Physics Description

May 2, 2006

Problem Formulation

An agent's main task is to control a single octopus arm to accomplish a given objective, such as bringing a piece of food into a mouth, or touching a series of point targets. The arm is immersed in a water-like fluid, with environmental forces such as gravity and buoyancy present. Pieces of food are treated as point masses, while the mouth is treated as an immaterial region (not subject to physics). Targets are similarly considered immaterial.

Physical model

The physical model we use is that of two-dimensional vector particle kinetics: all the entities subject to physics are represented using zero-size particles (point masses), and the motion of each particle is governed by the net force acting on it. (Some special particles are also subject to kinematic constraints.) Even a "solid body" such as the arm is modeled as a collection of particles, with its internal dynamics modeled using appropriate forces, as described in the section "Octopus arm model".

Under this model, the motion $\mathbf{r}(t)$ of each (unconstrained) particle is given by the second-order ordinary differential equation (ODE)

$$\ddot{\mathbf{r}}(t) = \mathbf{F}(t)/m$$

where m is the particle's mass, and $\mathbf{F}(t)$ represents the net force acting on the particle at time t. This force can depend on the particle's own position, as well as the positions of other particles. This ODE can be decomposed into two first-order ODE's as follows:

$$\dot{\mathbf{r}}(t) = \mathbf{v}(t) \quad (1)$$

$$\dot{\mathbf{v}}(t) = \mathbf{F}(t)/m \quad (2)$$

Now, by collecting together the ODE's for all the particles in the environment, the dynamics of the whole environment can be expressed in one (large) ODE system

$$\dot{\mathbf{y}}(t) = f(t, \mathbf{y}(t))$$

where $\mathbf{y}(t)$ is a vector of state variables that fully specifies the system state, and f is a function expressing the system dynamics. The solution of this ODE can be approximated using any technique. We used the fourth-order Runge-Kutta method.

The following subsections describe the forces that every particle is subject to. These forces represent various environmental phenomena, each of which is parameterized by one or more constants. These constants can be tweaked to produce good simulation behaviour, and need not be assigned their real-world values.

Gravitational force

For a particle of mass m, the gravitational force is given by

$$\mathbf{f}_g = -m a_g \hat{\mathbf{j}}$$

where $\hat{\mathbf{j}}$ is a unit vector along the y-axis, and a_g is a gravitational constant.

Buoyancy force

The buoyancy force represents the tendency of objects immersed in a fluid to float or move towards the surface. In our environment, we assume that all particles have about the same density as water, so the buoyancy force is defined to exactly cancel the gravitational force whenever a particle is in water. Specifically, for a particle with position $\mathbf{r} = [r_x, r_y]$, the buoyancy force is given by

$$\mathbf{f}_b = \begin{cases} m a_g \hat{\mathbf{j}} & y_{\text{surf}} > r_y \\ \mathbf{0} & \text{otherwise} \end{cases}$$

where y_{surf} is the water's surface level, and a_g is the gravitational constant.

Fluid friction force

The fluid friction force represents the "resistance" offered by a fluid to objects moving through it. For a particle with velocity $\mathbf{v}$, it is given by

$$\mathbf{f}_f = -k_f \left\|\mathbf{v}\right\|^2 \cdot \frac{\mathbf{v}}{\|\mathbf{v}\|}$$

where k_f is a positive fluid friction constant. (In our model, this force acts even on particles above the water's surface.)

Inter-particle repulsion force

The inter-particle force represents the tendency of particles to repel each other, and is similar to the electrostatic forces that exist between charges of the same polarity. It provides a simple but effective substitute for a full solid-body collision model. For a particle at position $\mathbf{r}$, the repulsion force due to another particle at position $\mathbf{r}'$ is given by

$$\mathbf{f}_r = \begin{cases} \frac{k_r}{\|\mathbf{r}-\mathbf{r}'\|^{p_r}} \cdot \frac{\mathbf{r}-\mathbf{r}'}{\|\mathbf{r}-\mathbf{r}'\|} & \|\mathbf{r}-\mathbf{r}'\| < T_r \\ \mathbf{0} & \text{otherwise} \end{cases}$$

where k_r, p_r, and T_r are constants.

Octopus arm model

The arm model used is based on a similar octopus arm of [1]. The arm is two-dimensional and is only allowed to move in the plane. It is divided into several quadrilateral compartments; the vertices of each compartment, called nodes, are particles, while the sides are massless springs, representing muscles.

The dynamics of the arm are modeled entirely by forces upon the nodes, described in the following sections. These forces act in addition to the ones described in the previous section.

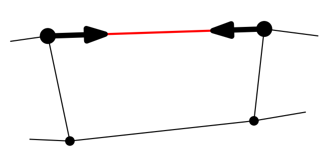
Muscle Force

To move, the octopus applies an action $a \in [0, 1]$ to each of its muscles, which represents the amount of contraction. We used the linear muscle model described in [1], where the total force of the muscle is composed of a passive force due to the properties of the muscle fibers, and an active force due to the contraction of muscle fibers.

The effect of an action a on a muscle is modeled as a force applied to each of the two nodes at the ends of the muscle. At each node, this force is given by

$$\mathbf{f}_m = \begin{cases} (a \times k_{\text{active}} + k_{\text{passive}}) \left(\frac{L}{L_{\text{rest}}} - k_{\text{length}} \right) \hat{\mathbf{m}} & \frac{L}{L_{\text{rest}}} > k_{\text{length}} \\ 0 & \text{otherwise} \end{cases}$$

where k_{active} is the muscle's contraction constant, k_{passive} is the reaction force of the muscle fibers, L is the muscle's current length, and L_{rest} is the muscle's length at rest. k_{length} is a constant defining the minimum *normalized length* (ratio of current to rest length) at which the muscle starts forcing. $\hat{\mathbf{m}}$ is an inward-pointing unit vector along the muscle, as shown below:



Note that each node can be part of up to three muscles and hence be subject to up to three different muscle forces.

Pressure Force

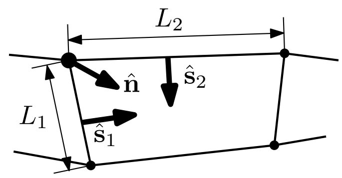
The area of each compartment is kept approximately constant by introducing a pressure force, which works to counteract deviations from the compartment's "desired area", on each of its four nodes. At each node, the force is given by

$$\mathbf{f}_p = k_p \left(A - A_{\text{des}}\right) \hat{\mathbf{n}}$$

where A is the compartment's area, A_{des} is its desired area, and k_p is a constant. $\hat{\mathbf{n}}$ is an inward "weighted" vertex normal given by

$$\hat{\mathbf{n}} = \frac{L_1\hat{\mathbf{s}}_1 + L_2\hat{\mathbf{s}}_2}{\|L_1\hat{\mathbf{s}}_1 + L_2\hat{\mathbf{s}}_2\|}$$

where L_1 and L_2 are the lengths of the edges connecting at the node, and , $\hat{\mathbf{s}}_1$ and $\hat{\mathbf{s}}_2$ are the surface normals of these edges, as illustrated below:



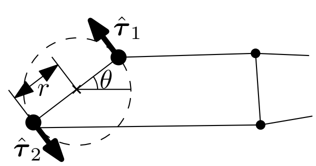
The presence of the scale factors L_1 and L_2 causes longer edges to receive more pressure than shorter ones.

Base of Arm

To keep the arm anchored at a fixed location, the two nodes on the base of the arm are kinematically constrained to move at diametrically opposite positions on a circle. We implement this constraint by considering only the angular motion of the two-node system around its center. This motion is driven by the tangential components of the net forces on the two nodes. Specifically, we compute the "net tangential force" on the system as

$$F_{\text{tan}} = \mathbf{F}_1 \cdot \hat{\tau}_1 - \mathbf{F}_1 \cdot \hat{\tau}_2$$

where $\mathbf{F}_1$ and $\mathbf{F}_2$ are the net forces on the two nodes, and $\hat{\tau}_1$ and $\hat{\tau}_2$ are unit tangent vectors along the circle, as shown below:



The angular acceleration α is then computed as

$$\alpha = \text{arctan2}(F_{\text{tan}}, r)$$

where arctan2 is the four-quadrant inverse-tangent function, and the angular position θ is updated according to the ODE's

$$\dot{\omega} = \alpha \tag{3}$$

$$\dot{\theta} = \omega \tag{4}$$

References

[1] A. E. H. F. Yekutieli, Sagiv-Zohar, "A dynamic model of the octopus arm. i. biomechanics of the octopus reaching movement." *Journal of Neurophysiology*, pp. 1459 – 1468, 2005.