

80-110 The Nature of Mathematical Reasoning

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1. TERMINOLOGY FOR PROOFS/ARGUMENTS

- **Statement/proposition:** A sentence that can be *true* or *false*. E.g., “Dogs are more intelligent than cats”, “ $3+4=7$ ”.
- **Argument/proof:** List of statements that are connected by inferences.
- **Premise:** Statement that is assumed to be true at the beginning of a proof.
- **Conclusion:** Statement at the end of a proof.
- **Inference rule/justification:** Truth preserving rule that connects two or more statements within a proof. ‘Truth preserving’ means that whenever the antecedents are true, then the conclusion is also true. (Thus, an inference rule might be understood as being a ‘little proof’ by itself—with premises and conclusion, but no intermediate steps.)

2. KINDS OF ARGUMENTS

Definition 1 (Deductively valid argument). *An argument is deductively valid if it is impossible for the premises to be true and the conclusion false at the same time.*

Note: *Validity* is a relation between the premises and the conclusion of an argument—it has nothing to do with how the world actually is!

Definition 2 (Sound argument). *An argument is sound if it is deductively valid, and it has true premises.*

Definition 3 (Inductive argument). *An argument is inductive if the truth of the premises and the intermediate statements make it more likely that the conclusion is also true, but they do not guarantee its truth.*

Definition 4 (Formal argument). *An argument is formal if the validity of the argument does not depend on the meaning of the symbols employed.*

Definition 5 (Irrelevant argument). *An argument is irrelevant if the validity of the argument does not depend on the statements used in the argument.*

Definition 6 (Fallacious argument). *An argument is fallacious if one of the inference rules used is not admissible.*