) False. Take $r = 1$ and $s = \varepsilon$. Then $\varepsilon$ belongs to the language of the left-hand side, but it does not belong to the language of the right-hand side.

(b) False. Take $s = 0$ and $r = 1$. Then every word in the language of the left-hand side starts with 0 while every word in the language of the right-hand side starts with 1.

(c) False. Take $s = 0$ and $r = 1$. Then 01 belongs to the language of the left-hand side, but not to the language of the right-hand side.

Q6. $X = r^*s$. 
Z. We start from the following GNFA:
Question 8 Assignment 1

Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA accepting $A$, where $Q = \{q_0, \ldots, q_m\}$. For $i = 0, \ldots, m$ define

$$A_i = \{w| M \text{ stops at } q_i \text{ with input } w\},$$

$$B_i = \{w| \text{With input } w, \text{ if we start at } q_i, \text{ then } M \text{ stops at an accept state}\},$$

and

$$C_i = \{w| \text{With input } w^R, \text{ if we start at } q_i, \text{ then } M \text{ stops at an accept state}\}.$$

Note that $A_i$ and $B_i$ are both regular. Indeed $A_i$ is accepted by $(Q, \Sigma, \delta, q_0, \{q_i\})$ and $B_i$ is accepted by $(Q, \Sigma, \delta, q_i, F)$. Since $C_i = \{w^R|w \in B_i\}$ and $B_i$ is regular, Question 4(b) shows that $C_i$ is also regular. Now note that

$$\{w|ww^R \in A\} = (A_0 \cap C_0) \cup (A_1 \cap C_1) \cup (A_2 \cap C_2) \cup \ldots \cup (A_m \cap C_m).$$

Since the set of regular languages is closed under unions and intersections, and $A_i$ and $C_i$ are regular, we conclude that $\{w|ww^R \in A\}$ is also regular.