COMP 330 - Fall 2016 - Assignment 1

Due 7:00 pm Sept 28, 2016

General rules: In solving these questions you may consult books but you may not consult with each other. There are in total 110 points, but your grade will be considered out of 100. You should either drop your solutions in the assignment drop-off box located in the Trottier Building on the 2nd floor, or submit it through mycourses. Late homeworks can be submitted until 48 hours after the deadline. Late submissions must be submitted through mycourses as the drop-off box will not be checked after the deadline. There will be a penalty of -25% for one-day delays, and -40% for two-day delays on late homeworks.

1. (20 points) Give DFA’s accepting the following languages over the alphabet \{0, 1\}: (For this question drawing the state diagram is sufficient, and you are not required to describe the corresponding 5-tuples \((Q, \Sigma, \delta, q_0, F)\)).

(a) The set of all strings \(w\) such that the number of ones in \(w\) is divisible by 3 and not divisible by 2 (for example 1011 is accepted but 10 and 111111 are not).

(b) The set of all strings of length at least two that start and end with the same symbol (for example 00, 1011 are accepted but 0, 01101 and 10 are not accepted).

(c) The set of all strings that contain 0110 as a substring (for example 10110 is accepted but 01010001 is not).

(d) The set of all strings of length at least three such that every three consecutive symbols contain at least two 1’s (for example 1101 is accepted but 1010111 is not).

2. (10 points) Give an NFA \(M = (Q, \Sigma, \delta, q_0, F)\) accepting the following language over the alphabet \{a, b, c, d\}: The set of all strings that contain at least two \(a\)’s with no \(c\) appearing between them (For example bcabbdaca and aa are accepted but abcedabcb and bd are not accepted).
In this question you are required to describe each one of \( Q \), \( \Sigma \), \( \delta \), \( q_0 \) and \( F \) in your construction. In particular give the table of \( \delta \).

3. (15 points) Let \( L \) be a finite language over \{0, 1\}. Describe a DFA that accepts \( L \). For this question you are not allowed to use any theorems or lemmas from the lecture or the book.

4. (20 points) Let \( M \) be a DFA accepting a language \( A \) over the alphabet \{0, 1\}. For each one of the following languages design an NFA (using \( M \)) that accepts the language.

   (a) \( A \setminus \{\varepsilon\} \).

   (b) \( \{w^R|w \in A\} \), where \( w^R \) is the reverse of the string \( w \). (For example the reverse of 01101 is 10110).

   (c) \( \{x_1x_2\ldots x_n|x_1, x_2, \ldots, x_n \in A \text{ and } n \text{ is divisible by } 3\} \).

5. (15 points) Let \( r, s \) and \( t \) be regular expressions. Prove or disprove each one of the following equalities.

   (a) \( (rs \cup r)^* = r(sr \cup r)^* \)

   (b) \( s(rs \cup s)^* = rr^*s(rr^*s)^* \)

   (c) \( (r \cup s)^* = r^* \cup s^* \)

6. (5 points) Let \( r \) and \( s \) be regular expressions. Give a regular expression \( X \) such that always \( L(X) = L(rX \cup s) \).

7. (10 points) Express the following set as a regular expression: The set of all strings of length at least three over \{0, 1\} such that every three consecutive symbols contain at least two 1’s.

8. (15 points) Show that if \( A \) is a regular language, then so is \( \{w|ww^R \in A\} \), where \( w^R \) is the reverse of the string \( w \).